Mathematical model of thermal aggregates

Imrich Pokorný [1] a *Karol Kostúr* [1]

Matematický model tepelných agregátov

Tepelné agregáty môžme charakterizovať jako priemyselné pece s veľkou spotrebou energie. Jednou z možných ciest zníženia spotreby energie je optimalizácia a priebežné riadenie priemyselných tepelných agregátov pomocou simulačných modelov. Východiskom pre tvorbu simulačných modelov je matematický model. Matematické modelovanie tepelných procesov je založené na riešení parciálnych diferenciálnych rovníc a nelineárnych algebraických rovníc popisujúcich základné procesy prenosu tepelnej energie.

V príspevku je popísaná základná metodika tvorby matematického modelu zónovou metódou vrátane efektívneho riešenia. Prínosom príspevku je rozpracovanie analytického postupu riešenia nelineárneho systému bilančných rovníc, ktorého použitie značne urýchľuje priebeh simulácie v porovnaní s numerickým riešením.

Introduction

Industrial furnaces belong to the group of the biggest appliances of energy and their heating regime influence of quality of metallurgical semi-products. For this reason it is necessary to optimize the design of heat aggregates. One of effective ways to design a new heat aggregate or to reconstruct the old heat aggregate is simulation (Dvořáček et al., 1990). The creation of simulation models requires high-professional knowledge of the furnace's heattechnics and considerably programmable capacity. For the creation of simulation models is needed typically long time, which is often in contradiction with the requirements of users. This contradiction leads to abandoning traditional simulation models as effective tool to construct heat aggregates.

Mathematical models of processes play an important role more and more in our time and they have a very important place as a software of Automated System of Coíntrol of Technological Processes (ASC TP). We can realise the control of processes more effectively because of the predicting property of the model. The purpose was to obtain the mathematical model of heating of batches in such a way that it makes possible optimisation of heating.

As a fuel we can use a mixed gas, which is composite by earth gas, coke gas and gas from blast-furnace. Input of fuel is shown on the Fig. 1 as a vertical arrow. The worked area of furnace we divided on the smaller part - modules. There are usually two modules together - top and bottom. We suppose the homogenity of processes in module. Then e.g. the temperature of fire and walls of furnace is constant in module. In the module each batch is described by one-dimensional thermal field, if it is possible.
⇐

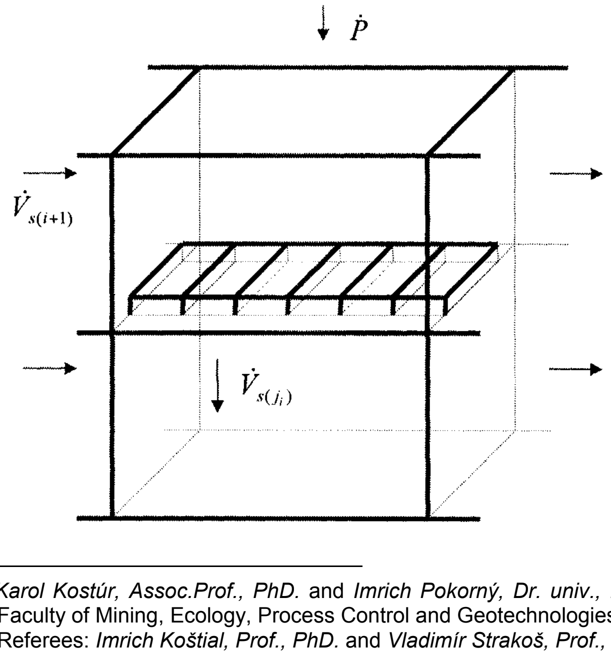

Fig.1. Scheme of pairs of modules.

Model of combustion

By stechiometric calculations we can define structure and quantity of fuel in the modules. Volume flow of combustion products in the i-th module can be given by following

[1] *Karol Kostúr, Assoc.Prof., PhD.* and *Imrich Pokorný, Dr. univ., PhD.* Department of Management and Control Engineering, Faculty of Mining, Ecology, Process Control and Geotechnologies, 042 00 Košice, Boženy Němcovej 3, Slovakia
(Referees: *Imrich Koštial, Prof., PhD.* and *Vladimír Strakoš, Prof., PhD.*, accepted in revized form April 29, 1998)

equations:

$$\dot{V}_{s(i)} = m.\dot{P}_{(i)} + k_{1(i+1)}\dot{V}_{s(i+1)} + k_{2(i+1)}\dot{V}_{s(j_i)} \qquad i = 1,\ldots,np \tag{1}$$

where m - quantity of fuel,
$\dot{P}$ - volume flow of fuel [$m^3.s^{-1}$]
$\dot{V}_s$ - volume of combustion products
$k_{1(i)}, k_{2(i)}$ - coefficients modifications of quantity (may be 0)
j_i - order number of corresponding top module to i-th module
np - number of modules.

Detailedly the streaming is described in (Strakoš et al., 1997).

Model of heat radiation and convection

We divided i-th module to n(i) zones. Namely b(i) batch surfaces , c(i) wall surfaces, f(i) fictitious zones and one volume zone (n(i)=b(i)+c(i)+f(i)+1). In each zone we solve the heat transmission by radiation and by convection by the following expressions.

Resulting flow by radiation in j-th zone of i-th module is (Dorčák, 1987)

$$Q_{s(i,j)} = \frac{\sum_{l=1}^{n(i)} QV_{(i.l)} - QV_{(i,j)}.Z_{(i)}}{\sum_{l=1}^{n(i)} A_{(i.l)} - A_{(i,j)}.Z_{(i)}}.A_{(i,j)} - QV_{(i,j)} \qquad [\,W\,] \tag{2}$$

where

$$A_{(i,j)} = \begin{matrix} S_{(i,j)}.\dot{\varepsilon}_{(i.j)} & pre \quad j = 1,\ldots,n(i)-1 \\ 4.V_{(i)}.K_{(i)} & pre \quad j = n(i) \end{matrix} \qquad [\,m^2\,]$$

factor of correction is

$$Z_{(i)} = \frac{\sum_{j=1}^{n(i)} A_{(i,j)}}{\sum_{j=1}^{n(i)-1} S_{(i,j)}}.Q_{(i)}$$

proper radiation flow is

$$QV_{(i,j)} = \sigma.T^4_{(i,j)}.A_{(i,j)} \qquad [\,W\,]$$

and proper convection flow is

$$Q_{k(i,j)} = \alpha.\left(T_{(i,k)} - T_{(i,j)}\right).S_{(i,j)} \tag{3}$$

for $i = 1,\ldots,np, \quad j = 1,\ldots,n(i)-1, \quad k = n(i)$

where i - index of module, j - index of zone
(j=1,...,b(i) - batches, j=b(i)+1,...,b(i)+c(i) - walls,
j=b(i)+c(i)+1,...,b(i)+c(i)+f(i) - fictive zones, j=n(i) - fuel)

S - area of zone $\left[m^2\right]$

V - volume of zone $\left[m^3\right]$

K - absorptive coefficient $\left[m^{-1}\right]$

ε - radiation

O - adaptive coefficient

σ - Stefan-Boltzman's constant $\left[W.m^{-2}.K^{-4}\right]$

T - thermodynamic temperature $\left[{}^\circ K\right]$

α - coefficient of heat transfer by convection $\left[W.m^{-2}.K^{-1}\right]$.

Model heat conduction

We can obtain thermal field of heat batches for j=1,...,b(i) by solving of system of Fourier equation of heat conduction

$$\frac{\partial(t_{(i,k)}.c_{(i,k)}.\rho_{(i,k)})}{\partial\tau} = \frac{\partial\left(\lambda_{(i,k)}\dfrac{\partial t_{(i,k)}}{\partial x_{(i,k)}}\right)}{\partial x_{(i,k)}} \qquad \text{for } i = 1,\dots,np, k = 1,\dots,b(i) \tag{4}$$

in the case of combined boundary conditions

$$\frac{Q_{s(i,k)} + Q_{k(i,k)}}{S_{(i,k)}} = -\lambda_{(i,k)}\cdot\frac{\partial t_{(i,k)}}{\partial x_{(i,k)}}$$

for

$$x_{(i,k)} = 0 \quad \text{in top modules,} \quad k = 1,\dots,b(i)$$

$$x_{(i,k)} = h_{(i,k)} \quad \text{in bottom modules,} \quad k = 1,\dots,b(i)$$

respectively

$-\lambda_{(i,k)}\cdot\dfrac{\partial t_{(i,k)}}{\partial x_{(i,k)}} = 0$ for $x_{(i,k)} = h_{(i,k)}$, if batch is in contact by real zone about some temperature

Thermal field of walls we obtain as a solution of stationer equation of heat transfer in stabilized heat regime

$$\frac{\partial t_{(i,k)}}{\partial\tau} = 0 \qquad \text{for } i = 1,\dots,np,\ k = b(i)+1,\dots,b(i)+c(i) \tag{5}$$

in the case of combined boundary conditions

$$\frac{Q_{s(i,k)} + Q_{k(i,k)}}{S_{(i,k)}} = -\lambda_{(i,k)}\cdot\frac{\partial t_{(i,k)}}{\partial x_{(i,k)}} \qquad \text{for } x_{(i,k)} = 0 \quad i = 1,\dots,np, k = b(i)+1,\dots,b(i)+c(i)$$

$$\alpha_o\left(t_{(i,k)} - t_o\right) = -\lambda_{(i,k)}\cdot\frac{\partial t_{(i,k)}}{\partial x_{(i,k)}} \qquad \text{for } x_{(i,k)} = h_{(i,k)} \quad i = 1,\dots,np, k = b(i)+1,\dots,b(i)+c(i)$$

where c - specific heat capacity of the batch $\left[J.kg^{-1}.K^{-1}\right]$

λ - mean heat conductivity of the batch $\left[W.m^{-1}.K^{-1}\right]$

ρ - density $\left[kg.m^{-3}\right]$

α_o - coefficient of heat transfer from external surface into surroudings $\left[W.m^{-2}.K^{-1}\right]$

t - temperature in degree of Celsius $\left[{}^\circ C\right]$

t_o - temperature of environs $\left[{}^\circ C\right]$

Balance of energy

From law of maintenance of energy follows the equation of balance of energy

$$\dot{P}_{(i)}.H+\dot{V}_{p(i)}.c_{p(i)}.t_{p(i)}+\dot{V}_{v(i)}.c_{v(i)}.t_{v(i)}+\dot{V}_{s(i+1)}.c_{(i+1)}.t_{(i+1,n(i+1))}-\dot{V}_{s(i)}.c_{(i)}.t_{(i,n(i))}-$$
$$-\sum_{k=1}^{b(i)+c(i)}\left(Q_{s(i,k)}+Q_{k(i,k)}\right)=0 \qquad \text{for } i=1,\ldots,np \tag{6}$$

where V_p, V_v - volume flow of fuel, air

c, c_p, c_v - specific heat capacity of combustion products, fuel, and combustion air.

We obtain after modifications from the equation of balance of energy a fourth order equation with the unknown variable $x=t_{(i,n(i))}$ (a temperature) :

$$a.x^4+d.x+e=0 \tag{7}$$

resp.

$$a.x^4+b.x^3+c.x^2+d.x+e=0 \tag{8}$$

These coefficients in a real situations are:
a - negative and its place value is 10^{-6}
b - negative and its place value is 10^{-6}, resp. 0,
c - positive and its place value is 10^{-2}, resp. 0,
d - negative and its place value is 10^{3},
e - positive and its place value is 10^{7}.

We can solve these type of equation by different ways (by numerical methods see e.g. (Lanczos, 1961), (Pavluš et al., 1991) and (Rektorys, 1988), or by exact ways see e.g. (Kuros, 1971) and (Rektorys, 1988)). For instance by iteration, but in a general case we can not guarantee convergence of this process. The authors of (Pavluš et al., 1991, p. 46) show a short analysis of such situation. The exact algebraic solution by radicals by comment 1 from (Rektorys, 1988, p. 69) is not very good for the numeric solution of the equation. Methods, which are shown in the following literature (Kuros, 1971, pp. 239-240 and Lanczos, 1961, pp. 36-37) we can not use directly because the coefficients, which we obtain during the solution are very big and hence these coefficients have not legal representation of value on computers. This is because of the big difference between the place value of coefficients a a e. After correction of our coefficients we can use methods from (Kuros, 1971 pp. 239-240).

We divide the equation (8) by coefficient a $(a\neq 0)$ and we make the following substitution

$$y=x+\frac{b}{4a} .$$

After the substitution we have:

$$y^4+P.y^2+Q.y+R=0 \ , \tag{9}$$

resp.

$$\left(y^2+\frac{P}{2}+s\right)^2-\left[2.s.y^2-Q.y+\left(s^2+P.s-R+\frac{P^2}{4}\right)\right]=0. \tag{10}$$

The basic idea is to obtain full quadratic expression in the square brackets by choosing the s . It will be only the case when s will be a solution of following equation:

$$Q^2 - 4.2.s.\left(s^2 + P.s - R + \frac{P^2}{4}\right) = 0\,. \tag{11}$$

Let s_o be a solution of the equation (11) , then we can rewrite the equation (10) .

$$\left(y^2 + \frac{P}{2} + s_o\right)^2 - 2.s_o.\left(y - \frac{Q}{4.s_o}\right)^2 = 0\,. \tag{12}$$

Because our coefficients are specific (comment 2) we have $s_o > 0$ and hence from the equation (12) we obtain the following two quadratic equations

$$y^2 - \sqrt{2.s_o}\,.y + \left(\frac{P}{2} + s_o + \frac{Q}{2\sqrt{2.s_o}}\right) = 0\,,$$

which has only two complex solutions and

$$y^2 + \sqrt{2.s_o}\,.y + \left(\frac{P}{2} + s_o - \frac{Q}{2\sqrt{2.s_o}}\right) = 0\,,$$

which has one positive and one negative equation root. Our solution will be this positive solution:

$$y = \frac{-\sqrt{2.s_o} + \sqrt{\dfrac{2.Q}{\sqrt{2.s_o}} - 2.s_o - 2.P}}{2}$$

and hence $x = y - \dfrac{b}{4.a}$ is our temperature.

Now we analyse the solution of the equation (11) . After modification we have

$$s^3 + P.s^2 + \left(\frac{P^2}{4} - R\right).s - \frac{Q^2}{8} = 0\,. \tag{13}$$

If we can find solution of this equation by computer we have a problem especially in this part of solution because during the solution there are coefficients where their place order are 10^{39} . And hence we normalise our equation by substitution $s = \gamma.E$, where E is auxiliary coefficient.

After that substitution and divided the equation by E we obtain

$$\gamma^3 + \frac{P}{E}.\gamma^2 + \frac{\dfrac{P^2}{4} - R}{E^2}.\gamma - \frac{Q}{8.E^2} = 0\,. \tag{14}$$

We choose E such that the maximum of absolute value of coefficients of equation (14) has the place value 1.

We can put $E = \max\left\{|P|, \sqrt{|R|}, \sqrt[3]{Q^2}\right\}$.

After the substitution $\gamma = \delta - \frac{P}{3.E}$ the equation (14) takes the following form

$$\delta^3 + P_1.\delta + Q_1 = 0 . \qquad (15)$$

Based on the specifics of the coefficients of the equations (7) resp. (8) the expression $DI = \frac{Q_1^2}{4} + \frac{P_1^3}{27}$ and the coicient P_1 are positive and hence only one real root of the equation (15) is

$$\delta = \sqrt[3]{\sqrt{DI} - \frac{Q_1}{2}} - \sqrt[3]{\sqrt{DI} + \frac{Q_1}{2}}$$

and

$$s_o = \left(\delta - \frac{P}{3.E}\right).E .$$

There are 3 situations in generally in the solution of the equation (15) : *DI=0, DI<0* and *DI>0.* In each case there will be a desirable real root of the equation (15). In our case *DI>0* and hence we do not analyse other situations.

The solution of the model was realised for pushing furnace. (Kostúr, K.& Pokorný, I., 1990).

Conclusion

In spite of some simplifications with our model we can simulate the process of heating for the different technologic properties (input power, performance, structure of fuel, ...). We can use this model great part in the following two basic areas:

1. For the finding the optimal heating from point of view minimise the consumption of fuel on conditions to have some temperature of batches and difference of temperature of material after finishing the heating.
2. For the projection, or reconstruction of thermal aggregate. For the simulation of the different situations of constructions we can choose the best variant of them.

References

Dorčák, Ľ., Koštial, I., & Benková, M.: Nové algoritmy pre prenos energie žiarením a ich využitie pri riadení tepelných procesov. (New algorithms for transmission of energy by radiation and its utilize at heat process control). *Hutnícke Aktuality, 2/1987, s. 45, (in Slovak).*

Dvořáček, J. & Šnapka, P.: Metódy vedeckého řízení, skriptá (Methods of science management. Book for postgradual study). *Ostrava, 1997, (in Czech).*

Kostúr, K., & Pokorný, I.: Matematický model ohrevu brám v narážacej peci. (Mathematical model of heating slabs in pushing furnace). *In : Automation in Control'90, DT ZVTS Košice, 1990, s.101-108.*

Koštial, I. et al.: Modely pre riadenie narážacích pecí, čiastková správa. (Models of controls of furnace. Partial report). *ÚVT VŠT and SAV, Košice, 1989, (in Slovak).*

Kuros, A.G.: Kurz z vyššej algebry (Course of high algebra). *Nauka, Moskva, 1971, s. 431, (in Russian).*

Lanczos, K.: Aplikovaná analýza (Applied analysis). GIFML, Moskva, 1961, s. 524, (in Russian).

Pavluš, M., Török, Cs. & Purcz, P.: Programovanie v Pascale a úvod do numerickej matematiky (Programming in Pascal and introduction to the numerical mathematics). *ES TU in Košice, 1991, s. 159, (in Slovak).*

Rektorys, K.: Prehľad užité matematiky (Review of applied mathematics). *SNTL, Praha, 1988, s. 1137, (in Czech).*

Strakoš, V. & Kebo, V.: Calculation of shafts ventilation balance during their closing. *In: Topical Mining Problems in China and Czech Republic. TU Ostrava, 1997.*