Transformation embedment of local networks into superior net

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Transformačné vloženie lokálnych sietí do nadradenej siete

Transformačné spojenie lokálnych geodetických sietí s nadradenou sieťou (napr. ŠTS v S-JTSK) nedáva jednoznačné výsledky. V závislosti od použitých transformačných faktorov vznikajú na homologických bodoch súradnicové rozpory, ktoré treba dodatočne zohľadniť pri výpočtoch transformovaných súradníc. Jednou z najvhodnejších transformačných spojení je tzv. priama (bezrozporová) transformácia. Pomocou nej sa získajú priamo definitívne hodnoty transformovaných súradníc.

Key words: national grid datum, transformation, heterogeneity.

Introduction

Local geodetic networks (LGN) built in a local datum for geodetic assurance of construction processes of various engineering works should usually be merged in a superior control that is commonly the national grid (NG). It is due to the fact that (plane) coordinates of objects in all engineering projects have to be expressed in the NG datum. Inserting LGN into NG, the datum problem for LGN is really solved because after connecting LGN with NG, new points of LGN should have coordinates from the NG datum.

Embedding LGN in NG can be done in the following ways:

- by tying LGN to NG within their processing (adjusting with constraints) in which the datum reguired is created,
- by LGN transformation, where first LGN is processed as a free network with local datum and then it changed in datum of NG by a convenient transformation,
- by common processing (in datum of NG) all observables obtained from original measurements of NG and from actual ones of LGN.

The first approach, realizable in some variants, is mostly used in cases when any great demands are put on the establishment of LGN with respect to its quality (accuracy, reliability). The third method can not be performed at all if the superior control is a NG. So, for inserting LGN into NG, the second approach is practically the most convenient one, especially if high demands are put on its quality.

However, using a transformation mergence for LGN into NG, one obtaints multisense results, i.e. different final values of transformed coordinates for various reasons.

Problems from this field and a new method for inserting LGN in NG, so called deviationless (direct) transformation, will be discussed in further. For demonstration, embedment of a 2-dimensional (plane) LGN in the Slovak NG with datum S-JTSK is treated. Let us assume, this LGN consists of H_i, i=1,...,m datum points coordinated in S-JTSK and of U_j, j=1,...,u new points which coordinates has to be determined in S-JTSK.

Problems of transformation inserting LGN

As results of the transformation of coordinates C_{Hi}^{L} and C_{Uj}^{L} from the local datum, i.e. from LGN, one obtains the transformated coordinates C_{Hi}^{Jt}, C_{Uj}^{Jt}. As known, coordinate values C_{Hi}^{Jt} and the

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available ones C^{L}_{Hi} of points H_i (they are homological points of the transformation) will not be identical and their differences, called coordinate discrepancies (deviations) $dC_{Hi} = C^{J}_{Hi} - C^{Jt}_{Hi}$ are indicators of the locally NG deformations in surroundings of the points H_i. According to the displacement of these deformations in NG, discrepancies dC_{Hi} can reach average values up to 30-40 mm in the mostly local regions.

These discrepancies in the point field of LGN (developed by points H_i of NG with unchangeable coordinates C^{J}_{Hi} and by points U_j with transformed ones C^{Jt}_{Uj}) indicate the actual lesser or greater heterogeneity in LGN. This LGN heterogeneity is a regular but undesirable phenomenon in network transformations that can bring considerable problems in survey realization required with high accuracy in the relevant region (laying out, measurements of movements and structural deformations, etc.).

It is therefore an essential question in the geodetic praxis, how to modify (to improve) the LGN heterogeneity, i.e. to redistribute the primary point (coordinate) heterogeneity to a more equable ones in this net using various convenient methods. The present methods can be divided in two groups:

- methods based on additional changes of the transformed coordinates C^{Jt}_{Uj} to their final values C^{Jtf}_{Uj} (posttransformative solution),
- methods based on direct determination of the final values C^{Jtf}_{Uj} within the transformation procedure (direct, deviationless solution).

Both groups take into consideration the coordinate discrepancies and the desirable level of the LGN unhomogeneity is reached by suitable manipulations or creation of C^{Jt}_{Uj} only, meanwhile the coordinates C^{J}_{Hi} remain constant.

We have to be but conscious that the methods of both solution groups for getting final coordinates give different values. That means, the transformation embedment of LGN into NG with simultaneous heterogeneity reduction, as a mathematical-logistical procedure give multisense results. This effect is caused by the following transformation axioms being valid in the accomplishment of all the solution methods:

- each kind of transformation (similarity, affine, conform, etc.) gives diverse values of the transformated coordinates C^{Jt}_{Uj},
- various number of homological points, their displacement in LGN and the configuration of points H_i and U_j as well, cause also different values of C^{Jt}_{Uj},
- every method of heterogeneity reduction leads to diverse final values C^{Jtf}_{Uj} too.

Methods of the first group, i.e. approaches of indirect (posttransformative) improving the transformed coordinates (methods of weighted corrections, finite elements methods, Jung's method, collocation procedures, multiquadratic interpolation and others) are well known in the transformation praxis. Fewer methods are used and are known at all from the second group even if these approaches offer interessant and full applicable transformation possibilities for the direct solutions giving immediately final values of the transformed coordinates. A new method from this area will be in the forthcoming outlined with its concept and properties.

LGN embedment by direct transformation

In direct transformation based on suitable estimating (adjusting) process, the transformation parameters are simultaneously created with such final values of the transformed coordinates which improves the LGN heterogeneity. The transformation model of this solution is given by transformation eqations (in this case for the similarity transformation) using coordinate differences among the points H_i and U_j (Sütti, 1996; Sütti, 1997; Sütti et al., 1997).

$$X^{J}_{Hi} - X^{J}_{Uj} = a(X^{L}_{Hi} - X^{L}_{Uj}) - b(Y^{L}_{Hi} - Y^{L}_{Uj}),$$
$$Y^{J}_{Hi} - Y^{J}_{Uj} = a(Y^{L}_{Hi} - Y^{L}_{Uj}) + b(X^{L}_{Hi} - X^{L}_{Uj}). \qquad \text{Eq.1}$$

The final coordinates C_{Uj}^{Jtf} wanted from this estimating procedure are formed on base of the estimation concept used that can be a LS (least squares) procedure

$$(\sum_{1}^{n} v_i^2)^{1/2} = \min \qquad \text{Eq.2}$$

or a Minimax (minimization of the maximal residual)

$$\lim_{p\to\infty}(\sum |v_i|^p)^{1/p} = \min, (\max|v_i| = \min) \qquad \text{Eq.3}$$

one (Kampmann, 1992). The last concept has better properties for inserting a LGN in NG as LS. As a quality measure of LGN mergence in NG the confidence circles (degenerated confidence ellipses for this kind of transformation) of points with transformed coordinates can be used. As more equable will be the size of these circles, the better, more acceptable is the remaining heterogeneity in LGN.

Other effective measures for the inserting quality may be applied from the comparison of the LGN elements measured (distances mostly) with their values computed from the coordinates C_{Hi}^{J} and C_{Hi}^{Jtf}.

The first empirical experiences and analysis of applying the direct transformation for LGN heterogeneity improvement (Sütti et al., 1997), in comparison with the other methods, point at unambiguous priorites of the direct solution both in case of similarity and affine transformation or both LS and Minimax estimator are used. In the direct determination of C_{Uj}^{Jtf}, the estimating machanism creates the adjusted final coordinates directly in contrast of standard transformation, where estimation gives only estimated transformation parameters. In this solution, the primary LGN heterogeneity should be favourably modified. The whole solution procedure can be arranged in such a manner that coordinates C_{Uj}^{Jtf} will only be determined without transformation parameters. The direct transformation is applicable in any measuring technology in LGN. Using GPS surveys, the coordinates $C_{Uj}^{L} = C_{Uj}^{Jt}$ received by the "main" transformation from WGS84 into S-JTSK are further transformed for points U_j by this direct method to values C_{Uj}^{Jtf}.

Conclusion

Concept, algorithm and variants of the direct transformation can be applied for 1D, 2D and 3D nets. The problem of the geodetic transformation embedding a LGN in NG can be considered as a needed coordinates connection of two incompatible networks. In such a junction a compromise has to be made in that the determination quality of points U_j is diminished on the one hand but an improved final coordinates heterogeneity in LGN can be archieved on the other hand. Connecting two nets by transformation, any approach from the direct or indirect method can be declared as a best suitable and universal one in general conditions and situations. In case of more important LGN, utilization of two transformation methods and convenient testing procedures for the quality of junction is the most suitable solution.

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