Some Problems Related to Correction of Parts of Conical Anchorages

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Niektoré problémy súvisiace s opravou časti kužeľových kotiev

Článok prezentuje problematiku opravy tvaru zvieracích a kotviacich prvkov pri použití simulácie spájania v systéme objímka – čeľusť – reťaz, zameranom na silové parametre. Články založené na teórii pružnosti, ktoré by posudzovali deformáciu vo vnútri kontaktnej zóny, nie sú známe. Na simuláciu spájania bola použitá Airy-ho napäťová funkcia. Získané výsledky umožňujú vyvinúť série typov svoriek a kotiev, ktoré sa vyrovnajú konštrukciam vedúcich európskych spoločností.

Key words: shape correction, grips and anchorage elements, mating simulation, Airy stress function.

Introduction

The conical clamps and anchorages are important elements of the prestressing system of the pre-stressed concrete constructions and anchoraged strengthenings of mining excavations. Their working and strength parameters determine the quality of constructions.

The paper presents the problem of the shape correction for anchorage elements based on the simulation of mating within the sleeve-jaws-string system oriented toward the strength parameters. The obtained results enable us to develop a series of types for clamps and anchorages, possessing design parameters of leading European companies.

Theoretical Background and Model Assumptions

After a careful inspection of literature regarding the problem of load distribution inside the contact zone for mating of a divided cone with a sleeve leads to the conclusion that these papers are rather practical applications of the strength calculation type (Franeck et al., 1993; Jurkiewicz et al., 1993; Mirowski, 1996). There are no papers based on the theory of elasticity, considering the deformation conditions inside the contact zone. In this paper the partly solved problem of load distribution inside the contact zone for the pin-hole joint and any contact angle and for the same and different materials (Fig.1) was applied. The Airy stress function (Franeck et al., 1993; Jurkiewicz et al., 1993) was also used. The sleeve-jaws-string system was divided into elementary systems fulfilling the above assumptions. An additional, significant simplification was related to a string which was treated as a even bar of circular cross section. In many applications, in real systems, in addition to a bar also a rope (rope strand) or a string made of composite materials are considered.

Contact in the Sleeve-Jaw System

As was mentioned above, the problem of load distribution inside the contact zone of the pinhole joint within a plate (Fig.1) was applied for description of mating between the cone clamp and string. This problem can be presented in a simplified way (Jurkiewicz et al., 1993). This enables us to use denotations and equations necessary for the further analysis.

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Fig.1. Diagram of pin joint.

The limiting contact angle for this case can be determined from the following formula:

$$\alpha_o = \frac{\pi}{2} tgh \left[k \frac{m}{m-1} \sqrt{\frac{P}{E_z \varepsilon t}} \right], \tag{1}$$

where: $k = f(t, B, \alpha, \varepsilon)$,

$$m = \frac{B}{D} = const - shape coefficient,$$

$$\varepsilon = D - d - radial clearance,$$

$$q(x) = const - loads along the contact line,$$

$$E_z - equivalent Young's modulus,$$

while the load distribution at the pin-circular hole in the plate can be found from the formula 2:

$$p_{(\alpha)} = p_o \sqrt{1 - (A)^2}$$
, (2)

where: $A = \frac{\alpha}{\alpha_o}$.

For known angle $\ \alpha_{o}$ a relationship between α and P can be written as:

$$P = \pi t \frac{d}{2} p_o \int_{-\alpha_o}^{\alpha_o} \sqrt{1 - (A)^2} \cos \alpha \, d\alpha \,.$$
(3)

The relationship (1) can be then converted into the form:

$$P = \varepsilon \ tE_z \left[\frac{1}{k} \frac{m-1}{m} \operatorname{ar} \operatorname{ctgh}(C) \right]^2, \tag{4}$$

where: $C = \frac{2}{\pi} \alpha_{o}$,

for elementary t = dx it follows from formula (4) that:

 $P_i = q_y(x)dx , (5)$

hence

$$q_{y}(x) = \varepsilon E_{z} \left[\frac{1}{k} \frac{m-1}{m} \operatorname{arctgh}(C) \right]^{2}.$$
 (6)

Similarly we can consider the model of mating elements for the self-locking cone clamps and a string (Fig.2).

By introducing the term of continuous load $q_y(x)$ we can write:

$$P = \int_{0}^{l_{c}} q_{y}(x) = \varepsilon \ E_{z} \int_{0}^{l_{c}} \left[\frac{1}{k} \frac{m(x) - 1}{m(x)} \operatorname{ar} \operatorname{ctgh}(C) \right]^{2} dx .$$
(7)



Fig.2. Force distribution between a jaw and a sleeve of the self-locking cone clamp.

This equation enables us among other things to determine the load distribution as a function of geometric and material parameters, which can be presented in the form:

$$q_{y}(x) = \varepsilon E_{z} \left[\frac{1}{k} \frac{m(x) - 1}{m(x)} \operatorname{ar} \operatorname{ctgh}(C) \right]^{2}.$$
(8)

For example, for the following clamp parameters:

Q=200 kN,
$$\alpha_o = \frac{\pi}{3}$$
, D₁=45 mm, D₂=34 mm,
 $\varphi = 5^\circ$, E=2,11*10¹¹ Pa, I_c=48 mm. (9)

According to the formula (7) for the above parameters the value of ϵ = 0.213 mm was obtained. This parameter is of crucial importance for the continuous load distribution. If this value is constant, invariable along the entire contact length l_c , i.e. ϵ = const we say about the zero shape correction (Fig.3) - curve a1. The continuous load distribution corresponding to this correction is presented in Fig.3 - curve a2. The relation (8) after a conversion can be rewritten in the following simplified form:

$$q_{y}(x) = \varepsilon \, \mathrm{E}_{z} \Big[k_{1} \Big(k_{2} + k_{3} x \Big) \Big]^{2} \,.$$
 (10)

In order to meet the condition $q_y(x)$ for the entire continuous load distribution it follows from the above equation that $\varepsilon = \varepsilon(x)$, as is illustrated in Fig.3 - curve b1. It should be noted that it will be very difficult to achieve such a type of shape correction in practice.

The function can be linearly approximated with:

ε

$$(x) = \varepsilon_o - ax \,. \tag{11}$$

After the conversions, for geometric parameters previously given, the coefficients of equation (11) can be determined and the correction line takes a form:

$$\varepsilon(x) = 0.39 - 6.25 \cdot 10^{-3} \cdot x$$
 [mm]. (12)

This solution is presented in Fig.3 - curve c1 and the corresponding continuous load distribution in shown in Fig.3 - curve c2.



Fig.3. Correction curves and corresponding load distributions $q_v(x)$.

Taking equations (2), (3) and (7) into account for the case under consideration, the formula describing the radial clearance between a jaw and a sleeve of the self-locking conical clamp takes a form:

$$p_{(\alpha,x)} = \frac{2}{\pi_2 D(x)} E_z \varepsilon \left(x\right) \frac{\left[k_1 \left(k_2 + k_3 x\right)\right]^2}{\int\limits_{-\alpha}^{\alpha_0} \sqrt{1 - (A)^2} \cos \alpha \ d\alpha} \sqrt{1 - (A)^2} .$$
(13)

Similarly we can describe the force distribution on an elementary segment of the jaw-sleeve contact.





As results from the force distribution presented in Fig. 4, the axial loads $p_x(x,\alpha)$ can be expressed as:

$$p_x(x,\alpha) = p(x,\alpha)tg(\varphi + \rho).$$
(14)

The selected examples of similar calculations carried out for the jaw-sleeve contact zone in the cases of $\varepsilon = const$ and $\varepsilon = \varepsilon_o - ax$ are presented in Fig. 5 and Fig.6.

The formulae presented above enables a spatial analysis of the load distribution inside the jaw-sleeve contact zone. Thus, it is possible to achieve an optimal load distribution by forming the contact surfaces of these clamp elements.

The clamps and anchorages constructed on the basis of calculations presented in this paper were positively assessed by the Research Institute for Roads and Bridges. The results of fatigue approving tests performed for anchorages designed for ropes of \emptyset 15.5 mm diameter are shown in Fig.7. Such anchorages were applied for stressing the Grunwald Bridge in Cracow.



Fig.5. Radial load distribution for the zero correction Fig.6. Radial load distribution for the non-zero correction $\varepsilon = c_{o} - a \cdot x \cdot$

Conclusions

- 1. The simulation of load distribution inside the sleeve-jaw contact zone enables the determination of the effect of type of correction on its value and spatial distribution.
- 2. The analysis of clamps of various types of correction indicates that the application of correction in the form $\varepsilon = \varepsilon_o ax$ leads to good effects. This is important because such corrections can be made even during the clamp manufacturing process.
- 3. Theoretical investigations are confirmed by results of strength examinations. For example, for clamps of zero correction (ϵ =const), the propagation of damage always starts from the point x=l_c, in accordance with results of simulations. The remaining cases of non-zero correction are also reflected by results of strength testing.
- 4. For example, the strength tests for anchorages Ø 15.5 mm have been completed at a number of cycles reaching 3,4 million without any visible damages. The similar anchorages made by leading European companies withstand 2 million of cycles (according to catalogue data).



Fig.7. Results of approving tests for anchorages of the rope of $\mathscr{O}15.5$ mm diameter.

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