Implicit model fllowing via suboptimal control for input delay system

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Suboptimálne riadenie systému s oneskorením na vstupe implicitným modelom

The stabilization by feedback control of systems with input delays may be considered in various frameworks. In this paper the problem of implicit model following for input delay systems via optimal control is approached. The optimal control is implemented by piecewise constant signals that make it suboptimal. The simulation results showed the robustness with respect to the delay.

Key words: *time delay systems, linear quadratic problem, model following, piecewise constant control.*

Introduction

Systems with input delay are of interest to control theorists and practitioners for various reasons. They originate from the simplest model of process control which assigns to the controlled plant a transfer function of the form $H(s)e^{-hs}$ where $H(s)$ is strictly proper rational function. To this transfer function one may associate one of the following state representations:

$$\dot{x} = Ax + bu(t-h), \quad y = c^* x \qquad \text{or} \qquad \dot{x} = Ax + bu(t), \quad y = c^* x(t-h)$$

where in both cases $c^*(sI - A)^{-1}b \equiv H(s)$. These representations have in common the obvious fact that the state space is finite dimensional but the input operator in the first case or the output one in the second case are defined on infinite dimensional extensions and are unbounded. Control problems have been formulated and solved for such systems since the classical (transfer function based) period (the '50ies) and the mostly known result is that based on Smith predictor. The systems thus designed could be either non-robust or unstable (some times). For this reason the more recent techniques based on state space ensuring feedback stabilization and optimality of some quadratic criterion were applied (Olbrot, 1978; Manitius and Olbrot, 1979; Mee, 1973).

These papers were followed by those of Pandolfi (see, e.g. (Pandolfi, 1991)) containing a strong criticism of the first ones; this criticism was motivated by the fact that the results had been obtained by *ad-hoc* methods and not in a standard way, by deduction from an abstract theory. The main idea of Pandolfi was to re-introduce in the model the *propagation effects*. Since pure delay occurs from lossless telegraph equations with matched boundary conditions, this model is re-introduced and the system with input delay becomes a boundary condition for the partial differential equations which replace the delay.

The aim of this paper is a more applied one, closed to the idea of sub-optimality. We consider the problem of implicit model following, that is where the model is an „abstract" one and its states are not available for control purposes. The objective is to control a given system in such a manner that its output „imitates" that of a prescribed model. Further, feedback control by piecewise constant signals is considered; this is a suboptimal control (see the pioneering paper (Halanay and Răsvan, 1977) that showed the suboptimality of the piecewise constant controls at the level of the performance in the standard linear quadratic optimal stabilization problem).

Implicit Model Following via Optimal Control

Given the system with input delay:

$$\dot{x}(t) = Ax(t) + Bu(t-h), \quad y(t) = Cx(t) \tag{1}$$

and the model

$$\dot{x}_m(t) = A_m x_m(t) \tag{2}$$

we want to find the control law $u(t)$ such that the system output $y \to x_m$ (the order of the model is choosen equal with the dimension of y). Define the performance index as

$$J(u) = \int_0^\infty [(\dot{y}(t) - A_m y(t))^T Q(\dot{y}(t) - A_m y(t)) + u^T(t)Ru(t)]dt \tag{3}$$

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where: R is positive definite and Q positive semi-definite (R and Q are symmetrical matrices). Replacing y from (1) in (3) we obtain

$$J(u) = \int_0^{\infty} [x^T(t)Q_0x(t) + 2x^T(t)S_0u(t-h) + u^T(t-h)R_0u(t-h) + u^T(t)Ru(t)]dt \qquad (4)$$

where:

$$Q_0 = (CA - A_mC)^T Q(CA - A_mC); \quad S_0 = (CA - A_mC)^T QCB; \quad R_0 = B^T C^T QCB \qquad (5)$$

Since on $(0, h)$ the input signal is determined exclusively by the initial condition $u_0(\theta), \theta \in [-h, 0)$, the trajectory $x(t)$ on $(0, h)$ cannot be influenced by optimisation. So, we obtain the following equivalent linear quadratic problem defined by

$$J_1(v) = \int_h^{\infty} [x^T(t)Q_0x(t) + 2x^T(t)S_0v(t) + v^T(t)R_1v(t)]dt \qquad (6)$$

with $v(t) = u(t-h), t > h,$ and $R_1 = R_0 + R$ (we note that $R_1 > 0$), along the solution of

$$\dot{x}(t) = Ax(t) + Bv(t) \qquad (7)$$

Now, using a new control variable

$$z(t) = S^T x(t) + R_1^{1/2} v(t), \quad \text{with} \quad S = S_0 R_1^{-1/2} \qquad (8)$$

we obtain the equivalent linear quadratic problem defined by

$$J_1(z) = \int_h^{\infty} [x^T(t)Q_1x(t) + z^T(t)z(t)]dt \qquad (9)$$

along the solution of

$$\dot{x}(t) = A_1x(t) + B_1z(t), \qquad (10)$$

where:

$$\begin{aligned} A_1 &= A - BR_1^{-1}B^TC^TQ(CA - A_mC); \quad B_1 = BR_1^{-1/2} \\ Q_1 &= (CA - A_mC)^T (Q - QCBR_1^{-1}B^TC^TQ)(CA - A_mC) \end{aligned} \qquad (11)$$

This problem has the solution (Mee, 1973)

$$z(t+h) = -B_1^T Px(t+h), \quad t > 0 \qquad (12)$$

where: P is the positive definite solution of the matrix Riccati equation

$$A_1^T P + PA_1 - PB_1B_1^T P + Q_1 = 0 \qquad (13)$$

Finnaly, the following optimal state feedback law is obtained

$$u(t) = F\left[x(t) + \int_{-h}^{0} e^{-A(\theta+h)} Bu(t+\theta)d\theta \right] \qquad (14)$$

where:

$$F = -(R + B^TC^TQCB)^{-1}[B^TP + B^TC^TQ(CA - A_mC)]e^{Ah} \qquad (15)$$

Discrete Time Implementation

The implementation of the designed compensator requires memorizing of a trajectory segment i.e. a set of data that has infinite size. The practical implementation is finite and based on a suitable discretization. So, we shall use *piecewise constant control signals* (Halanay and Răsvan, 1977), defined as follows

$$u(t) = u_k, \quad k\delta \le t < (k+1)\delta, \quad k = 0, 1, 2, \ldots \qquad (16)$$

where: $\delta = h/N$. For the system (1), we associate the discrete time system

$$x_{k+1} = \mathbf{A}(\delta)x_k + \mathbf{B}(\delta)u_{k-N}, \quad y_k = Cx_k \qquad (17)$$

where:

$$\mathbf{A}(\delta) = e^{A\delta}, \quad \mathbf{B}(\delta) = \left(\int_0^{\delta} e^{A\theta} d\theta \right) B \qquad (18)$$

Let $(x_0, u_0(\cdot))$ be the initial condition associated with (1). Since the discretized system is satisfied by $x_k = x(k\delta)$, $x(\cdot)$ being the solution of (1) with piecewise constant control, it is only natural to choose the discretized initial condition $(x_0; u_{-i}^0 = u_0(-i\delta), \quad i = \overline{0, N})$.

We deduce that the compensator

$$u_k = F\left[x_k + \sum_{-N}^{-1} \mathbf{A}(\delta)^{-(N+j+1)} \mathbf{B}(\delta) u_{k+j} \right] \tag{19}$$

is stabilizing for (17) provided $\delta > 0$ is small enough.

Finally, following the line in (Halanay and Răsvan, 1977) it can be shown that the optimal state feedback (14), (15) implemented by piecewise constant control (16), (19) and (18) it is stabilizing and suboptimal for the system (1) with the performance index (3).

Conclusions

The problem of implicit model following for input delay systems via optimal control was formulated and solved. The optimal control was implemented by piecewise constant signals that made it suboptimal. The simulation results showed the robustness with respect to the delay.

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