Economic factorial analysis: general theory and original approaches

***Sergei Chebotarev*[1]**

Ekonomická faktorová analýza: všeobecná teória a pôvodné prístupy

This research work is about the methodology of economic factorial analysis, which is an actual instrument for the analysis of different economic and technological models. The acceptance of control solutions becomes the result of model's study in this case. This fact defines the importance of this unit for the analysis of systems. It is possible to realise the different numerical experiments on simulation of different processes and their following analysis using the methods received in the course of research of static and dynamical factorial analysis. The given analytical tool is widely used in practice. Author has carried out the research using the methods of factorial analysis in conditions of real production on one of the largest metallurgical plant of Europe. This fact also confirms a practical importance of factorial analysis.

Key words: *economic factorial analysis, numerical methods, simulation.*

Introduction

Economic analysis first of all is the factorial analysis that allows to investigate the transition from the initial factorial system to the resulting factorial system with the evaluation of factors influence on changing of economic activities. Let's consider some methodological and practical questions of this popular scientific area.

Static and dynamic factorial analysis

The forms of deterministic or stochastic dependency between generalize function and factors and the values of factor's influence on function's change are studied in direct factorial analysis (Bakanov and Sheremet, 1995) Examples of direct deterministic factorial analysis are the analysis of influence of labour production and number of workers on the production volume (*multiplicative model*) or the analysis of influence of profit, cost factor and circulating assets on the profitability level (*fractional model*) etc.

As the resulting factor system we can consider function $y=f(x)$ with the set of influencing factors $x=(x_i)$, $i=1, \dots, n$. The situation is considered when the factors has received the increment $\Delta x= (\Delta x_i)$ from the initial values. So we have the change Δy from initial model value. It is required to define what part of increment (decrease) of function is caused by changes of each argument (factor).

$$\Delta y = \sum_{i=1}^{n} A_{x_i} = \sum_{i=1}^{n} K \cdot \Delta x_i$$

The whole class of methods of economic factorial analysis expands the model as the linear function from changes of arguments. But all these methods give the expansion of system which contains the irresolvable residual that is the result of synergetic effect from simultaneous changes of interconnected factors (1).

$$\Delta y = L(\Delta x) + \theta(\Delta x) \qquad (1)$$

However, we can overcome this complexity by using of well-known Lagrange mean value theorem (Fihtengolts, 1997; Chebotarev, 1999) or the integral mean value theorem (2) that is more suitable for practical using.

$$\Delta y = \sum_{i=1}^{n} A_{x_i} = \sum_{i=1}^{n} \int_0^1 f'_{x_i}(x_1 + \alpha\Delta x_1, x_2 + \alpha\Delta x_2, ..., x_n + \alpha\Delta x_n)\Delta x_i d\alpha \qquad (2)$$

The variety of methods for the evaluation of factors influences is connected with distribution of residual between arguments. However the analysis of model without distribution of irresolvable residual between the factors can be the most preferable approach in the case of impossibility to choice one or another method.

The next step in research work was to analyse the models in the dynamic case when the system can be considered on the various temporal gaps or the model amplified because of growth of its aggregate structure (3).

$$t = r \cdot s = \sum_i z_i = \sum_i x_i \cdot y_i \qquad (3)$$

[1] *Sergei Chebotarev:* Department of Applied Mathematics, Lipetsk State Technical University, Lipetsk, Russia sergcheb@lipetsk.ru
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In this case the increment of resumptive function can be presented in two forms (4-5, where index “av” means average value), admitting different meaningful interpretations as the practical evaluation procedures for the chain «time/structure» deterministic factorial analysis (Blyumin et al., 2000a).

$$\Delta t = \sum_i \Delta z = \sum_i y_{i\,av} \cdot \frac{\sum_i \left(y_{i\,av} \cdot \Delta x_i\right)}{\sum_i y_{i\,av}} + \frac{\sum_i \left(x_{i\,av} \cdot \Delta y_i\right)}{\sum_i \Delta y_i} \cdot \sum_i \Delta y_i = s_{av}\Delta r + r_{av}\Delta s, \tag{4}$$

$$\Delta t = \sum_i y_{i\,av} \cdot \left[\frac{\sum_i ((x_i + \Delta x)\cdot(y_i + \Delta y))}{\sum_i (y_i + \Delta y)} - \frac{\sum_i (x_i \cdot y_i)}{\sum_i y_i} \right] + \tag{5}$$

$$+ \frac{1}{2}\left[\frac{\sum_i (x_i \cdot y_i)}{\sum_i y_i} + \frac{\sum_i ((x_i + \Delta x)\cdot(y_i + \Delta y))}{\sum_i (y_i + \Delta y)} \right] \cdot \sum_i \Delta y_i = s_{av}\Delta \bar{r} + \bar{r}_{av}\Delta s.$$

Economic factorial analysis of the second order

In classic economic factorial analysis the influence of factor’s modifications Δx, Δy on the modification of index Δf is analyzed at the study of the elementary two-factor multiplicative model $f=x\cdot y$. All efforts of traditional approach are directed to keep the irresolvable residual $\Delta x\cdot\Delta y$ and, whenever possible, to its expansion using the ensemble of methods.

The modern investigations suggest that the irresolvable residual can be analyzed, for example, in assumption that it has the same structure as the initial model. Let’s assume that along with the planned values of factors x_0, y_0, there are the planned values of its increments Δx_0, Δy_0. These magnitudes have the meaningful interpretation, for example, as the tolerances on increments of factors. The given interpretation is not abstract and it is confirmed by researches of real industrial models, when the deviations of factor’s values from initially admissible have been analyzed.

The further development of such approach is the economic factorial analysis of the third order and etc. The practical application of economic factorial analysis of the second order in production conditions already has shown its large flexibility and has given more adequate outcomes, than the traditional approaches.

Original approaches

Considered above new method of economic factorial analysis based on the mean value theorem can be used for estimation of relative increments for elasticity in relative economic factorial analysis of production function. The main objectives in this case consist in finding the factor expansion using not absolute but relative increments of arguments. And we can write the exact expansion (6) using the classical Lagrange theorem.

$$\delta_m f = \frac{\Delta f}{f(x_m)} = \sum_{i=1}^{n} f_i'(x_m) \cdot \left(\frac{x_{m_i}}{f(x_m)} \right) \cdot \frac{\Delta x_i}{x_{m_i}} = f_i^{<E>}(x_m) \cdot \delta_m x_i, \tag{6}$$

where $f_i^{<E>} = f_i'(x) \cdot x_i / f(x)$ is the partial elasticity of production function.

Thus equation (6) is the formula of finite relative increments for elasticity.

Specifically, partial elasticity for Cobb-Duglas (7) production function are constant

$$y = a \cdot \prod_{i=1}^{n} x_i^{\alpha_i} \tag{7}$$

and equal to parameters α_i. Relative increment for Cobb-Duglas function is calculated according to (8).

$$\delta_m f = \sum_{i=1}^{n} \alpha_i \cdot \delta_m x_i \tag{8}$$

Applications

Obtained methodological results have been used in analysis of some production models that describe energy consumption process. For example, the general multiplicative-additive model (9) allows to plan the requirement level of electricity in ironworks.

$$W = \sum_{i=1}^{n} N_i \cdot Q_i + \sum_{j=1}^{m} L_j \cdot n_j \tag{9}$$

The possibility of control under the consumption of energy allows to find the ways for the gradual decrease of costs and increases the profitability of business.

Conclusions

Proposed theoretical approaches allow to analyse the large types of economic and technological models with any structure and any number of factors (Blyumin et al., 2000a). More substantial information on these researches can be found in (Blyumin et al., 2000b; Chebotarev, 2001).

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