Mathematical modelling breakpoints in the subsidences

Vladimír Sedlák[1]

Matematické modelovanie lomových bodov v poklesových kotlinách

V príspevku je prezentovaná teória matematického modelovania polynomiálnych lomových bodov pri analýze deformačných charakteristík poklesových kotlín. Teória určovania lomových bodov bola vyvinutá ako súčasť kinematických analýz horninového masívu dobývacieho ložiskového poľa magnezitu vo východoslovenskom regióne. Teoretické poznatky modelovania sú doplnené praktickými výsledkami deformačných meraní in situ.

Kľučové slová: deformačné modelovanie, lomový bod, pokles, matematické hypotézy.

Introduction

A gradual subsidences development at the Košice-Bankov mine region in Slovakia was minitored by geodetic measurements (levelling) from the beginning of mine underground activities in the magnesite mineral deposit. The stage, so-called the static analysis from long-term geodetic observations cumulated into every year periodical measurements can be indicated in the present time. The analysis of time factors of the gradual subsidences development continuing with underground exploitation allows a production of more exact model situations in each separate subsidence processes, and especially it provides an upper degree in a prevence of deformations in the earth surface (Knothe, 1984; Sedlák, 1992 a,b, 1993 a,b, 1994; Sedlák & Havlice, 1993).

A possibility of improving polynomial modelling subsidences is conditioned by the knowledge to detect position of so-called *breakpoints,* i.e. the points in the earth surface in which the subsidence border with a zone of breaches and bursts start to develope over the mineral deposit exploitation. It means that the breakpoints determine a place of the subsidences where it occurs to the expressive fracture of the earth continuous surface consistence. For this reason it is necessary to present some theoretical knowledge and practical procedures from a long-term exploration in deformation subsidence measurements at the magnesite mineral deposit mine field in the Košice-Bankov region.

Research overview

Problems of mine damages in the earth surface, dependent on the underground mine activities at the magnesite mineral deposit, did not receive a systematic research attention in Slovakia till 1976. After there, the requirements for a scientific motivation in the subsidences development following out from rising exploitations and from introducing progressive mine technologies were taken in consideration. This scientific exploration in the subsidences must be supported by systematic monitoring mine influences in the earth surface using precise geodetic methods.

The monitoring station project in the Košice-Bankov case was designed and realized by the research staff of the Department of Geodesy & Geophysics of the Technical University of Košice in 1976 (Kunák et al., 1985). The first monitoring data were taken from this monitoring station in the same year. The monitoring station is situated in the earth surface in the Košice-Bankov mine region near by the West shaft. The monitoring station is constructed from the geodetic network of the reference points and objective ones situated in seven profiles.

Geodetic deformation analysis

The deformation analysis based on geodetic methods may be divided into four steps:

[1] Katedra geodézie a geofyziky F BERG Technickej univerzity, 043 84 Košice, Park Komenského 19
(Recenzenti: Prof.Ing. Ladislav Kunák, CSc. a Doc.Ing. Jozef Novák, CSc. Revidovaná verzia doručená 20.2.1997)

1. Design of the geodetic monitoring network. First the monitoring network and an observation scheme are designed such that requirements following from the anticipated deformation pattern and the specified accuracy (precision and reliability) criteria are met.

2. Single-epoch analysis. The least-squares adjustment of the geodetic measurements in the free network mode is performed assuming that no deformations have taken place during this epoch. The main objective of this step is to establish a consistent mathematical model of the network, resulting in adjusted coordinates with a corresponding quality description in terms of precision and reliability.

3. Stability evaluation of reference points. Here a distinction should be made between absolute and relative deformation maesurements. Absolute deformations are studied relative to a set of points which are assumed to be stable whereas in the case of relative deformation measurements no stable points are available.

If present, reference points will be checked for instabilities. The whole set of available reference points throughout all epochs will be adjusted under the assumption that no deformation has taken place. Statistical testing is performed for the detection and identification of unstable reference points, which will be removed from the set. The final adjustment provides coordinates of the stable reference points with respect to which deformations of the object points will be described.

4. Deformation analysis. In practice the deformation pattern is often described by a polynomial model. Polynomial models in 1-, 2- or 3-dimensions (1D, 2D, 3D) are linear models and especially the parameters of the lower order models have a distinct physical meaning. In the least-squares adjustment the parameters of the mathematical model chosen are estimated and tested for their significance. Possible modifications of the deformation model will be based on the test results.

In the case of multi-epoch analysis a distinction can be made between a static and kinematic approach. In the case of static approach only two epochs are taken into consideration at the same time. In the case of a kinematic approach all available epochs are taken into account. This allows the detection of possible trends in the data, it allows the study of derivative functions such as velocity and it allows improvement of the identification of possible errors in the measurement data. The disadvantage of the kinematic approach, however, may be the large number of data to be processed.

1D deformation analysis from levelling networks

In accordance with the general phases of the geodetic deformation analysis the project at hand was defined to contain the following phases:

1. Single epoch evaluation of the levelling data available.
2. Stability evaluation of reference benchmarks.
3. Estimation of the most likely deformation model.

Obviously the design step is no element of the project since we are dealing with existing networks. The single epoch evaluation concentrates on the evaluation of the functional model, the observational data and the stochastic model. By means of the integration of hypothesis testing, including the outlier detection, and the variance component estimation a consistent mathematical model is obtained.

In the second phase of the project the assumption in the functional model of stable reference benchmarks is tested. Unstable benchmarks are removed from the set and will further be treated as objective-points. After establishing the correct functional model the stochastic model may be improved as well. Again a consistent mathematical model results.

The aim in the third phase is to arrive at the most likely mathematical model describing the deformation pattern underlying the data. The functional model part is restricted to 1D- or 3D-polynomials, which are either a function of time or a function of both positions (x,y) and time. The mathematical model is again balanced by modifications of the stochastic model.

Polynomial breakpoints

In the project described the third step consists again of three different steps, i.e.:

1. Estimation of 1D-polynomial model per benchmark.
2. Estimation of 3D-polynomial model per selection benchmarks.
3. Evaluation of possible external height-information available.

When evaluating the estimated time-dependent polynomials per benchmark, it become more and more apparent that such a polynomial could not accurately describe the behaviour of the benchmarks which came under the influence of the mineral deposit extraction some time after the start of the exploration. Such behaviour was described by higher order polynomials, whereas it was actually due to a break in the trend of the subsidence. This problem may be solved for allowing the polynomial function to have a so-called *breakpoint*, which is defined as: *A point in time at which a benchmark, due to the mineral deposit extraction, enters the subsidence area.* The present paper is mainly concerned with the estimation of breakpoints in 1D-polynomial models, i.e. polynomial breakpoints.

Modelling polynomial breakpoints

The estimation of polynomial breakpoints is a part of the procedure developed for establishing the most likely mathematical model, describing the subsidence behaviour of a specific benchmark in time. The procedure is based on the concept of least-squares estimation and multiplate hypothesis testing (Sedlák & Kunák, 1996).

Hypothesis testing

In general, the mathematical model under nullhypothesis may be modelled in terms of observation equations

$$H_0 : E\{\underline{y}\} = A_x ; \quad D\{\underline{y}\} = Q_y , \tag{1}$$

where $E\{\,.\,\}$ is mathematical expectation;
y is m-by-1 vector of observations;
A is m-by-n design matrix;
x is n-by-1 vector of unknowns;
$D\{\,.\,\}$ is mathematical dispersion;
Q_y is m-by-n variance covariance matrix of the observations;

and underlinement stands for stochasticity. Moreover, m equals the number of observations and n the number of unknowns.

The variability of the nullhypothesis may be tested against the widest possible alternative hypothesis, by means of the teststatistic

$$\underline{T} = \underline{\hat{e}}^* Q_y^{-1} \underline{\hat{e}} , \tag{2}$$

where $\hat{e}$ is m-by-1 vector of least-squares corrections of the observations.

Under the nullhypothesis this teststatistic has a central distribution χ^2 with m-n degrees of freedom. i.e. $\chi^2(m\text{-}n, 0)$. The teststatistic, Eq.(2), is known as the „overall model test".

In case of a rejection of the nullhypothesis, one will try to detect the cause of rejection by formulating a (number of) possible alternative hypothesis. In general, the model under the alternative hypothesis may be written as a linear extension of the model under the nullhypothesis

$$H_a : E\{\underline{y}\} = A_x + C\boldsymbol{L} ; \quad D\{\underline{y}\} = Q_y , \tag{3}$$

where C is m-by-q matrix;
$\boldsymbol{L}$ is q-by-1 vector;

and $C\boldsymbol{L}$describes the assumed model error. The dimension of the linear extension of the functional model q may vary from $q=1$ to $q=m\text{-}n$.

The validity of the alternative hypothesis may be tested by the teststatistic

$$\underline{T}_q = \underline{\hat{e}}^* \quad Q_y^{-1}C \quad [C^*Q_y^{-1}Q_{\hat{e}}\, Q_y^{-1}C]^{-1} \quad C^*Q_y^{-1}\underline{\hat{e}} \quad , \tag{4}$$

in which $Q_{\hat{e}}$ is the covariance matrix of the least-squares residuals. Under the nullhypothesis the teststatistic $\underline{T}_q$ has a central distribution χ^2 with q degrees of freedom, i.e. $\chi^2(q,0)$.

If $q = 1$ the C-matrix reduces to a m-by-1 vector c, and vector $\boldsymbol{L}$ reduces to a scalar, causing Eq.(4) to reduce to

$$\underline{T}_1 = (c^* Q_y^{-1} \underline{\hat{e}})^2 (c^* Q_y^{-1} Q_{\hat{e}} Q_y^{-1} c)^{-1} \quad (5)$$

which is described as $\chi^2(1,0)$ under the nullhypothesis. A well-known application of Eq.(5) is found in the method of data-snooping, where the data are checked for possible measurement errors by computing the so-called conventional alternative hypotheses. These hypotheses are of the form

$$c_i^* = [0 \ldots 010 \ldots] , \quad (6)$$

in which 1 is found at the position j. In general the data-snooping uses the w-teststatistic, which has a standard normal distribution under H_o and equals the square-root of Eq.(5).

In the Košice-Bankov case of estimation and testing it is custom to compute, next to the overall modell test all w-teststatistics for the conventional alternative hypotheses. In the present paper we will use all three types of tests, Eq.(2),(4) and (5).

The mathematical model under H_o

Given a certain benchmark, its height at the various epochs as computed after the stability analysis of the reference benchmarks from, together with their covariance matrix, the starting point for the evaluation of the benchmarks subsidence behaviour. The general form of a 1D time-dependent polynomials of order n for the benchmarks heights is given as

$$H_k = a_o t_k^o + a_1 t_k^1 + a_2 t_k^2 + \ldots + a_n t_k^n , \quad (7)$$

where H_k is height of the benchmark as determined at epoch k;
a_i is unknown coefficient, $i = 0, \ldots, n$;
t_k^i is measurement time of epoch k to the power i.

The mathematical model under the nullhypothesis assumes a linear subsidence, which is represented by a time.dependent polynomial of order 1. The assumption is based on the fact that in the Košice-Bankov case a large number of benchmarks shows a natural, linear subsidence.

Alternative hypotheses considered

In case of a rejection of the nullhypothesis a number of alternative hypotheses are to disposition. They are tested for their validity at the modelling subsidences. Alternative hypotheses follow out from three basic presumptions:

1. Height of the benchmark in one single epoch is incorrect.
2. The order of the polynomial should be increased by *1* or *2.*
3. Extension of the polynomial with a breakpoint is necessary.

Each one of the hypotheses considered may be written as a linear extension of the functional model under the nullhypothesis.

1. Heights of the benchmarks in one single epoch are incorrect (w-test). The alternative hypothesis considering the height of one benchmark H_j to be erroneous equals in fact the conventional alternative hypothesis. Hence the linear extension of the functional model is given as

$$c\boldsymbol{L} = [0 \ldots 0\ 1\ 0 \ldots 0]^* \boldsymbol{L}, \quad (8)$$

leading to a teststatistic of the form of Eq.(5).

2. The order of the polynomial should be increased by 1 or 2 (01- or 02-test). From Eq.(7) it is immediately clear that extension of the functional model with an extra parameter is a linear extension. Hence the mathematical model under H_a for testing the significance of a higher order term is given by

$$E\begin{bmatrix} H_1 \\ \vdots \\ H_k \\ \vdots \\ H_p \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ \vdots & \vdots \\ 1 & t_k \\ \vdots & \vdots \\ 1 & t_p \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} + \begin{bmatrix} t_1^2 \\ \vdots \\ t_k^2 \\ \vdots \\ t_p^2 \end{bmatrix} \begin{bmatrix} a_2 \end{bmatrix} ; \quad Q_H , \tag{9}$$

in which Q_H equals the covariance matrix of the benchmarks heights. This hypothesis leads to a teststatistic of the form of Eq.(5).

The alternative hypothesis allowing two extra higher order terms is given as

$$E\begin{bmatrix} H_1 \\ \vdots \\ H_k \\ \vdots \\ H_p \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ \vdots & \vdots \\ 1 & t_k \\ \vdots & \vdots \\ 1 & t_p \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} + \begin{bmatrix} t_1^2 & t_1^3 \\ \vdots & \vdots \\ t_k^2 & t_k^3 \\ \vdots & \vdots \\ t_p^2 & t_p^3 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} ; \quad Q_H , \tag{10}$$

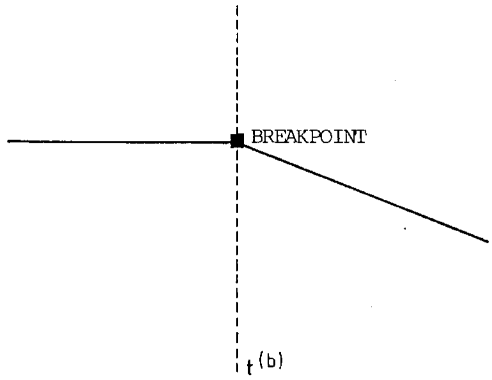
resulting in a teststatistic of the form of Eq.(4). In case the mathematical model under H_o equals a polynomial of order n the linear extensions of the functional model will become

$$c\boldsymbol{L} = \begin{bmatrix} t_1^{n+1} & \dots & t_k^{n+1} & \dots & t_p^{n+1} \end{bmatrix}^{*} \begin{bmatrix} a_{n+1} \end{bmatrix}$$

and

$$C\boldsymbol{L} = \begin{bmatrix} t_1^{n+1} & \dots & t_k^{n+1} & \dots & t_p^{n+1} \\ t_1^{n+2} & \dots & t_k^{n+2} & \dots & t_p^{n+2} \end{bmatrix}^{*} \begin{bmatrix} a_{n+1} \\ a_{n+2} \end{bmatrix} \tag{11}$$

respectively. The test for an extension of the polynomial model with two higher order terms was added since it may occur that one extra parameter is not significant, whereas two parameters are.



3. Extension of the polynomial with a breakpoint is necessary (B-test). For a mathematical formulation of the breakpoint hypothesis a number of assumptions need to be made. The restrictions follows from the type of polynomial functions that is searched for, which is illustrated in *Fig.1.* Here it is assumed that a breakpoint was detected at the point in time $t^{(b)}$. Although the point in time $t^{(b)}$ at which the breakpint may occur is in principle unknown, it is clear that *before* and *after* the breakpoint a different polynomial should be valid.

Fig.1. 1D polynomial with breakpoint in time $t^{(b)}$.

The assumptions are:

1. The polynomial order before the breakpoint is restricted to a maximum of one ($n_1 \leq 1$), which is also the case under the nullhypothesis. This assumption is based on the fact that a possibly natural subsidence in the Košice-Bankov case shows at the most a linear behaviour.

2. The polynomial order before the breakpoint does not exceed the polynomial order after the breakpoint, i.e. $n_2 \geq n_1$.

3. The function is required to be continuous in its breakpoint, meaning that the function values of both polynomials before and after the breakpoint should be the same.

In order to simplify the formulae used, the polynomial time scale is redefined such that at the origin is translated to the breakpoint $t^{(b)}$, i.e.

$$t_k = \frac{t^{(k)} - t^{(b)}}{t^{(p)} - t^{(1)}} , \qquad (12)$$

thus $t_b=0$. The assumptions made allow us to write the polynomial after the breakpoint as a linear extension of the one before the breakpoint, i.e.

$$H_k = \begin{cases} \sum_{i=0}^{1} a_i t_k^i , & \text{if } t^{(k)} \leq t^{(b)} \ (t_k \leq 0) \\ \sum_{i=0}^{1} a_i t_k^i + \sum_{i=1}^{n_2} da_i t_k^i , & \text{if } t^{(k)} > t^{(b)} \ (t_k > 0) \end{cases} , \qquad (13)$$

where H_k is benchmarks height as determined at epoch k;
n_2 is polynomial order after the breakpoint;
a_i are unknown polynomial coefficients before the breakpoint;
da_i is coefficient difference between the polynomial before and after the breakpoint;
t_k is measurement time of epoch k in the polynomial time unit as defined above;
$t^{(k)}$ is point in time at which epoch k was measured.

From Eq.(13) it follows that a breakpoint will be detected when a significant change in slope (da_1) appears. Testing for the presence of a polynomial breakpoint may therefore be based on the following alternative hypothesis

$$\begin{bmatrix} \underline{H}_1 \\ \cdot \\ \vdots \end{bmatrix} \quad \begin{bmatrix} 1 & t_1 \\ \cdot & \cdot \\ \vdots & \vdots \end{bmatrix} \quad \begin{bmatrix} 0 \\ \cdot \\ \vdots \end{bmatrix}$$

$$E\begin{bmatrix} \vdots \\ \underline{H}_k \\ \underline{H}_{k+1} \\ \vdots \\ \underline{H}_p \end{bmatrix} = \begin{bmatrix} \vdots & \vdots \\ 1 & 0 \\ 1 & t_{k+1} \\ \vdots & \vdots \\ 1 & t_p \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} + \begin{bmatrix} \vdots \\ 0 \\ t_{k+1} \\ \vdots \\ t_p \end{bmatrix}\begin{bmatrix} da_1 \end{bmatrix}, \qquad (14)$$

which is clearly a linear extension of the functional model under nullhypothesis, Eq.(7). In Eq.(14) it is assumed that the breakpoint appears at epoch k . Hence $t^{(b)}=t^{(k)}$ and thus $t_k=0$. The related teststatistic is of the form of Eq.(5). From Eq.(14) it follows that the breakpoint may not be detected in the first epoch ($t^{(1)}$) or the last one ($t^{(p)}$). Moreover, it is not possible to distinguish between a possible error in H_1 or H_p and a breakpoint before the second or after the last but one epoch respectively. Therefore possible breakpoints can only be detected within the interval $t^{(2)} < t^{(b)} < t^{(p-1)}$. In the case of Košice-Bankov subsidence, the interval is scanned for the most likely breakpoint by increasing $t^{(b)}$ with one or two years at a time and testing each related alternative hypothesis.

Results of testing for polynomial breakpoints

As mentioned before the estimation of a polynomial breakpoint is a part of a procedure to determine the most likely polynomial model for the subsidence of a benchmark in time. Therefore, the resulting mathematical model may or may not include a polynomial breakpoint.

Identification of a polynomial brakpoint

The aim of the procedure is to arrive at a consistent mathematical model, i.e. both the functional and the statistic model. In short the procedure is as follows. First a least-squares adjustment of the mathematical model under the nullhypothesis is performed. The validity of this model is tested by the application of the overall model test, given in Eq.(2).

Depending on the test result, the next steps are following:

1. Accept H_o : The estimated slope-coefficient (a_1) is tested for its significance. If the parameter is significant the functional model is replaced by a constant polynomial with implies stability of the benchmark considered.

2. Reject H_o : Test all alternative hypotheses as described above for their validity and determine the most likely alternative hypothesis. Depending on the most likely hypothesis selected, the following actions are taken:

a) w-test: Remove the observation concerned, i.e. the benchmark's height at the epoch which was identified by the largest *w-test* value.

b) 01- or 02-test: Adapt the mathematical model under the selected alternative hypothesis to be the new mathematical model under the nullhypothesis. Possibly more parameters are needed to describe the benchmarks behaviour accurately. Hence, the nullhypothesis is again tested for its validity. In the case of rejection the alternative hypotheses mentioned before are once more tested. However, the test for a possible breakpoint is now excluded from the procedure.

c) B-test: Adapt a breakpoint at the epoch which was identified by the largest *B-test* value. The order of the polynomial before and after the breakpoint is now determined for each part separately. Hence, the procedure restarts for each part separately. However, the test for a possible breakpoint is now excluded from the procedure.

After establishing the most likely functional mathematical model is balanced by modification of the stochastic model, by adaptation of the variances of the benchmark at all epochs. If the nullhypothesis is tested against a number of alternative hypotheses, which is often the case in practice, criteria are needed to select the most likely alternative hypothesis. In the present case all hypotheses considered lead to teststatistics that have as it is mentioned in Eq.(2), (4) and (5).

First consider the case where the dimensions of the hypotheses considered are equal. In our procedure this occurs when all *w-tests* or when all *B-tests* are compared. Since those teststatiscs $\underline{\boldsymbol{T}}^i$

are all of the form of Eq.(5) and thus all have the same central distribution with one degree of freedom, i.e.

$$\underline{\boldsymbol{T}}^{i} \sim \chi^{2}(1,0)\ \forall\ i, \qquad (15)$$

and the largest value implies the most likely alternative hypothesis. Hence, in this case the most likely alternative hypothesis is the one for which

$$\underline{\boldsymbol{T}}^{i} > \underline{\boldsymbol{T}}^{j}\ \ \forall\ j \neq i, \qquad (16)$$

where the indices *i* and *j* refer to hypothesis *i* and *j* respectively.

However, at a certain point in the procedure the most likely alternative hypothesis should be selected from a number of hypotheses with different dimensions. This is the case when it is necessary to discriminate between, for instance, the *01*- and *02-tests*. Although the related teststatistics χ^2 are again all χ^2 distributed, the number of degrees of freedom differs, i.e. we compare teststatistics of the form of Eq.(5) with teststatistics of the form of Eq.(4). Therefore the largest value does not automatically refer to the most likely alternative hypothesis.

In order to deal with this problem in the present case, a practical solution may be found, comparing the test quotients which are defined as

$$\underline{\boldsymbol{T}}_{q}^{i}\ /\chi_{a}^{2}\ (q_{i},0), \qquad (17)$$

where $\underline{\boldsymbol{T}}_{q}^{i}$ is teststatistics of the form of Eq.(4), refering to the *i*-th alternative hypothesis; $\chi_{a}^{2}\ (q_i,0)$ is a critical value of the central χ^2 distribution with q_i degrees of freedom for a certain choice of a_i.

Here it should be noted that the test quotients may only be used if the significance levels a_i of the tests involved are matched through an equal power.Those test quotients that are less than *1* are not taken into account, since the hypothesis in question is certainly not more likely than the nullhypothesis. For the order test quotients it is assumed that the most likely alternative hypothesis is the one which is rejected strongest, i.e. differs most from *1.* Hence, the most likely hypothesis is the one for which

$$\underline{\boldsymbol{T}}_{q}^{i}\ /\chi_{a}^{2}\ (q_{i},0) > \underline{\boldsymbol{T}}_{q}^{j}\ /\chi_{a}^{2}\ (q_{j},0)\ \forall\ j \neq i. \qquad (18)$$

Results in the Košice-Bankov case

It will be clear that both polynomials with and without a breakpoint may result from the procedure described in the previous paragraph. In this section examples of estimated polynomials in the Košice-Bankov case are presented and discussed. In the following the test quotient belonging to the overall model test is denoted by *OM-test.*

Benchmark No. 8 *(Fig.2):* The behaviour of this benchmark caused the original nullhypothesis to be rejected. The validation of the alternative hypotheses, as specified before, identified an extra parameter for the polynomial to be the most likely alternative hypothesis *(Tab.1).* After the adaptation of this alternative hypothesis as the new nullhypothesis, the overall model test value became 0.9733, which is clearly smaller than its critical value of 1.5479. Hence a quadratic polynomial model was accepted.

Point			QUOTIENTS			BREAKPOINT
No.	*w-test*	*OM-test*	*B-test*	*01-test*	*02-test*	*%*
8	1.8247	0.7791	1.9952	2.1890	1.5206	**0**
23	3.2209	1.2746	3.8987	4.1597	2.8840	**0**
27	3.9802	1.8011	3.9793	4.6063	3.2251	**0**
109	7.6910	2.2386	7.7961	4.3813	5.1463	**100**
110	7.5002	2.1923	7.5004	4.0027	4.8225	**90**
112	6.1754	2.0023	7.0129	4.1992	4.9026	**100**
113	6.0701	1.9077	6.5099	4.0558	4.2159	**70**

Tab.1. Overview of testquo-tients

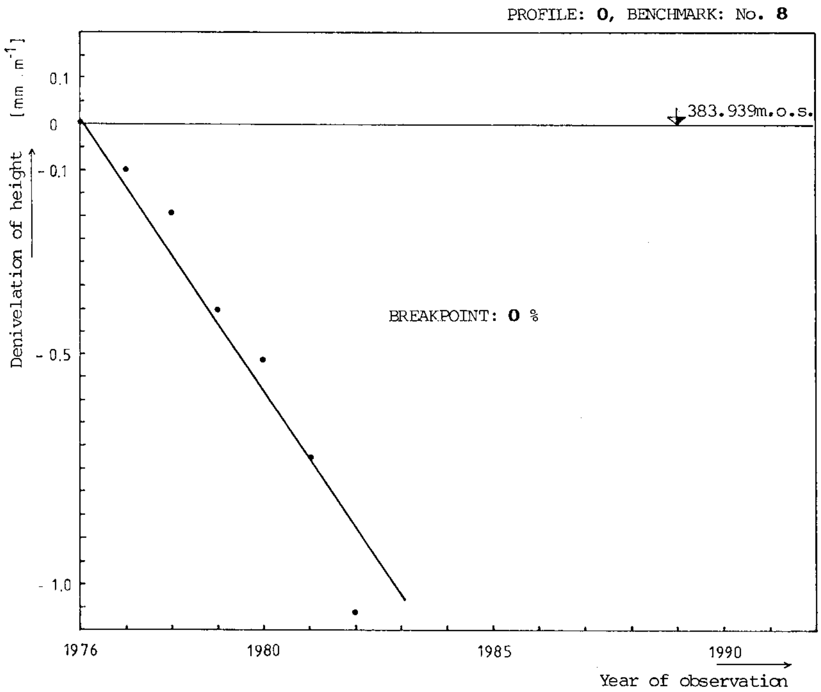


Fig.2 Polynomial model - Benchmark No. 8.

Benchmark No. 23 *(Fig.3):* The behaviour of this benchmark caused the original nullhypothesis to be rejected as well. However, in this case the most likely alternative hypothesis was determined to be
theone which assumed two extra parameters for the polynomial, which is also clear from the test quotients. After the adaptation of this alternative hypothesis, the new nullhypothesis of a cubic polynomial was accepted.

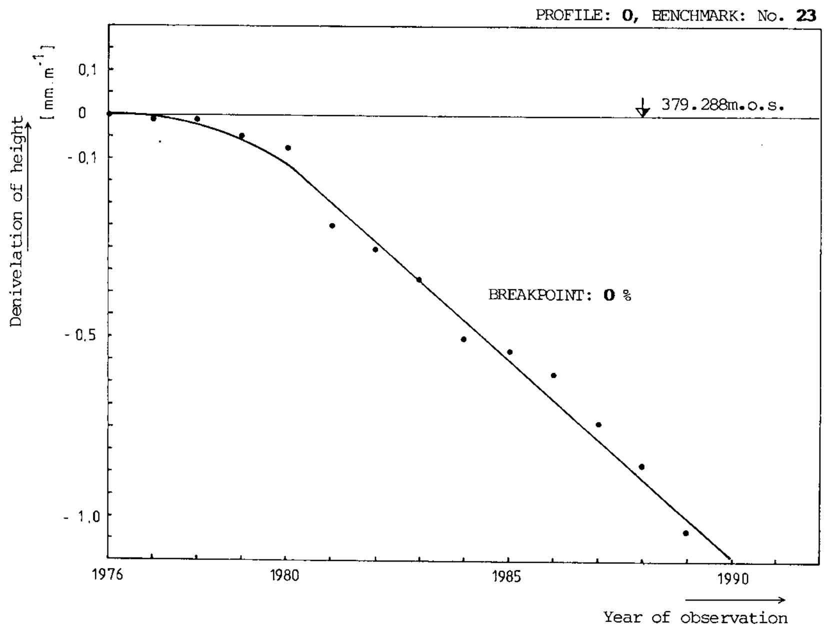


Fig.3. Polynomial model - Benchmark No. 23.

Benchmark No. 27 *(Fig.4):* This benchmark has a imiliar subsidence development of the benchmark No.8. The post likely sufficient alternative hypothesis trans-formed to the nullhypothesis, its overall model test value became 0.7506, which compared to its critical value (1.5479), is clearly accepted for a cubic polynomial model.

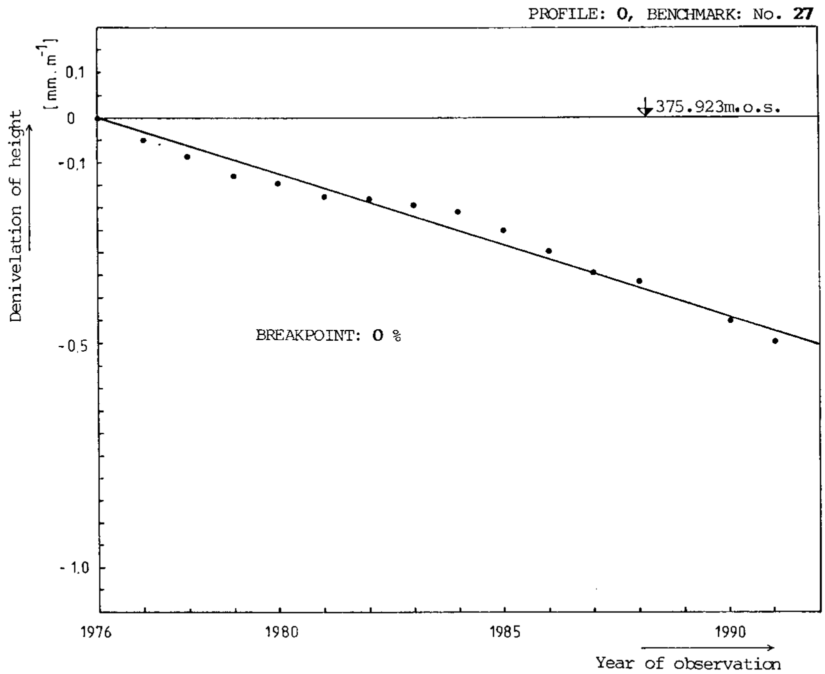


Fig.4. Polynomial model - Benchmark No. 27.

Benchmark No. 109 *(Fig.5):* This benchmark is a tipical example of the breakpoint estimation (see table of testquotients) at the point in time of 1986 (autumn). After adapting the model including a polynomial breakpoint as the nullhypothesis, the order of the polynomial after the breakpoint was determined to be of the order two.

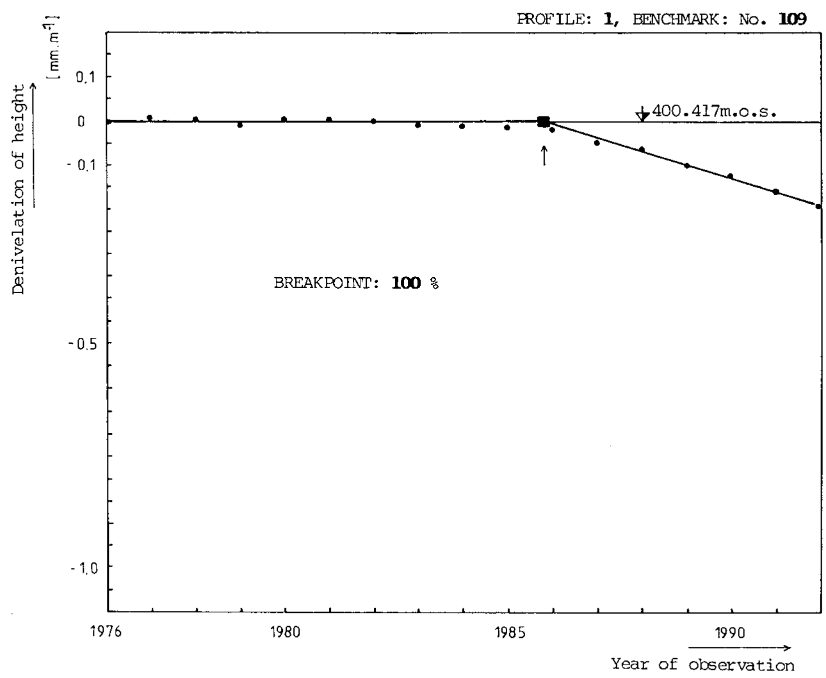


Fig.5. Polynomial model - Benchmark No. 109.

Benchmark No. 110 *(Fig.6):* The subsidence development of this benchmark indicates a breakpoint mostly (autumn 1987). The polynomial before the breakpoint can be accepted for a possibility of an alternative hypothesis partially; however, this fact fully excludes the nullhypothesis.

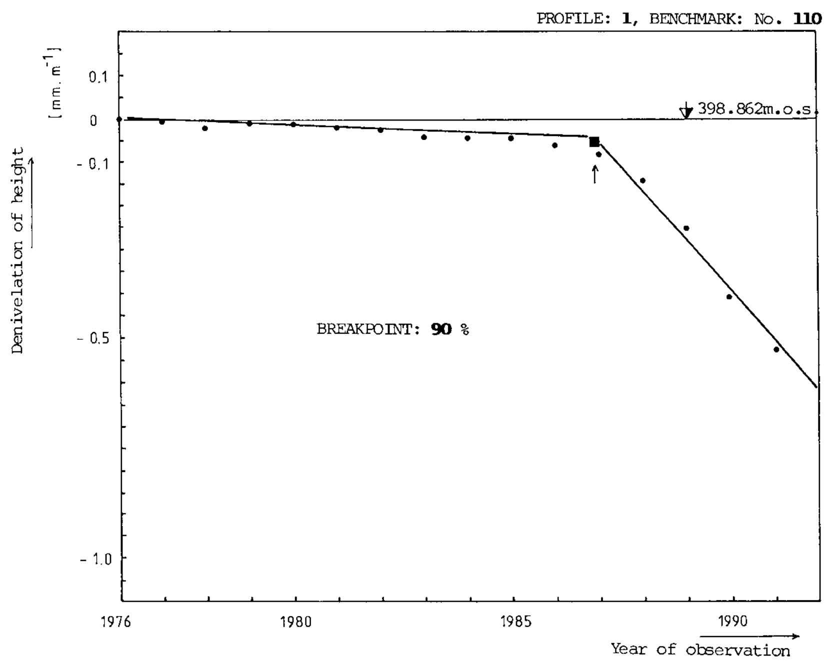


Fig.6. Polynomial model - Benchmark No. 110.

Benchmark No. 112 *(Fig.7):* This benchmark is a clear breakpoint. The nullhypothesis with the polynomial determined to be of order two can be again considered of the nullhypothesis in time of 1986÷1988. And the polynomial is determined to be of the order three after time of 1988.

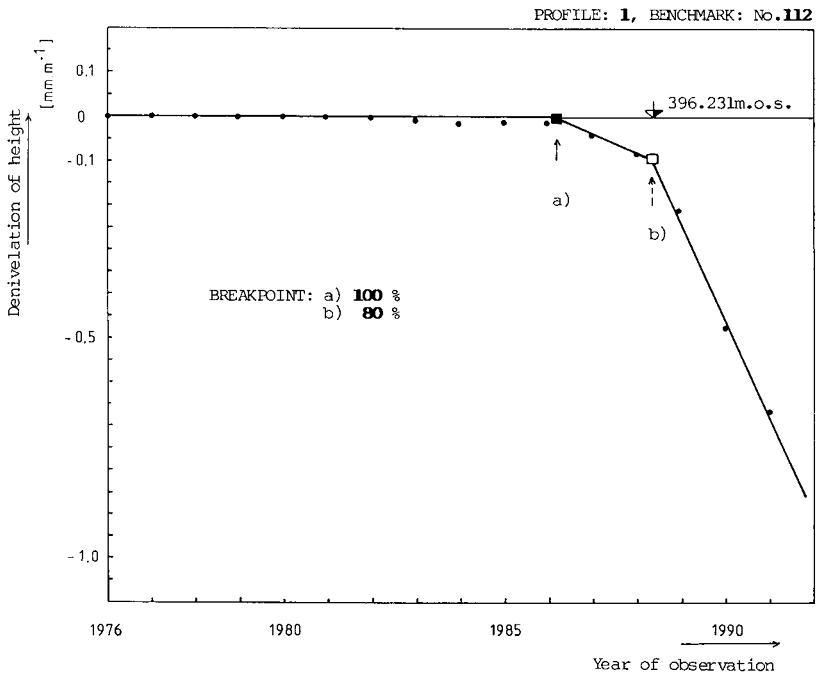


Fig.7. Polynomial model - Benchmark No. 112.

***Benchmark No. 113** (Fig.8):* For this tenchmark the original nullhypothesis, assuming a linear subsidence, was accepted. The overall model teststatistics was determined to be of 0.4681 which is clearly smaller than the critical value of 0.8497. However, the first epoch (spring 1986) was considered as a breakpoint possibility. And the alternative hypothesis after the breakpoint was accepted as the polynomial of order two.

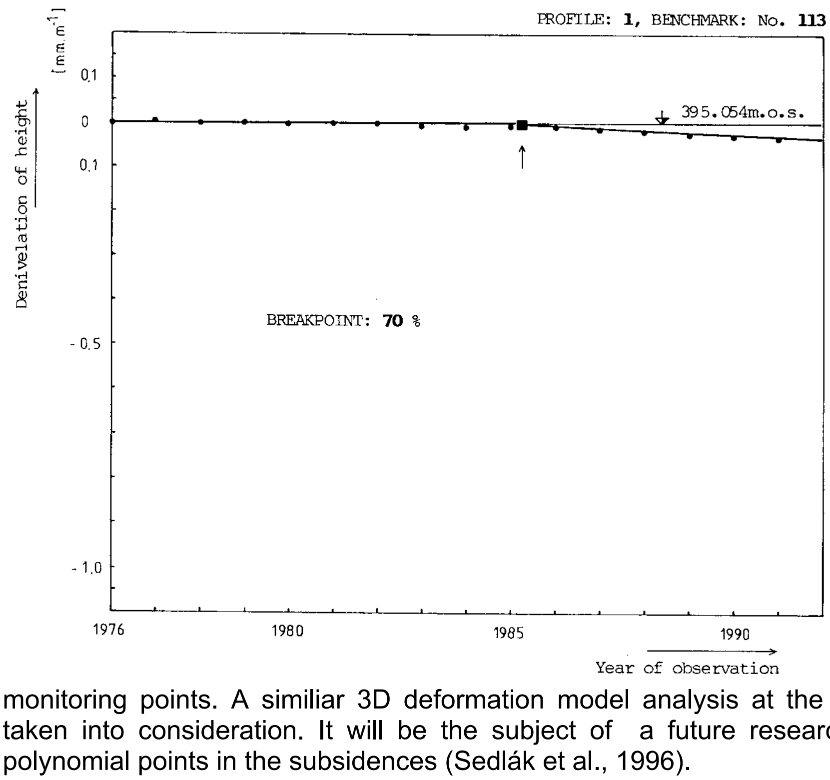


Fig.8. Polynomial model - Benchmark No. 113.

Conclusions

The examples of chosen benchmarks taken from the monitoring Košice-Bankov station can give an overview of some resulting polynomial models, representing trends in the deformation developments over an extracted mine space. The presented theory of the estimating the subsidence polynomial breakpoints follows out from a consideration of 1D deformation model of monitoring points. A similiar 3D deformation model analysis at the polynomial breakpoints can be taken into consideration. It will be the subject of a future research of the estimated differencial polynomial points in the subsidences (Sedlák et al., 1996).

References

Knothe,S.: Prognozowanie Wplywow Exploatacij Górniczej. *Katowice, Slask, 1984, p.160.*

Kunák,L., Okál,M., Dianiška,P., Krepelková,K., & Kuráň,Š.: Influences of exploitation on deformations around of mine works. *Research report, The Technical University of Košice, Košice, 1985, p.151*.

Sedlák,V.(a): Solving the time dependence of subsidence in mined areas. *Transactions of the Technical University of Košice*, ***2****(2), 1992, p.302-307.*

Sedlák,V.(b): Časový faktor pri vytváraní poklesov v handlovskom hnedouhoľnom ložisku. *Uhlí/Rudy,* ***40****, 11/1, 1992, p.398-401*.

Sedlák,V.(a): Metódy predurčenia poklesov na poddolovanom území. *Uhlí/Rudy*,***41****,3/2, 1993, p.90-95.*

Sedlak,V.(b): Modelling subsidence development-experiences from the Slovak Coalfields. In: *The Univ. of Calgary (red.): Seventh International FIG Symposium on Deformation. Measurments and Sixth Canadian Symposium on Mining Surveying, Proceed.,* Banf, *Alberta, 1993, p.354-363*.

Sedlák,V. & Havlice,K.: Časový faktor pri vytváraní poklesov na hnedouhoľných ložiskách a perspektívne možnosti geodetických meraní pre modelovanie poklesových kotlín. *Geodetický a kartografický obzor*, ***39/81****, 3, 1993, p.55-60.*

Sedlák,V.: Methods in subsidences detection mining exposed areas. In: *Kaláb,Z. (red.): New knowledge in seismology and engineering geophysics, Proceed., Institute of Geonics, Academy of Science of Czech Republik, Ostrava, 1994, p.123-131.*

Sedlák,V., Kunák,L., Havlice,K., & Šadera,M.: Moddeling deformations in land subsidence development at the Slovak coalfields. *Survey Ireland, 1996, p.14, (in press).*

Sedlák,V. & Kunák,L.: Mining Influences of Magnesite Exploitation with the Landscape Ecology, In: OMBKE (red.): Meeting of the ISM Commission No.5 - Mining and Environment, Budapest, September 17-19, 1996, p.14-26.