Modulus optimum for digital controllers

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Optimálny modul pre digitálne regulátory

The paper extends the popular modulus optimum with the original approach for the conventional digital controllers. The use of the D-transform allows obtaining the uniform computing formulas for the values of the digital and analog controller tuning parameters. The use is shown on an example.

***Key words:** modulus optimum, controller tuning, PID control.*

Introduction

For low order controlled plants without the time delay the modulus optimum (absolute value optimum criterion, Betrags Optimum) is often used for the conventional analog controller tuning (Åström & Hägglund, 1995; Kalaš, Jurišica & Žalman, 1978). It is so because of its simplicity and a suitable course of the step response from the practical point of view. It is widely used in the electrical drive control where the small time constants are substituted by their sum (Kalaš, Jurišica & Žalman, 1978; Szklarski, Jaracz & Víteček, 1989).

Modulus Optimum

The control system on the Fig. 1 is considered, where G_C is the controller transfer function, G_P - the plant transfer function, *W*, *V* and *Y* – the transforms of the desired *w*, disturbance *v* and controlled *y* variables.

For control systems with analog controllers the modulus optimum is based on the requirement for the control system transfer function to be in a form

$$G_{wy}(s) \to 1 \Rightarrow A_{wy}(\omega) \to 1 \Rightarrow A_{wy}^2(\omega) \to 1 \qquad (1)$$

where: $G_{wy}(s)$ - the control system L-transfer function, *s* – the complex variable in the L-transform, $A_{wy}(\omega)$ - the modulus of the control system F-transfer function $G_{wy}(j\omega)$, ω - the angular frequency.

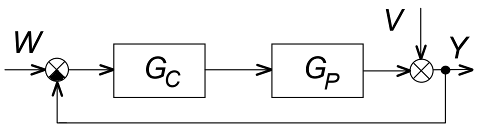

Fig.1. Control system.

The last relation in (1) is very important because it allows avoiding the radical, and additionally the following holds for the modulus square

$$A_{wy}^2(\omega) = G_{wy}(j\omega)G_{wy}(-j\omega) \qquad (2)$$

This paper considers only cases (see Tab. 2) in which the control systems with the analog controllers lead to the standard form of the control system L-transfer function for the modulus optimum after a compensation of the plant biggest time constants have been applied to them (Åström & Hägglund, 1995; Kalaš, Jurišica & Žalman, 1978; Szklarski, Jaracz & Víteček, 1989).

$$G_{wy}(s) = \frac{1}{T_w^2 s^2 + 2\xi_w T_w s + 1}, \qquad \xi_w = \frac{1}{\sqrt{2}} \doteq 0{,}707, \qquad T_w = T_i\sqrt{2} \qquad (3)$$

where: ξ_w is the damping coefficient, T_w - the control system time constant (*i* = 1 for 1st and 2nd row, *i* = 2 for 3rd and 4th row and *i* = 3 for 5th row in Tab. 2). It is interesting that the characteristic polynomial coefficients in (3) correspond to the coefficients of the standard forms by Whiteley, Naslin and ITAE criterion with the step response relative overshoot about 4.3%.

Further it is supposed that the control system D-transfer function is in the form (Vítečková at al., 2002)

$$G_{wy}(\gamma) = \frac{\beta_m\gamma^m + \dots + \beta_1\gamma + \beta_0}{\alpha_n\gamma^n + \dots + \alpha_1\gamma + \alpha_0}, \; n \ge m \qquad (4)$$

where: γ is the complex variable in the D-transform.

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For a small value of the sampling period T the two terms of the Taylor expansion for the approximation of the exponential function can be used

$$e^{Ts} \approx 1 + Ts \quad \Rightarrow \quad \gamma = \frac{e^{Ts} - 1}{T} \approx s \tag{5}$$

then the relation (2) can be written as

$$A_{wy}^2(\omega) \approx \frac{B_m \omega^{2m} + \mathrm{K} + B_1 \omega^2 + B_0}{A_n \omega^{2n} + \mathrm{K} + A_1 \omega^2 + A_0} \tag{6}$$

Tab.1. Conventional controller transfer function.

	TYPE	$G_C(\gamma)$	$G_C(s)$	$G_C(z)$
1	P	k_P	k_P	k_P
2	I	$\frac{\gamma T + 1}{T_I \gamma}$	$\frac{1}{T_I s}$	$\frac{T}{T_I} \frac{z}{z - 1}$
3	PD	$k_P \left(1 + \frac{T_D \gamma}{\gamma T + 1}\right)$	$k_P (1 + T_D s)$	$k_P \left(1 + \frac{T_D}{T} \frac{z - 1}{z}\right)$
4	PI	$k_P \left(1 + \frac{\gamma T + 1}{T_I \gamma}\right)$	$k_P \left(1 + \frac{1}{T_I s}\right)$	$k_P \left(1 + \frac{T}{T_I} \frac{z}{z - 1}\right)$
5	PID	$k_P \left(1 + \frac{\gamma T + 1}{T_I \gamma} + \frac{T_D \gamma}{\gamma T + 1}\right)$	$k_P \left(1 + \frac{1}{T_I s} + T_D s\right)$	$k_P \left(1 + \frac{T}{T_I} \frac{z}{z - 1} + \frac{T_D}{T} \frac{z - 1}{z}\right)$

Tab.2. Recommended controllers and their adjustable parameters for modulus optimum.

	CONTROLLED PLANT	CONTROLLER < ANALOG DIGITAL $T = 0$ $T > 0$-			
		TYPE	k_P^*	T_I^*	T_D^*
1	$\frac{k_1}{T_1 s + 1}$	I		$2k_1(T_1 - 0{,}5T)$	
2	$\frac{k_1}{s(T_1 s + 1)}$	P	$\frac{1}{2k_1 T_1}$	-	-
3	$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)}$ $T_1 \geq T_2$	PI	$\frac{T_1 - 0{,}5T}{2k_1 T_2}$	$T_1 - 0{,}5T$	-
4	$\frac{k_1}{s(T_1 s + 1)(T_2 s + 1)}$ $T_1 \geq T_2$	PD	$\frac{1}{2k_1(T_2 + 0{,}5T)}$	-	$T_1 - 0{,}5T$
5	$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)(T_3 s + 1)}$ $T_1 \geq T_2 \geq T_3$	PID	$\frac{T_1 + T_2 - T}{2k_1(T_3 + 0{,}5T)}$	$T_1 + T_2 - T$	$\frac{4T_1T_2 - 2T(T_1 + T_2) + T^2}{4(T_1 + T_2 - T)}$

In the case where the controller integral time T_I^* and derivative time T_D^* are determined from the conditions for the compensation, the controller gain k_P^* can be determined on the basis of the formula (Kalaš, Jurišica & Žalman, 1978)

$$A_1 = B_1 \quad \Rightarrow \quad \alpha_1^2 - 2\alpha_0\alpha_2 = \beta_1^2 - 2\beta_0\beta_2 \tag{7}$$

The Tab. 2 shows relations of the controller's adjustable parameters after the first order Páde approximation has been used on them.

The sampling period value T should not be bigger than about one third of the plant smallest compensated time constant.

The relations for the controller's adjustable parameters for $T > 0$ are valid for the digital controllers and for $T = 0$ are valid for the analog ones.

Conclusions

In the paper the popular modulus optimum is derived and simplified for the conventional digital controllers. Because the D-transform is used, the obtained formulas are uniform for digital and analog controllers. The simulation results show the practical usefulness of the proposed approach.

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