

# Using fuzzy logic to control active suspension system of one-half-car model

Antonín Stříbrský<sup>1</sup>, Kateřina Hyniová<sup>1</sup>, Jaroslav Honců<sup>1</sup> and Aleš Kruczek<sup>1</sup>

## Použitie fuzzy logiky na riadenie aktívneho tlmiaceho systému polovice auta

In the paper, fuzzy logic is used to control active suspension of a one-half-car model. Velocity and acceleration of the front and rear wheels and undercarriage velocity above the mentioned wheels are taken as input data of the fuzzy logic controller. Active forces improving vehicle driving, ride comfort and handling properties are considered to be the controller outputs. The controller design is proposed to minimize chassis and wheels deflection when uneven road surfaces, pavement points, etc. are acting on the tires of running cars. In the conclusion, a comparison of active suspension fuzzy control and spring/damper passive suspension is shown using MATLAB simulations.

**Key words:** vehicle, active suspension, one-half-car model, fuzzy logic control.

## Introduction

Replacement of the spring-damper suspensions of automobiles by active systems has the potential of improving safety and comfort under nominal conditions. But perhaps more important, it allows continuous adaptation to different road surface quality and driving situations. For the design of active suspension we know how to build a model and how to define the objective of the control in order to reach a compromise between contradictory requirements like ride comfort and road holding by changing the force between the wheel and chassis masses. In the recent past, it has been reported on this problem successively, about the base of optimization techniques, adaptive control and even, H-infinity robust methods. In this paper, fuzzy logic is used to control active hydro-pneumatic suspension of a one-half-car model. There are taken velocity and acceleration of the front and rear wheels and undercarriage velocity and acceleration above these wheels as input data of the fuzzy controller, and active forces  $f_1$  and  $f_2$  as its output data. The objective of the control is to minimize chassis and wheels deflections when road disturbances are acting on the running car.

## One-half-car Model

In this paper, we are considering a one-half-car model (Fig.1) which includes two one-quarter-car models (Hyniová, Stříbrský and Honců, 2001; Stříbrský, Hyniová and Honců, 2000) connected by a homogenous undercarriage. The undercarriage is determined by its mass  $m$  [kg] (taken as one half of the total body mass - 500 kg), length  $L$  [m] ( $L = L_1 + L_2 = 1,5m + 2,5m = 4m$ ), center of gravity position  $T$  [m] (given by  $L_1$  and  $L_2$ ) and moment of inertia  $J_p$  [ $\text{kgm}^2$ ] ( $2700 \text{ kgm}^2$ ).

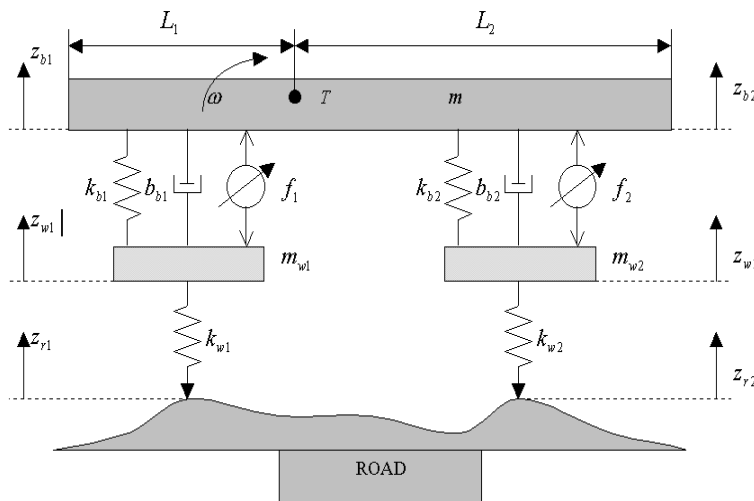


Fig.1. One-half-car model.

The motion equations of the wheels are as follows:

$$m_{w1} \ddot{z}_{w1} = -f_1 + k_{b1}(z_{b1} - z_{w1}) - k_{w1}(z_{w1} - z_{r1}) + b_{b1}(\dot{z}_{b1} - \dot{z}_{w1}) \quad (1)$$

$$m_{w2} \ddot{z}_{w2} = -f_2 + k_{b2}(z_{b2} - z_{w2}) - k_{w2}(z_{w2} - z_{r2}) + b_{b2}(\dot{z}_{b2} - \dot{z}_{w2}) \quad (2)$$

$$m \ddot{z}_{b1} = f_1 - k_{b1}(z_{b1} - z_{w1}) - b_{b1}(\dot{z}_{b1} - \dot{z}_{w1}) = F_1 \quad (3)$$

$$m \ddot{z}_{b2} = f_2 - k_{b2}(z_{b2} - z_{w2}) - b_{b2}(\dot{z}_{b2} - \dot{z}_{w2}) = F_2 \quad (4)$$

<sup>1</sup> Antonín Stříbrský, Kateřina Hyniová, Jaroslav Honců, Aleš Kruczek: Department of Control Engineering, Faculty of Electrical Engineering, Czech Technical University Karlovo náměstí. 13, 121 35 Prague, Czech Republic stribrsk@fel.cvut.cz (Recenzovaná a revidovaná verzia dodaná 19.11.2003)

with the following constants and variables which respect the static equilibrium position:

- $m_{w1}, m_{w2}$  ..... wheel masses (35 kg each),
- $k_{b1}, k_{b2}$  ..... passive suspension spring stiffness (16 000 N/m each),
- $k_{w1}, k_{w2}$  ..... tyre spring stiffness (160 000 N/m each),
- $b_{b1}, b_{b2}$  ..... passive suspension damping coefficients (980 Ns/m each),
- $f_1, f_2$  ..... active forces [N],
- $z_{r1}, z_{r2}$  ..... road displacements [m],
- $z_{b1}, z_{b2}$  ..... body (chassis) displacements [m],
- $z_{w1}, z_{w2}$  ..... wheel displacements [m].

The pitching motion equation is given as:

$$F_1 L_1 - F_2 L_2 - J_p \ddot{\alpha} = 0 \tag{5}$$

and motion of the gravity center as:

$$F_1 + F_2 - m_p \ddot{z} = 0 \tag{6}$$

Note, that:  $v_{b1} = v_T + \omega L_1$  (7)  
 $v_{b2} = v_T - \omega L_2,$

where: the meaning of constants and variables is the following:

- $v_T [m.s^{-1}]$  ..... gravity center velocity,
- $\omega [rad.s^{-1}]$  ..... angle velocity,
- $v_{b1} [m.s^{-1}]$  ..... undercarriage velocity above front wheel,
- $v_{b2} [m.s^{-1}]$  ..... undercarriage velocity above rear wheel.

### State-space Model

To model the road input let us assume that the vehicle is moving with a constant forward speed. Then the vertical velocity can be taken as a white noise process (for simulation results see Minarik, 2002) which is approximately true for most of real roadways.

To transform the motion equations of the one-half-car model into a state space model, the following state variables vector, input vector, and the vector of disturbances, are considered:

$$x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T$$

$$u = [f_1, f_2]^T$$

$$v = [\ddot{z}_{r1}, \ddot{z}_{r2}]^T,$$

where:

$$x_1 = z_{b1} - z_{w1}, \quad x_2 = z_{w1} - z_{r1}, \quad x_3 = \dot{z}_{w1}, \quad x_4 = z_{b2} - z_{w2},$$

$$x_5 = z_{w2} - z_{r2}, \quad x_6 = \dot{z}_{w2}, \quad x_7 = v_T, \quad x_8 = \omega$$

Then the motion equations of the one-half-car model for the active suspension can be written in state space form as follows:

$$\dot{x} = Ax + Bu + Fv, \tag{8}$$

where: for the given data

$$A = \begin{pmatrix} 0 & 0 & -1.0 & 0 & 0 & 0 & 1.0 & 1.5 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 457.1 & -4571.4 & -28.0 & 0 & 0 & 0 & 28.0 & 42.0 \\ 0 & 0 & 0 & 0 & 0 & -1.0 & 1.0 & -2.5 \\ 0 & 0 & 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 457.1 & -4571.4 & -28.0 & 28.0 & -70.0 \\ -32.0 & 0 & 2.0 & -32.0 & 0 & 2.0 & -3.9 & 2.0 \\ -8.9 & 0 & 0.5 & 14.8 & 0 & -0.9 & 0.4 & 3.1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -0.02860 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.02860 \\ 0.00200 & 0.00200 \\ 0.00055 & 0.00092 \end{pmatrix} \quad F = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Thanks to the negative real parts of all eigen values of  $A$ , the model is stable.

### Fuzzy logic controller

The control system itself consists of three stages: fuzzification, fuzzy inference engine and defuzzification. The fuzzification stage converts real-number (crisp) input values into fuzzy values while the fuzzy inference machine processes the input data and computes the controller outputs in cope with the rule base and data base. These outputs, which are fuzzy values, are converted into real-numbers by the defuzzification stage.

The fuzzy logic controller used in the active suspension has nine inputs:

$v_{b1}, a_{b1} = \mathcal{E}_{b1}, v_{w1}, v_{b2}, a_{b2} = \mathcal{E}_{b2}, v_{w2}, v_T, \mathcal{E}_T$ , and  $\omega$ , and two outputs  $f_1$  and  $f_2$ . All membership functions are triangular form.

Variables ranges are stated experimentally (Minarik, 2002) and given bellow.

Tab.1. Variables ranges.

$z_i$ [cm]	$ f_1 _{\max}$ [N]	$ f_2 _{\max}$ [N]	$ v_T _{\max}$ [m.s <sup>-1</sup> ]	$ \dot{v}_T _{\max}$ [m.s <sup>-2</sup> ]	$\omega$ [rads <sup>-1</sup> ]
5	2876.4	2949.4	0.3	6.6	0.1
10	5752.8	6062.5	0.5	13.5	0.2

$ v_{b1} _{\max}$ [m.s <sup>-1</sup> ]	$ \dot{v}_{b1} _{\max}$ [m.s <sup>-2</sup> ]	$ v_{w1} _{\max}$ [m.s <sup>-1</sup> ]
0.3	6.8	2.4
0.5	13.5	4.8

$ v_{b2} _{\max}$ [m.s <sup>-1</sup> ]	$ \dot{v}_{b2} _{\max}$ [m.s <sup>-2</sup> ]	$ v_{w2} _{\max}$ [m.s <sup>-1</sup> ]
0.3	6.8	2.2
0.5	13.5	4.9

The rule base used in the active suspension system for one-half-car model is represented by 160 rules with fuzzy terms derived by modeling the designer's knowledge and experience.

There are two types of rules for the one-half-car model in here. The "wheel" rules (several examples given in Tab.2) are corresponding to the rules of the one-quarter-car model (Stříbrský, Hyniová and Honců, 2000) and considering  $v_{b1}, a_{b1} = \mathcal{E}_{b1}, v_{w1}, v_{b2}, a_{b2} = \mathcal{E}_{b2}$ , and  $v_{w2}$  as fuzzy controller inputs, and  $f_1$  and  $f_2$  as controller outputs. The "undercarriage" rules (several examples given by Tab.3) are describing the mutual influence of the wheels and taking  $v_T, \mathcal{E}_T$ , and  $\omega$  as fuzzy controller inputs, and  $f_1$  and  $f_2$  as controller outputs.

The abbreviations used correspond to:

- NV ... Negative Very Big
- NB ... Negative Big
- NM ... Negative Medium
- NS ... Negative Small
- ZE ... Zero
- PV ... Positive Very Big
- PS ... Positive Small
- PM ... Positive Medium
- PB ... Positive Big
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The output of the fuzzy controller is a fuzzy set of control. As a process usually requires a non-fuzzy value of control, a method of defuzzification called "center of gravity method" is used here.

Tab.2. "Wheel" rules.

If [ $v_T = PS$ ] AND [ $\omega = PM$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = NV$ ] AND [ $f_2 = PM$ ]
If [ $v_T = NS$ ] AND [ $\omega = PM$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = NM$ ] AND [ $f_2 = PV$ ]
If [ $v_T = NM$ ] AND [ $\omega = PM$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = ZE$ ] AND [ $f_2 = PV$ ]
If [ $v_T = PS$ ] AND [ $\omega = PS$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = NB$ ] AND [ $f_2 = PS$ ]
If [ $v_T = ZE$ ] AND [ $\omega = PS$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = NB$ ] AND [ $f_2 = PB$ ]
If [ $v_T = NS$ ] AND [ $\omega = PS$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = NS$ ] AND [ $f_2 = PB$ ]
If [ $v_T = PS$ ] AND [ $\omega = NS$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = NV$ ] AND [ $f_2 = PV$ ]
If [ $v_T = NS$ ] AND [ $\omega = NS$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = PS$ ] AND [ $f_2 = NB$ ]
If [ $v_T = PS$ ] AND [ $\omega = NM$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = PM$ ] AND [ $f_2 = NV$ ]
If [ $v_T = NS$ ] AND [ $\omega = NM$ ] AND [ $\dot{v}_T = P$ ] THEN [ $f_1 = PV$ ] AND [ $f_2 = NM$ ]

Tab.3. "Undercarriage" rules.

If [ $v_{w1} = PM$ ] AND [ $v_{b1} = NM$ ] AND [ $\dot{v}_{b1} = P$ ] THEN [ $f_1 = PV$ ]
If [ $v_{w1} = PS$ ] AND [ $v_{b1} = NS$ ] AND [ $\dot{v}_{b1} = P$ ] THEN [ $f_1 = PB$ ]
If [ $v_{w1} = PM$ ] AND [ $v_{b1} = NM$ ] AND [ $\dot{v}_{b1} = ZE$ ] THEN [ $f_1 = PM$ ]
If [ $v_{w1} = PS$ ] AND [ $v_{b1} = NS$ ] AND [ $\dot{v}_{b1} = ZE$ ] THEN [ $f_1 = PS$ ]
If [ $v_{w1} = NM$ ] AND [ $v_{b1} = PM$ ] AND [ $\dot{v}_{b1} = P$ ] THEN [ $f_1 = NV$ ]
If [ $v_{w2} = PM$ ] AND [ $v_{b2} = NM$ ] AND [ $\dot{v}_{b2} = P$ ] THEN [ $f_2 = PV$ ]
If [ $v_{w2} = PS$ ] AND [ $v_{b2} = NS$ ] AND [ $\dot{v}_{b2} = P$ ] THEN [ $f_2 = PB$ ]
If [ $v_{w2} = PM$ ] AND [ $v_{b2} = NM$ ] AND [ $\dot{v}_{b2} = ZE$ ] THEN [ $f_2 = PM$ ]
If [ $v_{w2} = PS$ ] AND [ $v_{b2} = NS$ ] AND [ $\dot{v}_{b2} = ZE$ ] THEN [ $f_2 = PS$ ]
If [ $v_{w2} = NM$ ] AND [ $v_{b2} = PM$ ] AND [ $\dot{v}_{b2} = P$ ] THEN [ $f_2 = NV$ ]

### Simulation results

In this section, the controller was tested to compare the results of the designed fuzzy logic controller with a traditional passive suspension system. As an example, step responses of the undercarriage and front/rear wheels are shown in Figs. 2-6.

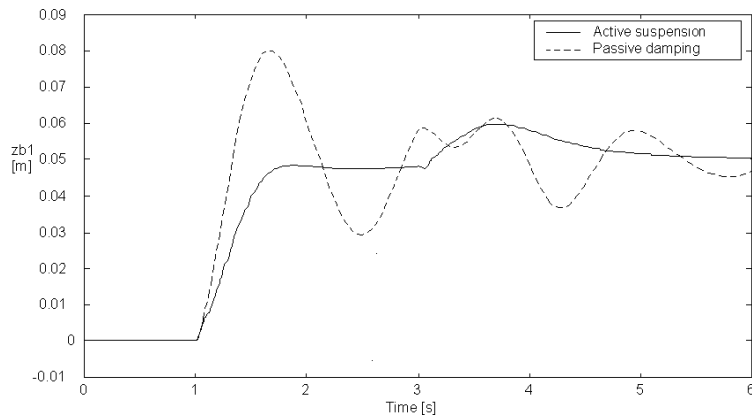


Fig.2. Undercarriage deflection above the front wheel.

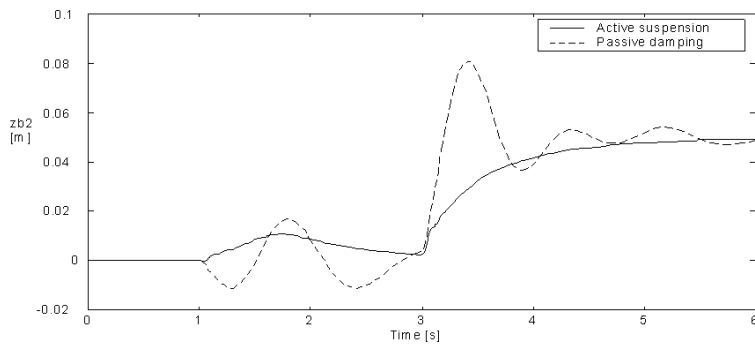


Fig.3. Undercarriage deflection above the rear wheel.

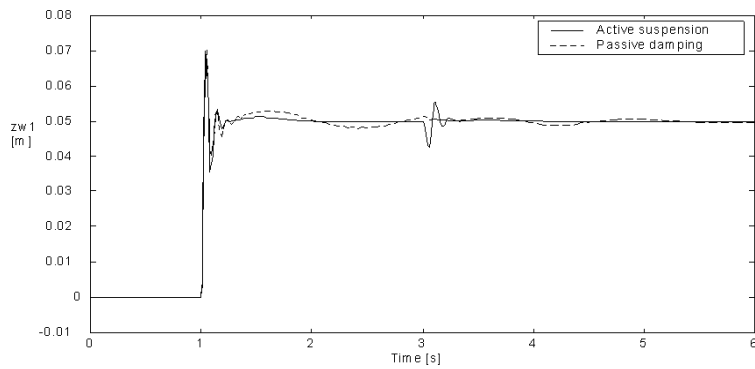


Fig.4. Front wheel deflection.

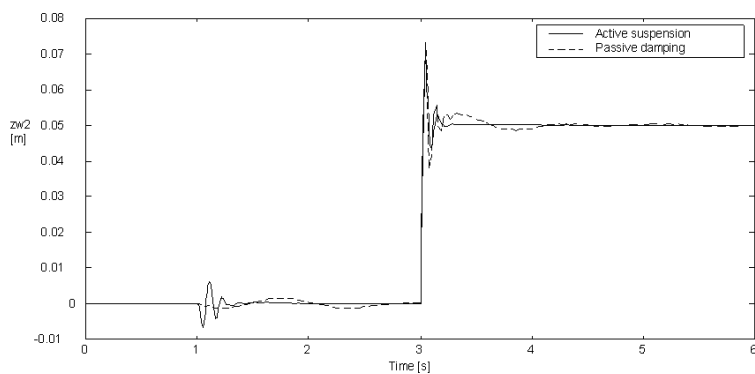


Fig.5. Rear wheel deflection.

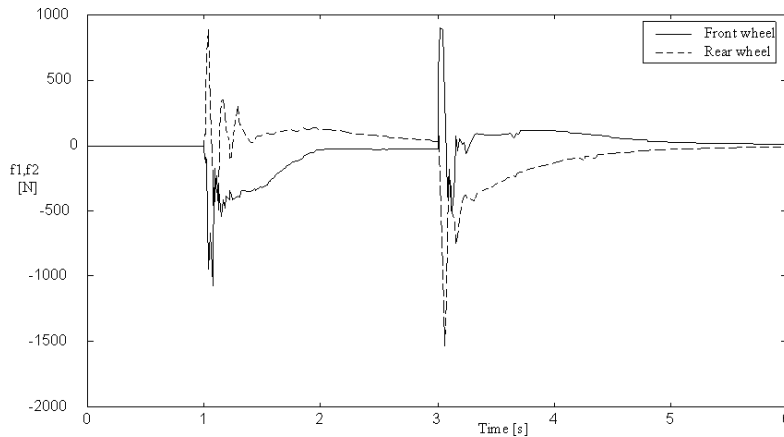


Fig.6. Active forces acting on front/rear wheels.

Many other interesting simulation examples are given in (Minarik, 2002).

### Conclusion

In this paper, the new active suspension control system is proposed to achieve both ride comfort and good handling. This aim was achieved with respect to the simulation results.

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