Determining Rheological Parameters of Generalized Yield-Power-Law Fluid Model

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Abstract

The principles of determining rheological parameters of drilling muds described by a generalized yield-power-law are presented in the paper. Functions between tangent stresses and shear rate are given. The conditions of laboratory measurements of rheological parameters of generalized yield-power-law fluids are described and necessary mathematical relations for rheological model parameters given. With the block diagrams, the methodics of numerical solution of these relations has been presented. Rheological parameters of an exemplary drilling mud have been calculated with the use of this numerical program.

Key words: determining rheological parameters, rheological model parameters

Introduction

Assumption of an appropriate rheological model influences the correctness of the obtained results of flow pressure losses measurements; besides it enables rational designing of drilling fluids pumping technological operations. The measurement of rheological parameters of drilling fluids can be made with the use of a rotary 6- or 12-range viscosity meter Fann. Such viscosity meters enable determining tangent stresses values for various rotary speeds.

The selection pricioples for an optimum rheological model of drilling fluids as well as determining rheological parameters of drilling fluids described with the Newton, Bingham, Cassson, Oswald-de Waelle and Herschel-Bulkley models with the use of statistical analysis methods are described in the paper [5].

Methodics of determining rheological parameters

Determining rheological parameters of drilling fluids is possible after establishing tangent stresses in the fluid in the presence of definite shear rate values [1], [3], [4]. Depending on the assumed rheological model of the drilling fluid, one or some measurements of tangent stresses have to be made, corresponding to the given shear rates. The assumed shear rate values should stay within the range $\dot{\gamma} \in [\dot{\gamma}_{MIN}, \dot{\gamma}_{MAX}]$. For the applied flowing mud volumes, well diameters, elements of the string and casing, the following assumptions should be made:

for the flow inside the pipes:

$$\dot{\gamma} \in [\dot{\gamma}_{100}, \dot{\gamma}_{600}] \tag{1}$$

for the flow in the annular space:

$$\dot{\gamma} \in [\dot{\gamma}_{1}, \dot{\gamma}_{100}]. \tag{2}$$

API standards [2] limit the range of shear rate values registered inside the pipes to $\dot{\gamma} \in [\dot{\gamma}_{300}, \dot{\gamma}_{600}]$

The principles of determining rheological parameters of Newtonean, Bingham, Casson, pseudoplastic and Herschel-Bulkley fluids on the basis of measurement results with a Fann viscosity meter are presented in paper [6].

Analyzing the behavior of actual drilling muds, a generalized yield-power-law model should be applied:

$$\tau^A = \tau_0^A + k \cdot \dot{\gamma}^C \tag{3}$$

Assuming a generalized yield-power-law model, its rheological parameters A, C, τ_0, and k are determined from the following system of equations:

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$$\begin{cases} \tau_{600}^{A} = \tau_{0}^{A} + k\left(\dot{\gamma}_{600}\right)^{C} \\ \tau_{300}^{A} = \tau_{0}^{A} + k\left(\dot{\gamma}_{300}\right)^{C} \\ \tau_{200}^{A} = \tau_{0}^{A} + k\left(\dot{\gamma}_{200}\right)^{C} \\ \tau_{100}^{A} = \tau_{0}^{A} + k\left(\dot{\gamma}_{100}\right)^{C} \end{cases} \tag{4}$$

After transformations, the system of equations (4) assumes the following form:

$$\begin{cases} \dfrac{\tau_{600}^{A} - \tau_{300}^{A}}{\tau_{600}^{A} - \tau_{200}^{A}} = \dfrac{\dot{\gamma}_{600}^{C} - \dot{\gamma}_{300}^{C}}{\dot{\gamma}_{600}^{C} - \dot{\gamma}_{200}^{C}} \\ \dfrac{\tau_{600}^{A} - \tau_{300}^{A}}{\tau_{600}^{A} - \tau_{100}^{A}} = \dfrac{\dot{\gamma}_{600}^{C} - \dot{\gamma}_{300}^{C}}{\dot{\gamma}_{600}^{C} - \dot{\gamma}_{100}^{C}} \end{cases} \tag{5}$$

The Authors propose to solve the system of equations (5) with numerical methods. For this purpose, a special algorithm was worked out at the Department of Drilling and Geoengineering at the Faculty of Drilling, Oil and Gas, AGH-UST (fig. 1).

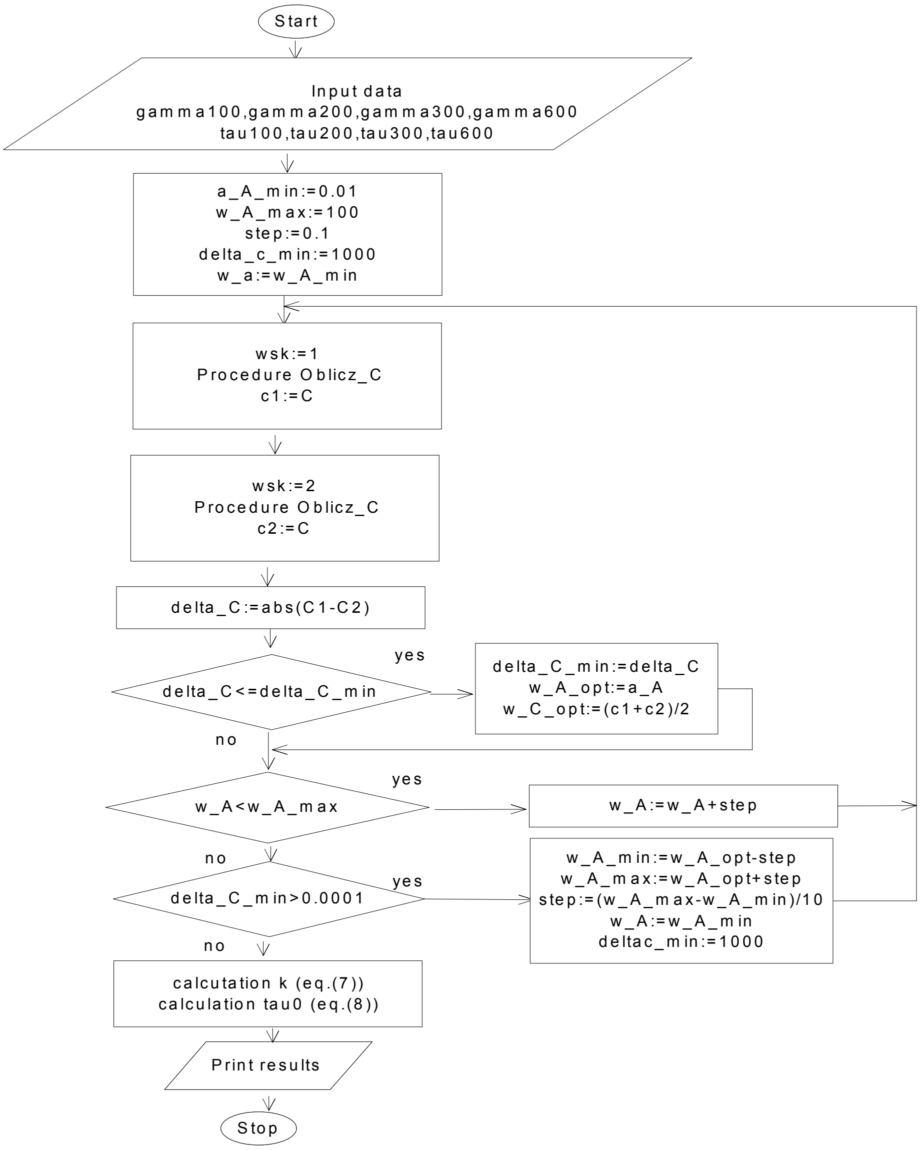

Fig. 1. Block diagram of determining rheological parameters of a generalized yield-power-law fluid

The nature of the proposed methodics lies in sequential solving of C value.

Thus, for the given minimum A value, the first and the second equations in the system are solved numerically (5), determining C1 and C2 elements and establishing differences between them (delta_C). The CI and C2 values can be determined employing an arbitrary numerical method of solving algebraic equations.

To determine C1 and C2 elements, a function g(x) is construed and its zero point is determined. A range halving method is applied in the proposed algorithm (fig.2).

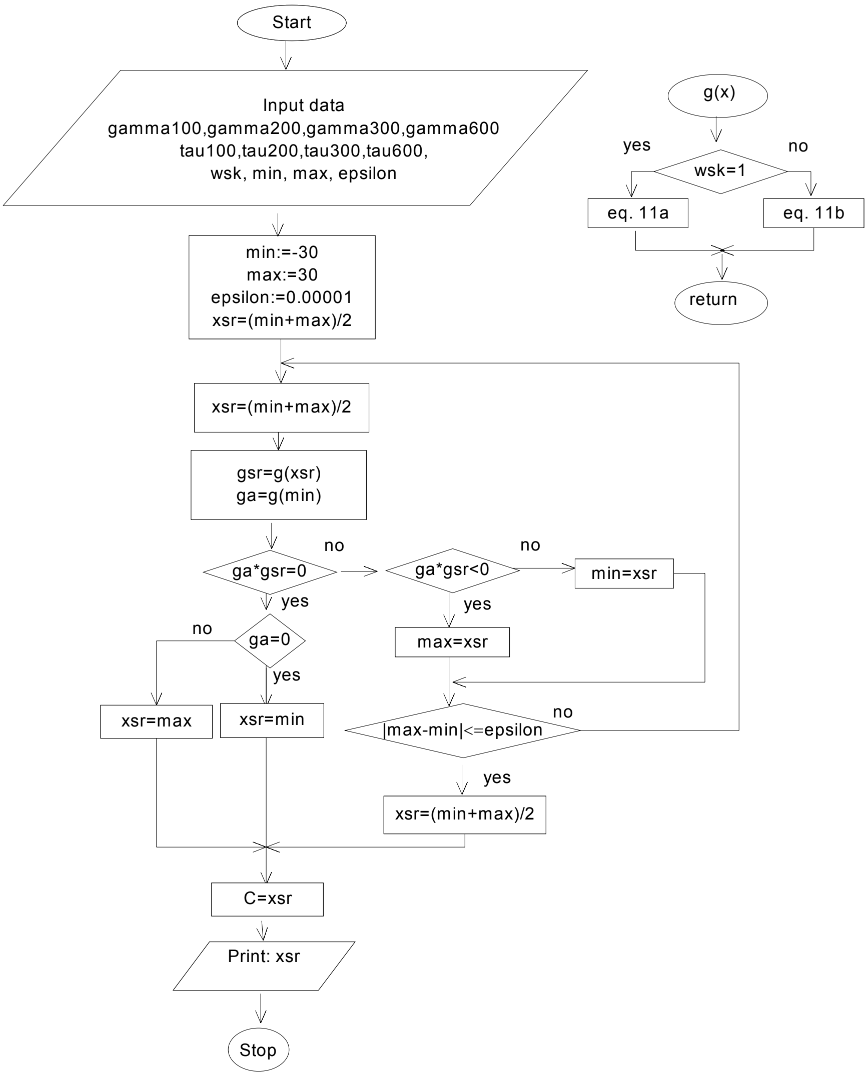

Fig. 2. Algorithm for C1 or C2 parameters determination, employing the range halving method (Procedura Oblicz_C)

When calculating C1 value, the following equation is applied:

$$g(x) = \frac{\tau_{600}^{A} - \tau_{300}^{A}}{\tau_{600}^{A} - \tau_{200}^{A}} - \frac{\dot{\gamma}_{600}^{x} - \dot{\gamma}_{300}^{x}}{\dot{\gamma}_{600}^{x} - \dot{\gamma}_{200}^{x}} \tag{6a}$$

To define C2 value, the below relation is employed:

$$g(x) = \frac{\tau_{600}^{A} - \tau_{300}^{A}}{\tau_{600}^{A} - \tau_{100}^{A}} - \frac{\dot{\gamma}_{600}^{x} - \dot{\gamma}_{300}^{x}}{\dot{\gamma}_{600}^{x} - \dot{\gamma}_{100}^{x}} \tag{6b}$$

Knowing the values of rheological parameters A and C, the remaining data can be defined. The parameter k can be determined from:

$$k = \frac{\tau_{600}^{A} - \tau_{300}^{A}}{\dot{\gamma}_{600}^{C} - \dot{\gamma}_{300}^{C}} \qquad (7)$$

whereas the parameter τ_0 from:

$$\tau_0 = \left(\tau_{600}^{A} - k\dot{\gamma}_{600}^{C}\right)^{\frac{1}{A}} \qquad (8)$$

A –coefficient of generalized yield-power-law model;
C - coefficient of generalized yield-power-law model;
η - plastic viscosity [Pa•s];
$\dot{\gamma}_i$- shear rate measured for i-th rotary speed, [s^{-1}];
τ - tangent stress, [Pa];
τ_i- -tangent stress measured for i-th rotary speed, [Pa];
τ_0 – yield point, [Pa];

Summing up

The correct selection of drilling mud rheological model enables proper designing of its pumping technology. Rheological parameters of fluids should be determined in the conditions corresponding to the range of shear rate changes in the course of the fluid flow.

One of the models having qualities of both yield-power-law and pseudoplastic fluids in its characteristic is a generalized yield power model. This model should be applied in an industrial practice.

Rheological parameters of the generalized yield-power-law model should be determined on the basis of the methodics presented in the paper.

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