Accuracy and reliability of plane networks transformed from WGS84 into S-JTSK

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Presnosť a spoľahlivosť geodetických polohových sietí transformovaných z WGS84 do S-JTSK

Lokálne polohové siete, zamerané metódami GPS a spracované v systéme WGS84, musia sa pre praktické použitie v geodézii (vytyčovacie práce, kontrolné merania a iné) transformovať do dátumu S-JTSK. Na posúdenie presnosti takto vzniklého bodového poľa je potrebné transformovať z WGS84 do S-JTSK aj charakteristiky presnosti bodov. Charakteristiky spoľahlivosti transformovanej siete v S-JTSK sú identické s charakteristikami vo WGS84.

Key words: accuracy and reliability measures of networks in WGS84, transformation of variance-covariance characteristics into S-JTSK, network accuracy measures in S-JTSK.

Introduction

Establishing local plane networks for setting out procedures, deformation and geodynamic measurements or networks with multipurpose functions, is a frequent task in geodesy (Weiss, 1997, 1998). Having established, measured and processed a such network, measures of its point accuracy (standard deviations of coordinates, confidence ellipses and others) as well as its reliability are needed for further use.

These demands on the a posteriori precision measures of a local network can be satisfied without difficulties if the network is measured terrestrically, that is, the customary geometrical elements or coordinate differences are surveyed (Weiss, 1997; Weiss, 1997). In this case the network is adjusted in a local frame or in S-JTSK (datum of the state plane control in Czech and Slovak Republic) and therefore the adjusted coordinates (and other quantities derived from them) as well as their accuracy measures will be directly derived in these coordinate frames. One can meet a different situation if the network is observed by GPS techniques. For practical use of the adjusting results, they have to be transformed into S-JTSK. However, in this case no accuracy measures required can be computed from the transformed network coordinates because the network was adjusted in WGS84. Then, besides the coordinate transformation, a further transformation of accuracy measures (above all of the variance-covariance matrix) for the coordinate estimates should be carried out from the datum WGS84 into the datum S-JTSK as well. The present paper is devoted to this transformation approach.

Accuracy of the network

Accuracy of the network points in WGS84

Accuracy of points determined by GPS can be described both on the basis of various a priori analyses and characteristics (see e.g. Blewitt et al., 1998; Hefty et al., 1994; Hoffman-Wellenhoff et al., 1994, 1996; Leick 1995; Mervart et al., 1997) and also by a lot of a posteriori measures derived from the factually realized GPS surveying. The a priori measures characterize generally the survey accuracy reachable by various GPS techniques and methods. They point at the positive and negative factors that affect the accuracy and at the rules to be respected for a surveying of quality. Having surveyed a point field by GPS under certain conditions, accuracy of the observables may be expressed by some measures derived from the obtained surveying data that are required for description of the network point accuracy and other accuracy measures of the adjusted network.

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Using a certain GPS surveying method, the output results from the surveyed data primarily processed (applying producers' software modules) involve also necessary information about the stochastic properties of the computed quantities. This information (for double and triple phase differences and single baseline solution) can be given for a GPS vector by the a posteriori variance factor - rms, standard deviations of the GPS vector components - $\sigma_{\Delta X}$, $\sigma_{\Delta Y}$, $\sigma_{\Delta Z}$ and by a correlation matrix for the vector components (e.g. SOKKIA GSR 2100 receiver, output listing files)

$$\begin{bmatrix} 1 & & \\ r_{yx} & 1 & \\ r_{zx} & r_{zy} & 1 \end{bmatrix} \tag{1}$$

or directly by a cofactor matrix (e.g. WILD-LEICA GPS - System 200 receiver)

$$\begin{bmatrix} q_{xx} & q_{xy} & q_{xz} \\ & q_{yy} & q_{yz} \\ & & q_{zz} \end{bmatrix}. \tag{2}$$

Applying these informations, the required cofactor matrix for a baseline vector $\Delta C_{ij}^{W} = [\Delta X\ \Delta Y\ \Delta Z]_{ij}^{T}$

$$Q_{\Delta Cij} = \begin{bmatrix} q_{\Delta X\Delta X} & q_{\Delta X\Delta Y} & q_{\Delta Z\Delta Z} \\ q_{\Delta Y\Delta X} & q_{\Delta Y\Delta Y} & q_{\Delta Y\Delta Z} \\ q_{\Delta Z\Delta X} & q_{\Delta Z\Delta Y} & q_{\Delta Z\Delta Z} \end{bmatrix}_{ij} \tag{3}$$

can always be formed. Then, for all the GPS vectors in the network, that is for the "observation vector" $L = [\Delta C_{12}^{w}\ \ \Delta C_{23}^{w} \ldots \Delta C_{ij}^{w} \ldots\ ]$, the total cofactor matrix becomes

$$Q_L = \mathrm{diag}(Q_{\Delta C12}\ Q_{\Delta C23}\ \ldots\ Q_{\Delta Cij}\ \ldots\ ). \tag{4}$$

After adjusting the network, the accuracy information of the estimated coordinates is involved in the cofactor matrix $Q_{\hat{C}w}$ or in the corresponding covariance matrix $\Sigma_{\hat{C}w}$, in that the 3x3 diagonal blocks

$$\Sigma_{\hat{C}i} = \begin{bmatrix} \sigma^2_{\hat{x}\hat{x}} & r_{\hat{x}\hat{y}}\sigma_{\hat{x}\hat{x}}\sigma_{\hat{y}\hat{y}} & r_{\hat{x}\hat{z}}\sigma_{\hat{x}\hat{x}}\sigma_{\hat{z}\hat{z}} \\ r_{\hat{y}\hat{x}}\sigma_{\hat{y}\hat{y}}\sigma_{\hat{x}\hat{x}} & \sigma^2_{\hat{y}\hat{y}} & r_{\hat{y}\hat{z}}\sigma_{\hat{y}\hat{y}}\sigma_{\hat{z}\hat{z}} \\ r_{\hat{z}\hat{x}}\sigma_{\hat{z}\hat{z}}\sigma_{\hat{x}\hat{x}} & r_{\hat{z}\hat{y}}\sigma_{\hat{z}\hat{z}}\sigma_{\hat{y}\hat{y}} & \sigma^2_{\hat{z}\hat{z}} \end{bmatrix}_{i} \tag{5}$$

represent the covariance matrices for single network points.

Accuracy of network points in S-JTSK

Cartesian coordinate estimates $\hat{C}^{W}$ of the network points in WGS84 are transformed to coordinates $\hat{C}^{Jt}$[2] in S-JTSK for the practical use in stages as follows
where the 3D reference frame (ellipsoid centric system for the Bessel ellipsoid) is denoted by the index R and the frame S-JTSK by J. Accuracy of $\hat{C}^{W}$ is described by $Q_{\hat{C}w}$ or $\Sigma_{\hat{C}w}$ (5) and for the transformed $\hat{C}^{Jt}$, the relevant covariance matrix $\Sigma_{\hat{C}Jt}$ is to be derived. This measure can be obtained by using transformation stages analogously as (6) in that the law of variance-covariance propagation has to be used for computing the corresponding matrices

[2] a sufficient precise linear transformation of coordinate estimates gives the transformed ones on the same level

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}_i^W \Rightarrow \begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}_i^R \Rightarrow \begin{bmatrix} \hat{\phi} \\ \hat{\lambda} \\ \hat{h} \end{bmatrix}_i^R \Rightarrow \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix}_i^J , \tag{6}$$

$$\Sigma_{\hat{C}W} \Rightarrow \Sigma_{\hat{C}R} \Rightarrow \Sigma_{(\hat{\varphi},\hat{\lambda},\hat{h})R} \Rightarrow \Sigma_{\hat{C}Jt} . \tag{7}$$

In the first transformation stage we use the Molodenskij - Badekas transformation model transcribed into the Gauss - Markoff estimation model (Klein,1997; Sütti et al.,1997)

$$\hat{C}_i^R = A\hat{\Theta} + C_i^W , \tag{8}$$

where A is the coefficient matrix and $\hat{\Theta}$ is the vector of 7 transformation para-meter estimates.

Applying the variance propagation law to (8) gives

$$\Sigma_{\hat{C}R} = A\Sigma_{\hat{\Theta}}A^T + \Sigma_{\hat{C}W} \tag{9}$$

where $\Sigma_{\hat{\Theta}}$ is the covariance matrix of the adjusted transformation parameters. As can be seen, the accuracy both of the accomplished, factual surveying determination of network points ($\Sigma_{\hat{C}w}$) and of the transformation parameters determined for this case ($\Sigma_{\hat{\Theta}}$) are projected into the matrix (9). Elements of $\Sigma_{\hat{C}R}$ are dependent on the reached survey accuracy, on number and choice of the homological points and on the local deformations in S-JTSK.

In the second stage the following transformations are performed

$$\hat{\varphi} = f(\hat{C}_R,...) , \ \hat{\lambda} = g(\hat{C}_R,...) \ , \ \hat{h} = h(\hat{C}_R,...) \tag{10}$$

for which closed formulas or expressions for iterative solution can be used. We shall assume, the coordinates $\hat{C}_R$ are random variables only in the arguments. Then, after using the variance propagation law, the covariance matrix of the geodetic coordinates $(\hat{\varphi},\hat{\lambda},\hat{h})_R$ reads

$$\Sigma_{(\hat{\varphi},\hat{\lambda},\hat{h})R} = F_R \Sigma_{\hat{C}R} F_R^T \tag{11}$$

where

$$F_R = \begin{bmatrix} \frac{\partial\varphi}{\partial X^R} & \frac{\partial\varphi}{\partial Y^R} & \frac{\partial\varphi}{\partial Z^R} \\ \frac{\partial\lambda}{\partial X^R} & \frac{\partial\lambda}{\partial Y^R} & \frac{\partial\lambda}{\partial Z^R} \\ \frac{\partial h}{\partial X^R} & \frac{\partial h}{\partial Y^R} & \frac{\partial h}{\partial Z^R} \end{bmatrix} \tag{12}$$

is the Jacobian of $(\hat{\varphi},\hat{\lambda},\hat{h})_R$ with respect to $\hat{C}_R$.

The third stage is created by a transformation chain

$$\begin{bmatrix} \hat{\varphi} \\ \hat{\lambda} \end{bmatrix}_i^R \Rightarrow \begin{bmatrix} U \\ V \end{bmatrix}_i \Rightarrow \begin{bmatrix} \check{S} \\ D \end{bmatrix}_i \Rightarrow \begin{bmatrix} R \\ D' \end{bmatrix}_i \Rightarrow \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix}_i^{Jt} \tag{13}$$

which consists of the stepped coordinate transformations from the 3D reference frame (R) into S-JTSK. The explicit transformation equations are known from the Křovak's cartographic projection (Daniš,1960; Kuska,1960)

$$U=u(\phi,...)\ ,\ V=v(\lambda,...)\ ,\ \check{S}=s(U,V,...)\ ,\ D=d(U,V,...)\ ,$$
$$R=r(\check{S},...)\ ,\ D'=p(D,...)\ ,\ X=x(R,D',...)\ ,\ Y=y(R,D',...). \quad (14)$$

in which only the coordinates $(\phi, \lambda)^R$ and from them derived other cartographic quantities are random variables. Then, accuracy of the transformed coordinates $\hat{C}^{Jt}$ can be described by the covariance matrix

$$\Sigma_{\hat{C}Jt} = F_J F_P F_K F_S\ \Sigma_{(\hat{\phi},\hat{\lambda})R}\ F_S^T F_K^T F_P^T F_J^T = F_{CP}\ \Sigma_{(\hat{\phi},\hat{\lambda})R}\ F_{CP}^T \quad (15)$$

where the coefficient matrices (the corresponding Jacobians) are

$$F_S = \begin{bmatrix} \dfrac{\partial U}{\partial \varphi} & \dfrac{\partial U}{\partial \lambda} \\ \dfrac{\partial V}{\partial \varphi} & \dfrac{\partial V}{\partial \lambda} \end{bmatrix}, F_K = \begin{bmatrix} \dfrac{\partial D}{\partial U} & \dfrac{\partial D}{\partial V} \\ \dfrac{\partial \check{S}}{\partial U} & \dfrac{\partial \check{S}}{\partial V} \end{bmatrix}, F_P = \begin{bmatrix} \dfrac{\partial R}{\partial D} & \dfrac{\partial R}{\partial \check{S}} \\ \dfrac{\partial D'}{\partial D} & \dfrac{\partial D'}{\partial \check{S}} \end{bmatrix}, F_J = \begin{bmatrix} \dfrac{\partial X}{\partial R} & \dfrac{\partial X}{\partial D'} \\ \dfrac{\partial Y}{\partial R} & \dfrac{\partial Y}{\partial D'} \end{bmatrix}, \quad (16)$$

the covariance matrix of the geodetic coordinates reads

$$\Sigma_{(\hat{\varphi},\hat{\lambda})R} = \begin{bmatrix} \sigma^2_{(\hat{\varphi})R} & r_{\hat{\varphi},\hat{\lambda}}\sigma_{\hat{\varphi}}\sigma_{\hat{\lambda}} \\ r_{\hat{\lambda},\hat{\varphi}}\sigma_{\hat{\lambda}}\sigma_{\hat{\varphi}} & \sigma^2_{(\hat{\lambda})R} \end{bmatrix} \quad (17)$$

and F_{CP} contains coefficients resulting from multiplication of the Jacobians (16).

The components of matrices (16), partial derivatives of the functions (14), exist due to their continuous types at all points of the variables. Usable expressions for computer supported calculations of these derivatives can be available too (Sütti, 1998; Tomášová, 1997).

Using equations (9), (11) and (15), the required covariance matrix $\Sigma_{\hat{C}Jt}$ of the transformed coordinates $\hat{C}^{Jt}$ (applying the covariance matrices $\Sigma_{\hat{C}W}$ and $\Sigma_{\hat{\Theta}}$ as arguments), can be then computed by the formula

$$\Sigma_{\hat{C}Jt} = F_{CP} F_R\ (A\Sigma_{\hat{\Theta}}A^T + \Sigma_{\hat{C}W})\ F_R^T F_{CP}^T\ . \quad (18)$$

The matrix (18) gives the most universal description of the transformed coordinate precision in the datum S-JTSK. From ths expression all further accuracy measures for netpoints, layout elements and for network quality analysis are derivable. All these measures are generated not only by positioning accuracy of a certain, factual GPS surveying but by its modifications (changes) owing to the used transformations.

As it is known, the coordinates C_H^J of the homological netpoints and their coordinates $\hat{C}_H^{Jt}$ obtained from the transformation are not identical values. Their differences, so called coordinate discrepancies $dC_H = \hat{C}_H^{Jt} - C_H^J$, can be either negligibly small or considerable large (some tens mm) according to the rate of the local coordinate deformations of the datum S-JTSK. In most cases the discrepancies have to be taken for computing changes of coordinates $\hat{C}^{Jt}$ to their new values $\hat{C}^{Jtc}$. This is necessary for improvement of the mutual compatibility of the local network and the state plane control (ŠTS), that is, for increasing their homogeneity. One should be aware of that the modification of coordinates $\hat{C}^{Jt}$ to $\hat{C}^{Jtc}$ realizable by various methods (computing the corrections $\hat{C}^{Jtc} - \hat{C}^{Jt}$ by weighted mean approaches, multiquadratic or spline aproximations, by determining $\hat{C}^{Jtc}$ using similarity or affine transformations and others, see e.g. Sütti, 1979), does not heighten the point accuracy reached by GPS surveying and given by $\Sigma_{\hat{C}Jt}$. This modification of $\hat{C}^{Jt}$ translate the related netpoints to new positions $\hat{C}^{Jtc}$ in the frame of the used homological points only, saving for them the same accuracy characteristic $\Sigma_{\hat{C}Jt}$. The new coordinate positions $\hat{C}^{Jtc}$ coincides better with the unchangeable coordinates of the homological points ("stable points") in the state plane control. This should be emphasized, because the discrepancies dC_H are often used for a general, informative accuracy description of the transformed network points.

Reliability of the network transformed

As known, the internal reliability measures are connected with analysis of the observables in the surveyed network (testing the residuals of the measured quantities to detect gross data errors - outliers) while the external reliability measures describe the internal reliability effects in the adjusted coordinates. These reliability measures are functions of non-coordinate arguments as covariance matrix of the measured quantities, redundancy matrix, testing parameters and others (Mürle et al., 1984, Schädlich, 1983). Thus, their transformation into S-JTSK is not realizable. The reliability measures based on surveys in a network can be then determined for the "surveyed and adjusted status" of a network only and not for a "derived status", that is, for a transformed network. Because the both network status are related to the same physically existing pointfield, for reliability characterising "the transformed network", if it is required, the reliability measures valid for the surveyed state of the network can be used.

Conclusion

For setting out processes but also for other applications of networks surveyed by GPS in the engineering surveying, it is necessary to know the accuracy and reliability measures of the transformed networks. To acquire the relevant matrices describing these properties of the network transformed into S-JTSK, it is needful to transform the covariance matrices $\Sigma_{\hat{C}w}$ and $\Sigma_{\hat{\Theta}}$ for getting information of the transformed network accuracy while for the network reliability description in S-JTSK the same measures as in WGS84 can be used. The outlined solution is conveniently realizable from the mathematical point of view and the development of a suitable software program causes difficulties nor from the numerical viewpoint.

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