Methods of modeling and optimization of work effects for chosen mineral processing systems

Tadeusz Tumidajski*[1], *Daniel Saramak, Dariusz Foszcz a Tomasz Niedoba

Metódy modelovania a optimalizácie práce pre zvolené úpravnícke systémy

The methods being used in the mineral processing modeling are reviewed in this paper. Particularly, the heuristic approach was presented. The new, modern techniques of modeling and optimization were proposed, including the least median squares method and genetic algorithms. The rules of the latter were described in details.

***Key words**: mathematical modeling, least median squares method, genetic algorithms*

Introduction

The processing of minerals increasingly takes an advantage of the automation development and of the computer-steering course of technological processes. It requires applying of suitable measuring and controlling apparatus, computer data collecting and then their processing in order to build desirable mathematical models of processes and their systems. In industrial conditions, a phenomenological approach based on the analysis of active and passive experimental results is frequently applied (Goldberg, 2003). It allows to register dependencies between the course of indexes and conditions in which a process is conducted. It seeems, that in many events a heuristic approach in the mathematical modeling of industrial mineral processing is applied. This heuristic approach, which is based on laws of the course of phenomena determining the satisfactory course of the process, enables to define a justified form of the model. A work effect of an industrial mineral processing system depends on the performance of the whole system of processes, namely the cooperation between its individual components. That fact requires a modeling approach to the system of processes, which takes into consideration their combinations and which ensures the work optimization of the whole system (Maczka, Trybalski, 1976; Tumidajski et al., 2004).

A review of applied mineral processing models

The mathematical models of mineral processing industrial processes applied so far can be divided into several groups depending on: the method of building the model, the structural construction of the model, the specificity of the process and the level of the using process information. Heuristic models mentioned above are in general based on the mass balance and the energy balance equations, as well on equations describing considered physiochemical processes. Let's consider a single grain feature w and therefore assume that a separation of elementary fractions $(w, w + dw)$ into the product $(x, x + dx)$ occurs, where x denotes a continuous variable dividing the set of grains (a feed) into products. The random variable W is described by the density function $f(w)$, the participation of elementary fraction $(w, w + dw)$ denotes the product of $f(w)$ and dw. Let's then $p(w, x)$ denote the probability of the grain transfer (or a part of the grain) from the fraction $(w, w + dw)$ into the product $(x, x + dx)$ and $g(x)$ denote a function of the continuous random variable X density. We can write that (Tumidajski, 2004):

$$\int_D p(w,x) f(w) dw = g(x) \tag{1}$$

where D characterizes the changeability area of variable W, namely $D = (w_{min}, w_{max})$. It is a concise denotation of a crucial law for processes of mineral processing – the mass balance law. Considering models of mineral processing, processes which use a mass or transport preservation law are:

[1] *prof., Tadeusz Tumidajski, Daniel Samarak, PhD., Darius Foszcz, PhD, M.Sc. Tomasz Niedoba*, University of Science and Technology, Department of Mining and Geoengeenering, Tadeusz.Tumidajski@agh.edu.pl, dsaramak@agh.edu.pl, foszcz@agh.edu.pl, tniedoba@isk.net.pl

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a) a model of comminution kinetics (Bass, 1954; Lynch, 1997)

$$P(x,t) = P(x,0) + \int_0^t \int_{w=x}^{w_{\max}} \Delta(w) B(x,w) \frac{\partial P(w,t)}{\partial w} dw dt \tag{2}$$

where: *P(x,t)* is the yield of grains with dimensions smaller than *x, P(x, 0)* is a share of such grains in time *t = 0*, *S(w)* is the function describing a participation (probability) in crushing of grains with dimension *w, B(x, w)* is the probability of transfer of grain particles from the class *(w, w + dw)* into class *(x, x + dx)*

b) a set of differential equations describing the kinetics of one-way processes (Siwiec, 1982).

$$\begin{cases} y_1'(t) = k_1 \gamma_1(t) \\ y_p'(t) = k_p \gamma_p(t) + \sum_{i=1}^{p-1} k_{ip} \gamma_i(t) \\ y_n'(t) = \sum_{i=1}^{n-1} k_{in} \gamma_i(t) \end{cases} \tag{3}$$

where k_{ip} and k_{in} are coefficients describing the transfer of grains from the coarse class (with yield γ_i) into the finer class (subsequent products).

c) phenomenological discrete models built on the basis of discretisation of the signal gathered from a measuring apparatus applied in mineral processing plants. Such models use an auto-regression of the analyzed value, a moving average, and so-called exogenous values. As a result, we obtain ARMA, ARMAX or ARX-type models (Tumidajski, 2004).

d) Discrete models taking advantage of the processes laws (semi-heuristic). Let's assume that the amount of feed (production capacity) and the density of the overflow in a spiral classifier has a parabolic influence on the classification results both in classifiers and in hydrocyclones, where an overflow mass is a feed to the process of flotation. After taking into account an existence of delays in time (a necessary modeling condition for discrete models) and the previously confirmed auto-correlation, the following form of the model is proposed:

$$\begin{aligned} X_{-0,071HC}(t) &= c_1\left[F_{MP}(t-2) - F_{MP0}\right]^2 + c_2\left[F_{MP}(t-1) - F_{MP0}\right]^2 + c_3\left[F_{MP}(t) - F_{MP0}\right]^2 + \\ &+ c_4\left[F_{MP}(t-2) - F_{MP0}\right] + c_5\left[F_{MP}(t-1) - F_{MP0}\right] + c_6\left[F_{MP}(t) - F_{MP0}\right] + \\ &+ c_7\left[D_{KZ}(t-2) - D_{KZ0}\right]^2 + c_8\left[D_{KZ}(t-1) - D_{KZ0}\right]^2 + c_9\left[D_{KZ}(t) - D_{KZ0}\right]^2 + \\ &+ c_{10}\left[D_{KZ}(t-2) - D_{KZ0}\right] + c_{11}\left[D_{KZ}(t-1) - D_{KZ0}\right] + c_{12}\left[D_{KZ}(t) - D_{KZ0}\right] + \\ &+ c_{13}P_{HC}(t) + c_{14}X_{-0,071HC}(t-1) + c_{15} \end{aligned} \tag{4}$$

where $F_{MP}(t)$ is the feed amount; $D_{KZ}(t)$ is the density of overflow in spiral classifier; $P_{HC}(t)$ is the tension of feed in hydrocyclones; $X_{-0,071HC}$ is the content of –0,071 mm class in the overflow of the hydrocyclone; t-1 is the 5-minute delay; t-2 is the a 10-minute delay; D_{KZ0} and F_{MP0} are the values of density and processing accepted as optimal (normative, given values*)*.

The results obtained in empirical tests confirmed a conformity of model with the reality.

e) a work model of specific appliance. The model describing the work of hydrocyclones is given as an example (d_{50} – split size) (Karr, Yeager, 2003)

$$d_{50} = \frac{C_1 D_c^{C_2} D_i^{C_3} D_o^{C_4} \exp[C_5 \phi]}{D_u^{C_6} h^{C_7} Q^{C_8} \rho^{C_9}} \tag{5}$$

where D_c is the diameter of the hydrocyclone, D_i is the diameter of the slurry input, D_o is the diameter of the overflow, D_u is the diameter of the underflow, h is the height of the hydrocyclone, Q is the volumetric flow rate into the hydrocyclone, ϕ is the percent solids in the slurry input, ρ is the density of the solids, C_i are empirical constants.

f) mathematical models of separation curves based on the heuristic analysis of the separation processes. The example of such curve may be the curve obtained and described in the position (Brożek, Turno, 2004), with the form:

$$T(\rho) = \frac{1}{1 + \exp\left[-\frac{K_1(\rho - \rho_r)gd}{K_2\left(\rho_r v_r^2 + c\overline{\rho} v_p^2\right)} \right]} \tag{6}$$

where K_1 and K_2 are the shape coefficients, d is the grain size, c is the volumetric concentration of grains in the enrichment chamber, v_r^2 is the average square speed of turbulent liquid motion, ρ_r is the density of liquid which is a separation density for coarse grains.

g) models taking into consideration convection and diffusion phenomena, which lead to the Focker-Planck-Kolmogorov equation (Siwiec, 1982)

$$\frac{\partial p}{\partial t} = -k_1 \frac{\partial p}{\partial x} + \frac{1}{2} - k_2 \frac{\partial^2 p}{\partial x^2} \tag{7}$$

where $p(x,t)$ is the probability of the grain position (x, x+dx) in the moment t; k_1 and k_2 are constants.

Applied approximation methods

The most popular approximation method is the least squares method (LS), which is based on the minimization sum of squares of residual errors. This methods has a fundamental disadvantage, namely it is sensitive to excessive errors. The next and more effective method, applied in the determination of the models' parameters, is the least median squares method (LMS) (Karr *et al,* 1995). LMS method limits to a large extent the influence of extensive errors to the obtained modeling results as it is an extensive-error-proof procedure. In the approximation we use only a part of data taken from the whole n-element data set, creating $k = \binom{n}{p}$ p-element subsets from the initial data set. Then we make k-number of auxiliary approximations, and finally, in the base of them, compute the final estimator by using either a repetitive or a single median procedure. Formulas are presented below. For the repetitive procedure:

$$\hat{\theta} = \underset{i_1}{med}\left(\ldots \left(\underset{i_{p-1}}{med}\left(\underset{i_p}{med}\, \tilde{\theta}(i_1, \ldots, i_p) \right) \right) \right) \tag{8}$$

For the single procedure:

$$\hat{\theta} = \underset{i_1, \ldots, i_p}{med}\, \tilde{\theta}(i_1, \ldots, i_p) \tag{9}$$

It is obvious that with a change of feed characteristics connected with the mine works' progress and the conditions of devices' working, the model coefficients will change their values too. Because of this, to ensure the conformity of the model with the reality, its adaptation is needed, together with the resulting monitoring algorithms adaptation. Assuming the linear general model form, it may be presented:

$$\hat{y} = f(X, C) = \sum_{j=1}^{k} c_j \varphi_j(X) \tag{10}$$

where X is the dependent variables vector; C is the vector of coefficients c_j, j = 1, ..., k, N is the number of cases – measured data sets $X(x_{1i}, x_{2i}, \ldots, x_{ki}, y_i)$, i = 1, ...,N; $N \geq k$, $\varphi_j(X)$ is the multi variable functions system.

The coefficients of this model may be obtained by the least squared method and its adaptation to the changed conditions by the antigradient method. This is done by the actual selection of coefficients minimizing the square of difference between the value y determined by the model and the new measured value (Tumidajski, Kunysz, 2002).

Another solution of the adaptation problem may be the recurrent least squared method, in which the error measure (divergence from reality) is assumed (Rutkowska *et al.,* 1997; Tumidajski, 1994; Tumidajski, Kunysz, 2002):

$$Q(n) = \sum_{t=1}^{n} \lambda^{n-t} \left[y_i - f(X, C) \right]^2 \tag{11}$$

where λ is the forgetting coefficient, chosen from the range (0,1).

If we confront with non-linear functions, which cannot be simplified to the linear form, then we have to determine the loss function by estimating the value of the forecasted results deviation from the empirical ones. These functions may be as follows:

a) $L(C_1,\ldots,C_k)=\sum_{i=1}^{n}\left(y_i-f(x_{1i},\ldots,x_{mi};C_1,\ldots,C_k)\right)^2$ - quadratic loss function,

b) $L(C_1,\ldots,C_k)=\sum_{i=1}^{n}w_i\left(y_i-f(x_{1i},\ldots,x_{mi};C_1,\ldots,C_k)\right)^2$ - scaled loss function,

where w_i are scales added to the empirical results

c) $L(C_1,\ldots,C_k)=\sum_{i=1}^{n}\left|y_i-f(x_{1i},\ldots,x_{mi};C_1,\ldots,C_k)\right|$ - modular loss function.

When searching for the minimum of the function L, we give the values of parameters $C_1,...,C_k$.

The procedure of the estimation is usually conducted by using various methods (it depends on the type of the researched function). This methods are e.g. the quasi Newton method (method of minimisation in the direction of the highest slope) the simplex method, the simplex and quasi Newton method, the Hooke-Jeeves method, the Rosenbrack method, the Hooke-Jeeves and the quasi Newton method; the Rosenbrack and the quasi Newton method, the Powell method, the Zangwill method, the coupled gradient method, the Fletcher and Reeves method (Dryja et al*.,* 1982, Magiera, 2002). Some of these methods are available in the STATISTICA program package.

Furthermore, in case of the affirmation of changeability of the regression function parameters in certain intervals of dependent random variables, it is possible to divide this function into intervals. This operation is called the segment regression.

New methods of model coefficients and optimisation of processing plants work

Nevertheless, in the industrial mineral processing is difficult to apply empirical computer models both in the project (description) as well as in the control application stage. This is connected with very limited possibilities of applying traditional statistical methods and a long time connected with calculations (collecting data).

The genetic algorithms were applied to solve difficult researching problems and occurred to be very efficient in the adaptation of empirical models in the mineral processing systems. This approach lowered significantly the time consumed to find computer model coefficients based on empirical values and adjusted the preciseness of given models (Goldberg, 2003; Karr, Yeager, 1995; Michalewicz, 2003; Roberts, 1976; Rutkowska et al, 1997; Svedensten, Evertsson, 2005).

To prepare conditions of the genetic algorithm application for determining parameters of the searched equations describing the course of researched process, a searching of these parameters needs to be applied. This is done by generating their following values and accepting the function which determine the model conformity with the existing data. This is so-called the adaptation function its value decides what parameters‘ values should be chosen. The coding leads to the generation so-called of the chromosomes (numbers in the binary system) their crossing and adjustment (mutation) is realized by the genetic algorithm. The way of coding and the genetic operators form (mutations and crossings) depends exactly on the problem which is solved. As the result of the crossing operator, a new specimen is created, whose chromosomes are composed from randomly chosen fragments of two or more specimens‘ chromosomes. The classical mutation operator consists of the negation of the randomly chosen chromosome bit with some very low probability. The purpose of mutation is to give new parts of solutions (in fact new schemes). It is worth to say that a too high probability of mutation leads to a chaos in the genetic process. The lack of mutation may lead to the closure of algorithm in the solutions subspace. The assertion of the unambiguous character of the adaptation function is not always possible. However, it is recommended to estimate the same specimen during the whole time of genetic algorithm operation similarly. In the same time, the basic feature of the adaptation function must be retained so that the specimens better adapted must be evaluated to be higher. There are many methods of the selection. To apply the theorem of schemes, a method must be used in which the number of representatives of the specimen from the previous generation in ther new one is proportional to the ratio of this specimen adaptation to the mean adaptation of specimens of its generation. The basic method of this type is the roulette method. It relyes on the division of the roulette circle into specimens of the previous generation by assuring the probability of drawing certain specimen to be proportional to their adaptation value. Next, we turn the roulette and allow the ball to find the specimen, which will be transferred to the next generation. We turn the roulette as long as we fill the population of the new generation. To secure the precision of selection, the super specimens of very high values of adaptation in comparison with the mean value of adaptation, being created from time to time, should not dominate the new generation. This would close the genetic process round the local minimum. A similar problem may occur when the genetic process approach to the certain solution and the values of specimens‘

adaptation are very similar. This would make difficult to select correctly the next generation and, probably, to adjust the solution. The solution of this problem is the rescaling of each adaptation values (Goldberg, 2003; Michalewicz, 2003).

In the paper (Karr, Yeager, 1995), an equation describing the hydrocyclone work was presented as an example of equation parameters determination (5). It occurred that the application of the genetic algorithm allows to achieve satisfying parameters' values just after 10 minutes, while the statistical package needs 9 hours to do the same.

In many cases, because of existing equation parameters, the complicated non-linear equations forms cause that the traditional approach by the least squares method is not effective. Furthermore, the technique connected with the optimum searching is not effective too. In these cases, the genetic algorithms, whose purpose is to select stable models, are not only convenient but also may be absolutely necessary.

The genetic algorithms take a huge part the modelling in industrial processing plants work. It concerns both the process course simulation and the optimisation of the whole plant work. The purpose of their application is to identify the composition of optimal work conditions of technological system parts by ensuring the most efficient work of the whole system. In this approach, many mathematical models of processes (devices), being the system parts, are applied which are connected by allowing the creation of a system model (describing its general activity). It contains values which make possible the plant work optimisation. In this case, the term "optimisation" means the maximization of the profit function with taking into consideration the imposed limitations on parameters determining the products quality and on the monitoring parameters.

The most commonly used optimisation concerns the achieved economic effects, which are the profit maximization or the loss minimization. So, the optimisation in the mineral enrichment process is generally connected with the optimal processing. The problem of optimisation is a dual task what means that its purpose may be the profit maximisation or the loss minimisation. The selection of the conducted optimisation direction is the decision maker's choice. The values of these parameters given by the maximisation of the profit function (or the minimisation of the loss function) are the optimising values. In the enrichment processes, the optimisation means also the process stabilisation. The leading of the enrichment process in this way ensures the stability of its basic parameters, like e.g. the efficiency, the feed amount or the quality of a processed ore. So, they are stable in some previously predicted and approved range. This ensures that in a stable enrichment process the contents of useful component in the produced concentrate is characterized by low deviations from its assumed value.

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