Testing 3D displacement vectors by confidence ellipsoids

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Testovanie deformačných vektorov
Testovanie 3D deformačných prejavov štandardnou procedúrou je možné aj geometrizovať , t.j. posúdiť vlastnosti vektorov posunov pomocou relatívnych konfidenčných elipsoidov. Sú uvedené dve metódy : priestorová a priesečníková a ich použitie ilustrované príkladom.

Key words : Deformation surveying,3D discrepancies, relative confidence ellipsoids, testing.

Introduction

Geodetic survey technics, making possible 3D measurements of points (GPS technologies, geodetic total stations, IMS, analytical photogrammetry) and following 3D processing networks in a convenient 3D cartesian reference frame S^S, are increasingly used in the present time. Many of these 3D measuring technics are applied for various quality controls of products, regular checking of object movements, all kinds of deformation measurements, i.e. for threedimensionally determining the network object points P_k ,$k\in\langle 1,p\rangle$ in epochs t_i,t_j. The computed coordinate estimates, $C^S_{ki}=[XYZ]_{ki}^T$ and $C^S_{kj}=[XYZ]_{kj}^T$ of P_{ki} and P_{kj} (two positions of the same point P_k obtained from epochs t_i,t_j) are determined with accuracy described by absolute confidence ellipsoids E_{ki} and E_{kj} and their coordinate differences (discrepancies) $\delta C_{kij}^S=C_{kj}^S-C_{ki}^S=[\delta X\ \delta Y\ \delta Z]_{kij}^T$ are obtained with accuracy given by a relative confidence ellipsoid E_{kij} with the center in position P_{ki} (Fig.1).

As known, using the discrepancy vector δC_{kij}^S (giving informations about the movement of P_k between epochs t_i,t_j), one can accept a decision based on its testing by statistical hypotheses, whether the moving P_k, expressed by δC_{kij}^S, is significant, i.e. if the position change of P_k from P_{ki} to P_{kj} can be hold as its deformation displacement or if δC_{kij}^S is a manifestation of the measuring errors in C_{ki}^S and C_{kj}^S. An analogous information source for a such decision can be created by a relative confidence ellipsoid E_{kij} together with the vector δC_{kij}^S. This ellipsoid may be therefore used (instead of conventional testing procedures) to deside on the character of the 3D vector δC_{kij}^S in the same way as the confidence ellipses are applied for testing 2D position changes of points (Heck et al., 1977; Hetényi, 1982, 1984).

The presented paper gives methods of using E_{kij} for testing δC_{kij} and outlines the visualisation possibilities of these procedures.

Testing the discrepancy vector

Initial data for testing

A convenient procedure for a common adjusting network observations from both epochs t_i,t_j give the coordinate estimates C_i^S and C_j^S of P_k, the discrepancy vector δC_{kij}^S and its cofactor matrix

$$Q_{\delta Cij}=Q_{Ci}+Q_{Cj}-Q_{Cij}-Q_{Cji} \qquad (1)$$

where the matrix arguments are submatrices of the cofactor matrix of the estimates (obtained within the network adjustment)

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$$Q_{Cij} = \begin{bmatrix} Q_{Ci} & Q_{Cij} \\ Q_{Cji} & Q_{Cj} \end{bmatrix} , \qquad (2)$$

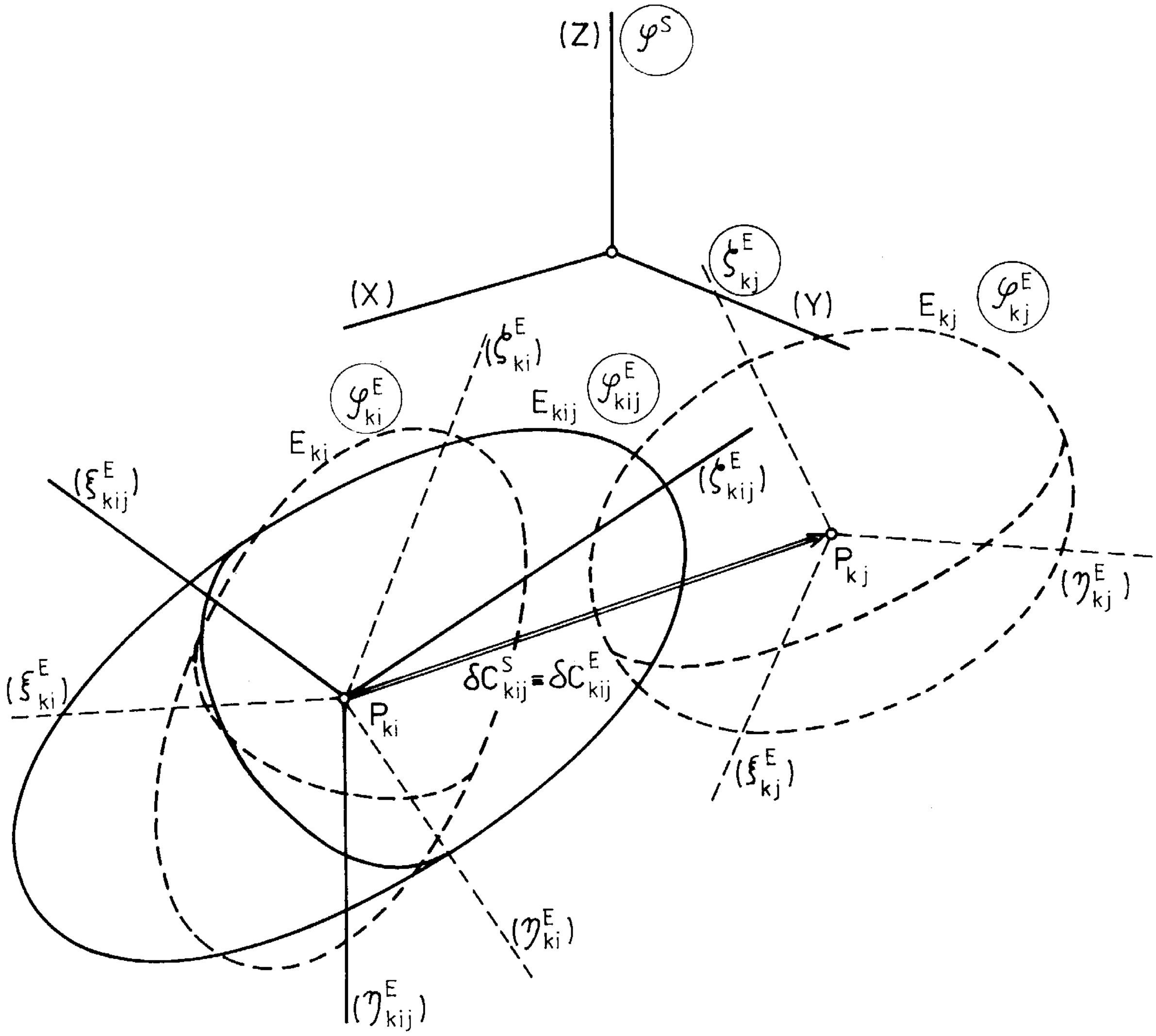

Fig. 1. Relative confidence ellipsoid at $P_{ki.}$

describing their accuracy.In addition, the aposteriori variance factor s_o^2 and other measures of the adjusted network are available among them the covariance matrices of the estimates and of the difference vector

$$\Sigma_{Cij} = s_o^2 Q_{Cij} = \left[\begin{array}{ccc|ccc} s^2_{Ci1} & L & s_{Ci1p} & s_{Cij11} & L & s_{Cij1p} \\ M & & M & M & & M \\ s_{Cip1} & L & s^2_{Cip} & s_{Cijp1} & L & s_{Cijpp} \\ \hline s_{Cji11} & L & s_{Cji1p} & s^2_{Cj1} & L & s_{Cj1p} \\ M & & M & M & & M \\ s_{Cjip1} & L & s_{Cjipp} & s_{Cjp1} & L & s^2_{Cjp} \end{array}\right] \qquad (3a)$$

$$\Sigma_{\delta Cij} = s_o^2 \begin{bmatrix} Q_{\delta C1} & \cdots & Q_{\delta C1p} \\ \vdots & Q_{\delta Ck} & \vdots \\ Q_{\delta Cp1} & \cdots & Q_{\delta Cp} \end{bmatrix}_{ij} = \begin{bmatrix} \Sigma_{\delta C1} & \cdots & \Sigma_{\delta C1p} \\ \vdots & \Sigma_{\delta Ck} & \vdots \\ \Sigma_{\delta Cp1} & \cdots & \Sigma_{\delta Cp} \end{bmatrix}_{ij} . \qquad (3b)$$

Spectral decomposition of the submatrices Σ_{Cki}, Σ_{Ckj} and $\Sigma_{\delta Ckij}$ (Forsythe et al., 1977; Linkwitz, 1988; Pelzer, 1980, 1985) give

$$\Sigma_{Cki}=M_{ki}\,\Lambda_{ki}\,M_{ki}^T, \quad \Sigma_{Ckj}=M_{kj}\,\Lambda_{kj}\,M_{kj}^T \qquad (4a)$$

$$\Sigma_{\delta Ckij}=M_{\delta Ckij}\,\Lambda_{\delta Ckij}\,M_{\delta Ckij}^T , \qquad (4b)$$

where

$$\Lambda_{ki}=\mathrm{diag}(\lambda_{ki1}\ \lambda_{ki2}\ \lambda_{ki3}), \quad \Lambda_{kj}=\mathrm{diag}(\lambda_{kj1}\ \lambda_{kj2}\ \lambda_{kj3}) , \qquad (5a)$$

$$\Lambda_{\delta Ckij}=\mathrm{diag}(\lambda_{\delta Ck1}\ \lambda_{\delta Ck2}\ \lambda_{\delta Ck3}) \qquad (5b)$$

are spectral matrices and

$$M_{ki} = \begin{bmatrix} \cos(X\xi) & \cos(X\eta) & \cos(X\zeta) \\ \cos(Y\xi) & \cos(Y\eta) & \cos(Y\zeta) \\ \cos(Z\xi) & \cos(Z\eta) & \cos(Z\zeta) \end{bmatrix}_{ki}, M_{kj} = \begin{bmatrix} \cos(X\xi) & \cos(X\eta) & \cos(X\zeta) \\ \cos(Y\xi) & \cos(Y\eta) & \cos(Y\zeta) \\ \cos(Z\xi) & \cos(Z\eta) & \cos(Z\zeta) \end{bmatrix}_{kj} , \qquad (6a)$$

$$M_{\delta Ckij} = \begin{bmatrix} \cos(X\xi) & \cos(X\eta) & \cos(X\zeta) \\ \cos(Y\xi) & \cos(Y\eta) & \cos(Y\zeta) \\ \cos(Z\xi) & \cos(Z\eta) & \cos(Z\zeta) \end{bmatrix}_{kij} , \qquad (6b)$$

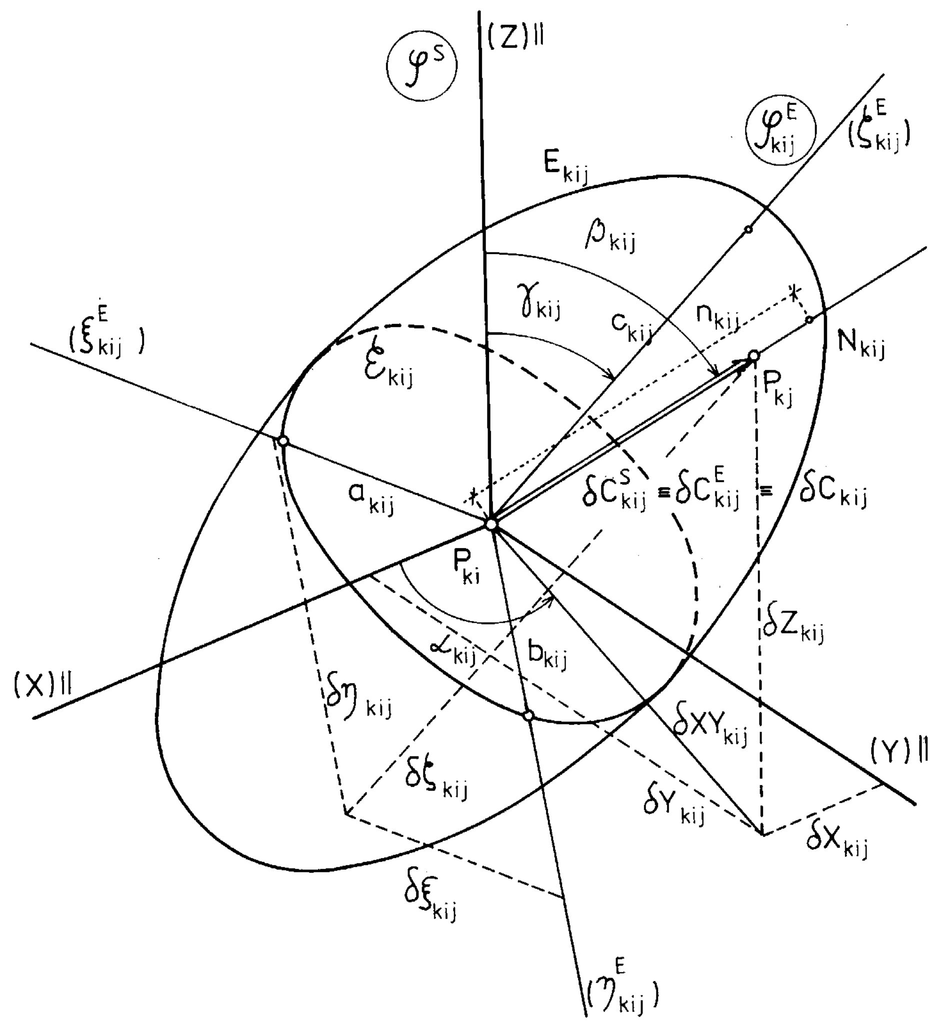

Fig. 2. Geometric interactions of absolute ellipsoids at P_{ki} and P_{kj} with the relative ellipsoid at P_{ki}.

modal matrices. The spectral matrices include eigenvalues ofΣ_{Cki}, Σ_{Ckj} and $\Sigma_{\delta Ckij}$, the modal matrices in the columns give eigenvectors needed for the axes orientation of the ellipsoids E_{ki} , E_{kj} and of the relative ellipsoids E_{kij} in relation to the coordinate axes (X),(Y),(Z) of the system s^S (Fig.2).

Standard method of testing

The conventional testing the vector δC_{kij} of P_k is made by the known procedure. The null hypothesis H_o :$E(\delta C_{kij})=0$ is formulated , i.e. a hypothesis assuming that δC_{kij} occur due to accidental measuring errors, and this presumption is tested on the level α using the statistics (Koch, 1987; Linkwitz, 1988)

$$T_k=(\delta C_{kij}^T\, Q_{\delta Ckij}^{-1}\, \delta C_{kij})/(\nu_1 s_o^2) \sim F(\nu_1,\nu_2) \qquad (7)$$

with a central Fisher-Snedecor distribution of probability (if H_o is valid) and with degrees of freedom ν_1= redundancy of the network,ν_2=3. If $T_k \geq F_k(1-\alpha;\nu_1,\nu_2)$, where $F_k(1-\alpha;\nu_1,\nu_2)$ is the critical value of the F-distribution , H_o is on the level α rejected, otherwise, if $T_k \langle F_k(1-\alpha;\nu_1,\nu_2)$, H_o can be admitted. Rejection of H_o indicates that the vector δC_{kij} does not possess a stochastical character. Such vector may be with probability 1-α considered as output of significant coordinate changes of P_k (in the initial point P_{ki}) ,i.e. as a deformation displacement vector of P_k from position P_{ki} (in epoch t_i) to position P_{kj} (in epoch t_j).

Testing with a relative confidence ellipsoid

Absolute confidence ellipsoid

As known (Hustopecký, 1987; Koch, 1987; Pelzer, 1980, 1985), the probability of positioning non accidental 3D coordinates C_{ki} (true,theoretical position of P_{ki}) in an accidental absolute point-ellipsoid E_{ki} [3)] is

$$P\{(C_{ki}-C_{eki})^T \Sigma_{Ceki}^{-1} (C_{ki}-C_{eki}) \leq F(1-\alpha;\nu_1,\nu_2)\} = 1-\alpha \tag{8}$$

where C_{eki} are estimates of C_{ki} .The equation

$$(C_{ki}-C_{eki})^T \Sigma_{Ceki}^{-1} (C_{ki}-C_{eki}) = F(1-\alpha;\nu_1,\nu_2) \tag{9}$$

can be interpreted as that of a three-axis ellipsoid E_{ki} with centre in P_{ki}. By the covariance matrix Σ_{Ceki} in Eq.(9) the accuracy of coordinates C_{eki} and by the factor $F(1-\alpha;\nu_{1,2})$, the confidence volume of E_{ki} are given.

Each ellipsoid E_{ki} by axes $(\xi_{ki}),(\eta_{ki}),(\zeta_{ki})$ creates an own 3D cartesian coordinate system s_{ki}^E (Fig.2) and then its equation may be defined according to (Linkwitz,1988; Pelzer, 1980, 1985)

$$\frac{\xi_{ki}^2}{\sigma_{\xi ki}^2} + \frac{\eta_{ki}^2}{\sigma_{\eta ki}^2} + \frac{\zeta_{ki}^2}{\sigma_{\zeta ki}^2} = F(1-\alpha;\nu_1,\nu_2) \tag{10a}$$

or

$$\frac{\xi_{ki}^2}{a_{ki}^2} + \frac{\eta_{ki}^2}{b_{ki}^2} + \frac{\zeta_{ki}^2}{c_{ki}^2} = 1. \tag{10b}$$

In Eq.(10) it holds that

$$\sigma_{\xi ki}^2 = s_o^2\,\lambda_{ki1},\ \sigma_{\eta ki}^2 = s_o^2\,\lambda_{ki2},\ \sigma_{\zeta ki}^2 = s_o^2\,\lambda_{ki3} \tag{11}$$

and semiaxes of E_{ki} (determining its dimension and confidence space) are

$$\begin{aligned} a_{ki}&=s_o(\lambda_{ki1}F(1-\alpha;\nu_1,\nu_2))^{1/2},\\ b_{ki}&=s_o(\lambda_{ki2}F(1-\alpha;\nu_1,\nu_2))^{1/2},\\ c_{ki}&=s_o(\lambda_{ki3}F(1-\alpha;\nu_1,\nu_2))^{1/2}, \end{aligned} \tag{12}$$

where eigenvalues λ_{ki} may be obtained according to Eq.(5).

If $F(1-\alpha;\nu_1,\nu_2) = 1$ (when $1-\alpha$=0,199), the confidence ellipsoid may be called the standard (Helmerts') ellipsoid.

Relative confidence ellipsoid

Formal applying the vector $C_{kj}-C_{ki} = \delta C_{kij}$ and their covariance matrix $\Sigma_{\delta Ckij}$ (3b) in Eq.(9) instead of $C_{ki}-C_{eki} = v_{cki}$, one can write the equation of a relative confidence ellipsoid E_{kij}

$$\delta C_{kij}^T \Sigma_{\delta Ckij}^{-1} \delta C_{kij} = F(1-\alpha;\nu_1,\nu_2) \tag{13a}$$

or

$$\frac{\delta\xi_{kij}^2}{a_{kij}^2} + \frac{\delta\eta_{kij}^2}{b_{kij}^2} + \frac{\delta\zeta_{kij}^2}{c_{kij}^2} = 1 \tag{13b}$$

with its centre in P_{ki} and with axes $(\xi_{kij}),(\eta_{kij}),(\zeta_{kij})$ creating an own cartesian coordinate system s_{kij}^E. Ellipsoid E_{kij} with lengths of its semiaxes

$$\begin{aligned} a_{kij} &= s_o(\lambda_{\delta k1}\ F(1-\alpha;\nu_1,\nu_2))^{1/2},\\ b_{kij} &= s_o(\lambda_{\delta k2}\ F(1-\alpha;\nu_1,\nu_2))^{1/2},\\ c_{kij} &= s_o(\lambda_{\delta k3}\ F(1-\alpha;\nu_1,\nu_2))^{1/2}, \end{aligned} \tag{14}$$

[3]statements and relations introduced in 3.1 for P_{ki} are analogous for P_{kj}

is convenient for testing the vector δC_{kij} and therefore it can be applied for this purpose.

The concept of the testing procedure is based on a judgement on the mutual space position of E_{kij} and the vector δC_{kij} with the length

$$|\delta C_{kij}| = |\delta C_{kij}^{E}| = |\delta C_{kij}^{S}| = |(\delta\xi_{kij}^{2} + \delta\eta_{kij}^{2} + \delta\zeta_{kij}^{2})^{1/2}| = |(\delta X_{kij}^{2} + \delta Y_{kij}^{2} + \delta Z_{kij}^{2})^{1/2}|. \qquad (15)$$

If δC_{kij} with its initial point in P_{ki} breaks through the ellipsoid surface (with centre in the same point P_{ki}), H_o is rejected , on the contrary, when δC_{kij} lies inside the ellipsoid E_{kij}, H_o can be admitted.

Further, two methods will be introduced for determination of δC_{kij} space positioning in relation to E_{kij}. For this, the vector δC_{kij} has to be transformed from the system s_{kij}^{S} into the system s_{kij}^{E} (of the ellipsoid coordinates) according to

$$\delta C_{kij}^{E} = \begin{bmatrix} \delta\xi \\ \delta\eta \\ \delta\zeta \end{bmatrix}_{kij}^{E} = M_{\delta Ckij}^{-1}\ \delta C_{kij}^{S} = M_{\delta Ckij}^{-1} \begin{bmatrix} \delta X \\ \delta Y \\ \delta Z \end{bmatrix}_{kij}^{S} , \qquad (16)$$

where the transformation matrix is defined by Eq.(6b).

Testing methods proposed

Space method

Each point in s^{E}_{kij} (e.g. P_{kj} with coordinates $\delta\xi_{kij},\delta\eta_{kij},\delta\zeta_{kij}$) fulfilling Eq.(13b) lies on the ellipsoid surface and each of these points with $\delta\zeta_{kij}$=0 has to be simultaneously situated in the centric-ellipse E_{kij} of E_{kij} (Fig.1)

$$\frac{\delta\xi_{kij}^{2}}{a_{kij}^{2}} + \frac{\delta\eta_{kij}^{2}}{b_{kij}^{2}} = 1 \qquad (17)$$

on the plane $\{(\xi_{kij}^{E})\ (\eta_{kij}^{E})\}$, perpendicular to the axis (ζ_{kij}^{E}) in the point P_{ki}.If components $\delta\xi_{kij}$ and $\delta\eta_{kij}$, instead of Eq.(17), give an unequality

$$\left(\frac{\delta\xi_{kij}^{2}}{a_{kij}^{2}} + \frac{\delta\eta_{kij}^{2}}{b_{kij}^{2}}\right) \rangle\ 1 , \qquad (18a)$$

the point P_{ki} will be situated outside of the area bounded by E_{kij}. This is equivalent to the statement, that the space position of the point P_{ki} is outside E_{kij}. The vector $\delta\dot{C}_{kij}^{S} \equiv \delta\dot{C}_{kij}^{E} \equiv \delta\dot{C}_{kij}$ in this case breaks through the ellipsoid surface and its end-point P_{kj} is out of E_{kij}.This is the finally conclusion relating
to the mutual space positions of δC_{kij}, P_{ki} and E_{kij} in situations with validity of Eq.(18a).

On the contrary, if components $\delta\xi_{kij}$, $\delta\eta_{kij}$ fullfil the relation

$$\left(\frac{\delta\xi_{kij}^{2}}{a_{kij}^{2}} + \frac{\delta\eta_{kij}^{2}}{b_{kij}^{2}}\right) \langle\ 1 , \qquad (18b)$$

i.e. when P_{kj} will be positioned inside of E_{kij} , it has to be investigated further whether P_{kj} will be found inside of E_{kij} (in the upper or lower halfellipsoide with regard to the plane of E_{kij}), or P_k will be situated out of E_{kij}. To decide upon these possibilities, Eq. (13b) can be used , from which we have

$$\delta\zeta_{kij} = c_{kij}\sqrt{1-(\frac{\delta\xi_{kij}^2}{a_{kij}^2}+\frac{\delta\eta_{kij}^2}{b_{kij}^2})}$$

or

$$f(\delta\zeta_{kij}) = 1-\frac{\zeta_{kij}^2}{c_{kij}^2} = \frac{\delta\xi_{kij}^2}{a_{kij}^2}+\frac{\delta\eta_{kij}^2}{b_{kij}^2} \quad . \tag{19}$$

According to Eq.(19), the point P_{kj} with coordinates $\delta\xi_{kij}$ and $\delta\eta_{kij}$ inside the ellipse E_{kij}, and, simultaneously with $\delta\zeta_{kij}$ or $f(\delta\zeta_{kij})$ satisfying Eq. (19), is positioned on the surface of E_{kij}.

If the relation

$$\left|f(\delta\zeta_{kij})\right| \rangle (\frac{\delta\xi_{kij}^2}{a_{kij}^2}+\frac{\delta\eta_{kij}^2}{b_{kij}^2}) \tag{20a}$$

is valid, P_{kj} will be found out of the space demarcated by E_{kij}, i.e. the vector δC_{kij} will be breaking through the ellipsoid. Otherwise, if

$$\left|f(\delta\zeta_{kij})\right| \langle (\frac{\delta\xi_{kij}^2}{a_{kij}^2}+\frac{\delta\eta_{kij}^2}{b_{kij}^2}) \quad , \tag{20b}$$

the point P_{kj} and the vector δC_{kij} will be situated in E_{kij}.

Intersection method

This investigation is based on comparing length of the vector $\delta C_{kij}^E = \left|(\delta\xi_{kij}^2 + \delta\eta_{kij}^2 + \delta\zeta_{kij}^2)^{1/2}\right| \equiv \delta C_{kij}^S \equiv \delta C_{kij}$ with the distance $n_{kij} = \overline{P_{ki}N_{kij}}$ (Fig.1), where N_{kij} is the intersection point of the straight line defined by P_{ki},P_{kj} (direction of the vector δC_{kij}) with the ellipsoid surface.

The distance n_{kij} in s_{kij}^E may be determined by

$$\begin{aligned} n_{kij} &= \sqrt{(\xi_{Nkij}-\xi_{Pki})^2+(\eta_{Nkij}-\eta_{Pki})^2+(\zeta_{Nkij}-\zeta_{Pki})^2}, \\ n_{kij} &= \sqrt{\xi_{Nkij}^2+\eta_{Nkij}^2+\zeta_{Nkij}^2} \end{aligned} \tag{21}$$

because $\xi_{Pki} = \eta_{Pki} = \zeta_{Pki} = 0$. Coordinates of the intersection point can be obtained by solving a linear equation system, consisting of Eq.(13b) and the equation for the straight line through P_{ki},P_{kj}

$$\begin{aligned} &\frac{\delta\xi_{kij}^2}{a_{kij}^2}+\frac{\delta\eta_{kij}^2}{b_{kij}^2}+\frac{\delta\zeta_{kij}^2}{c_{kij}^2} = 1 \ , \\ &\frac{\xi_{Nkij}}{\delta\xi_{kij}} = \frac{\eta_{Nkij}}{\delta\eta_{kij}} = \frac{\zeta_{Nkij}}{\delta\zeta_{kij}} \end{aligned} \tag{22}$$

with regard to the null coordinate values of P_{ki}. The solution of Eq. (22) yields

$$\xi_{Nkij} = 1/\sqrt{k},$$
$$\eta_{Nkij} = \frac{\delta\eta_{kij}}{\delta\xi_{kij}}\frac{1}{\sqrt{k}}, \qquad (23)$$
$$\zeta_{Nkij} = \frac{\delta\zeta_{kij}}{\delta\xi_{kij}}\frac{1}{\sqrt{k}}$$

where

$$k = \frac{1}{a_{kij}^2} + \frac{\delta\eta_{kij}^2}{b_{kij}^2\delta\xi_{kij}^2} + \frac{\delta\zeta_{kij}^2}{c_{kij}^2\delta\xi_{kij}^2} \quad . \qquad (24)$$

Comparing δC_{kij} and n_{kij}, if

$$|\delta C_{kij}| > n_{kij} \qquad (25a)$$

the end point P_{kj} of the vector δC_{kij} is positioned out of E_{kij} ,i.e. the vector breaks through the ellipsoid surface. On the contrary, if

$$|\delta C_{kij}| < n_{kij} \;, \qquad (25b)$$

the point P_{kj} (and at the same time the whole vector δC_{kij}) will be found inside E_{kij}.

Visualisation of results

Except for getting numerical results and statements from the testing procedure, a convenient graphical presentation of these results is expedient. Using it, one can obtain a global survey of mutual relations of the vectors δC_{kij} and the ellipsoids E_{kij} in the point field of a network. Such a presentation may enable for the observer an immediate glance over the situation, i.e. over dimensions and orientation of the ellipsoids E_{kij}, space relations between E_{kij} and δC_{kij} (whether δC_{kij} is going through E_{kij}, in which direction, with what length etc.). By a suitable visualisation of these informations, a good visual survey of displaying the vectors δC_{kij} may be possible in the whole space of the deformation network.

The standard computer possibilities of a 3D visualisation using mesh surfaces,grids,contour plots and other forms are not always the most convenient ones from the mentioned points of view because of unsufficiently information content of such presentations. It seems that various 2D line-form plotting of E_{kij} and its relation to δC_{kij}, completed with convenient and needful numeric or symbol data, is the most acceptable 3D graphic description of these objects and their mutual interactions. From such 2D visualisations of the computed results in the 3D space, e.g. the following ones can be applied: scaled and oriented drawing E_{kij}, plotting the scaled and oriented true length of δC_{kij} in the plane {(X) (Y)} with data of its inclination (related to (Z) in $\langle 0,\pi\rangle$) and using different possible designations for δC_{kij} being inside or outside of E_{kij} (in Fig.3 double line, full one).

All data necessary for plotting and designation of the above mentioned geometrical characteristics are computable within computer supported solving scalar and vector quantities for performing test. Using these data and a suitable software for the automatical drawing, a graphical output can be done showing at which points of the network are 3D deformation displacements.

Other ways of 2D plotting or combinations of 3D and 2D visualisations with various numerical and symbolic complements (increasing the lucidity of these plots) are possible and applicable too.

Example

In a 3D landslide deformation network (Fig.3), surveyed by a geodetic total station (spatial distances to 3 km,horizontal and zenit angles, trigonometrical height differences measured) and processed in a 3D cartesian system for epochs t_o (start) and t_1 (next) using the Gauss-Markoff model,

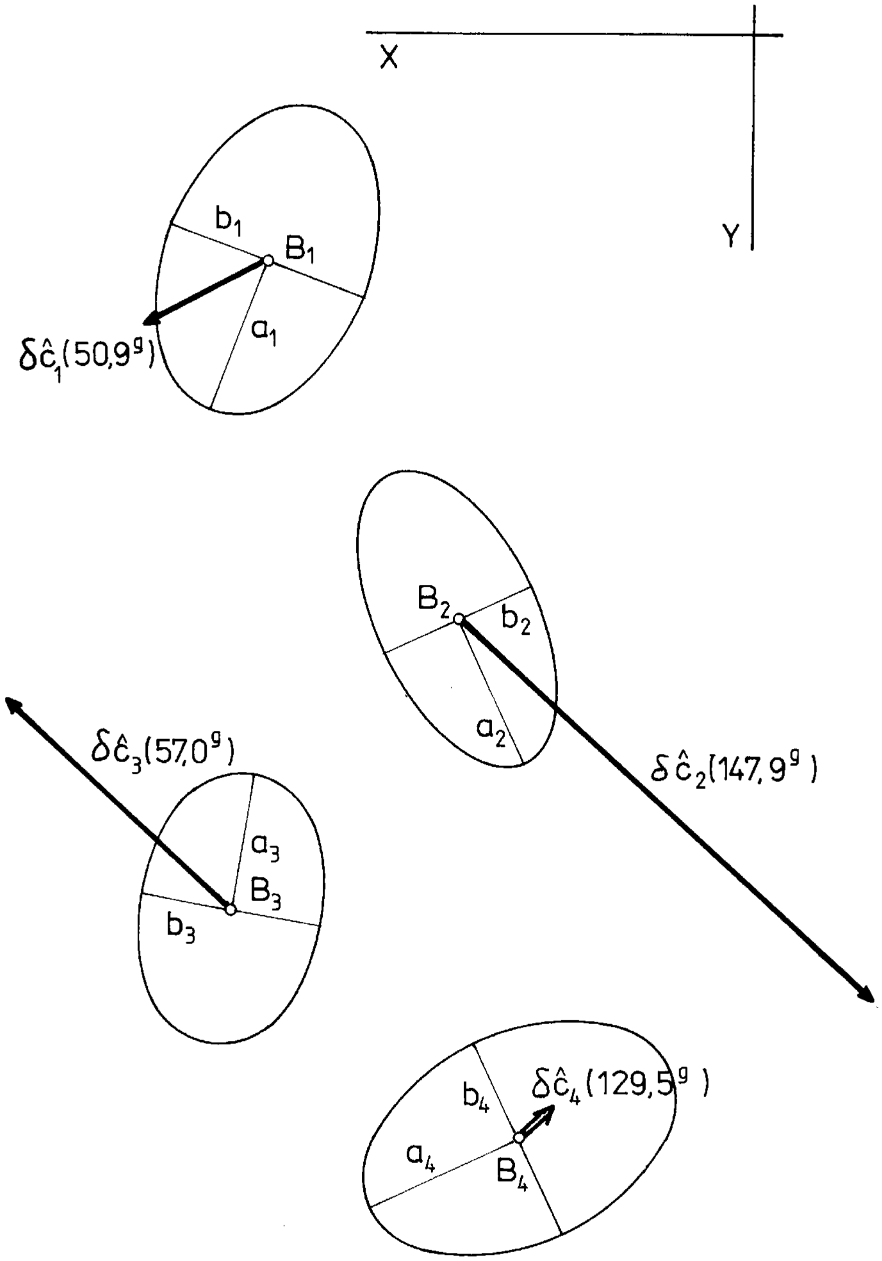

Fig. 3. Visualisation of the geometric positions between discrepancy vectors and the relative ellipsoids at the network points (scale: network 1 : 2000, vectors and ellipsoids 2 : 1).

coordinate estimates C_o and C_1 of the object points $B_k, k \in \langle 1,4 \rangle$, the vector δC_k , the aposteriori variance factor s_o^2=89,354 and the cofactor matrix $Q_{\delta C}$ have been determined. The accuracy of the adjusted coordinates is given by the average standard deviation 5.56 mm.

The coordinates of the vectors δC_{kij}^S (of points C_1), related to points with C_o in s^S, are (Tab.1)[4] :

Point	δX^S	δY^S	δZ^S	$\delta\xi^E$	$\delta\eta^E$	$\delta\zeta^E$
1	6.23	3.77	7.08	-1.04	6.60	7.65
2	-25.62	18.27	-29.44	-3.55	-42.78	-3.81
3	13.05	-12.88	14.70	5.17	19.90	-11.39

[4]all numeric data are in mm

4	-1.98	-2.06	-1.43	-1.80	-2.59	0.52

Tab.1 Tab.2

Using $Q_{\delta C}$,the corresponding spectral (Eq.(5b)) and modal matrices (Eq.(6b)) for the points B_k , by transformation (16) of the coordinates $\delta\xi^E$,$\delta\eta^E$,$\delta\zeta^E$ of the end points C_1 of discrepancy vectors in $s_{k01}{}^E$ (Tab.2), and the lengths of the vectors δC_{k01} according to Eq.(15) (Tab.3), were further computed.

Point	δC_{k01}	n_{ko1}
1	10.16	5.65
2	43.09	5.78
3	23.50	6.31
4	3.20	8.89

Tab.3 Tab.6

Lengths of the semiaxes of the standard relative ellipsoid E_{k01} (14) are (Tab.4):

Point	a	b	c	ξ_{Nk01}	η_{Nk01}	ζ_{Nk01}
1	12.44	8.04	4.79	0.58	-3.67	-4.26
2	2.35	5.88	12.48	0.48	5.74	0.51
3	4.20	7.94	10.26	1.39	5.34	-3.06
4	6.07	8.12	12.63	5.02	7.20	-1.45

Tab.4 Tab.5

For testing by the intersection method, the length n_{k01} (Eq.(21)) (Tab.6) and the coordinates for the intersection points of the vectors δC_{k01} through E_{k01} were computed using Eq.(23) and (24) (Tab.5).
The comparison of n_{k01} with δC_{k01} yields

$n_1 < \delta C_1$,
$n_2 < \delta C_2$,
$n_3 < \delta C_3$,
$n_4 > \delta C_4$

and, as may be declared, movements of B_1,B_2 and B_3 should be hold as their significant position changes, i.e. these movements should be taken for deformation displacements only the movement of B_4 may not be considered as a deformation displacement of this point.

The standard relative ellipsoids with the corresponding vectors are visualised in the way introduced in chapter 5 (Fig.3) to gain a good view upon the point field of the network with the maximum of needed information.For each point B_k :

- E_{k01} of E_{k01} given by their scaled true semiaxes a_{k01}, b_{k01} (Tab.4) and oriented by bearings σ_{ak01}, was drawn in the plane {(X) (Y)},

- the scaled projections of the vector δC_{k01} (Tab.3) with orientations given by bearings $\sigma_{\delta Ck01}$ were plotted in the plane {(X) (Y)} with their inclinations $\beta_{\delta C}$ to (Z).

Bearings of semiaxes "a" are computable from the modal matrices (Eq.(6b)), bearings and inclinations of δC_{k01} from the coordinates (Tab.1).

Conclusion

The conventional numerical testing procedures in the 3D deformation measurements can be supplied by equivalent graphical testing the discrepancy vectors.For this reason, various methods may be applied that investigate the mutual space relations of the vectors and the corresponding relative confidence ellipsoids. To this form of testing a suitable graphic description of the space situation of testing results should be joined that could given a good visual presentation.

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