The distribution of settling velocity of non-spherical mineral particles

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Distribúcia usadzovacej rýchlosti nesférickej minerálnej častice

The settling velocity is an independent variable of the hydraulic separation performed for instance by means of jigs. Therefore, the settling velocity characterizes the material forwarded to the separation process.

The paper presents a method of determining the distribution of settling velocity in the sample of non-spherical particles for the turbulent particle motion in which the settling velocity is expressed by the Newton formula. Because it depends on thee size, density and shape factors of the particle, which are random variables of certain distributions, the settling velocity is a random variable too. Applying theorems of probability calculation concerning the functions of random variables, formulae for the frequency fractions of settling velocity were presented.

Key words: *settling velocity, distribution of settling velocity, random variables, function of random variables, non-spherical particle, particle shape*

Introduction

The terminal velocity of a particle is the settling velocity of the uniform motion, when the geometrical sum of all forces acting upon the particle is equal to zero. There are many methods of determining this terminal velocity (Sztaba, 1992). In this paper a theoretical method resulting from the solution of the particle motion equation will be used.

In the turbulent motion, the force of resistance is expressed by Newton's formula. Therefore, the force balance equation and the terminal settling velocity are as follows (Finkey, 1924; Sztaba, 2004):

$$P_\Psi = P_N = \frac{\pi}{12} K_2 \rho_0 v^2 d^2 = \frac{\pi d^3}{6} K_1 (\rho - \rho_0) g \qquad (1)$$

$$v_n = \sqrt{2g} \sqrt{\frac{K_1}{K_2} d \frac{\rho - \rho_0}{\rho_0}} = K_N z^{1/2} x^{1/2} d^{1/2} \qquad (2)$$

where: $K_N = \sqrt{2g} = 4{,}43 \left[m^{-1/2} s^{-1} \right]$ is a constant, $x = \dfrac{\rho - \rho_0}{\rho_0}$ denotes the reduced relative density, d is the particle size, g is the acceleration due to gravity, ρ is the particle density, ρ_o is the liquid density, v is the terminal settling velocity of a spherical particle, K_1 is the volume shape factor, K_2 is the dynamic shape factor equal to the ratio of particle drag coefficient to the sphere drag coefficient (Ganser, 1993; Thomson et al., 1991), and $z = \dfrac{K_1}{K_2}$ denotes the quotient shape factors.

It results from Eq.(2) thatthe settling velocity of a non-spherical particle is a function of particle size, its density, shape factors and properties of the medium in which the particle motion takes place (ρ_o, μ). During the separation of heterogeneous materials (from the point of view of their physical and geometrical properties – such as the enrichment of coal and ores), the particle size, its density and shape factors are random variables of fixed distributions. As a result, the particle settling velocity will be a random variable being a function of random variables such as particle density, size and shape factor. The form of distribution of this random variable results from Eq.(2) and from distributions of the particle size, density and shape factors. For spherical particles, $K_1=1$ and $K_2=1$and the settling velocity will be expressed by the following expression:

$$v_s = K_N x^{1/2} d^{1/2} \qquad (3)$$

This paper presents methods of calculating the non-spherical particle settling distribution according to Newton's formula because the separation in classifying devices including jigs takes place in a turbulent motion, for which the particle settling velocity is calculated from Eq.(2).

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(Recenzovaná a revidovaná verzia dodaná 23. 9. 2005)

The distribution of settling velocity according to Newton's formula for spherical particles

The reduced relative density of a particle *x,* occurs in the formulas of the particle settling velocity as:

$$x = \frac{\rho - \rho_0}{\rho_0} \tag{4}$$

Therefore:

$$\rho = \rho_0 x + \rho_0 = \rho(x) \tag{5}$$

If the random variable ρ has the distribution expressed by a frequency function $f(\rho)$, the frequency function of the random variable x is expressed according to the theorem of functions of random variables (Gerstenkorn et al. 1972) by the following formula:

$$f_1(x) = f[\rho(x)]\left|\frac{d\rho(x)}{dx}\right| \tag{6a}$$

$$f_1(x) = f(\rho = \rho_o x + \rho_o)\rho_o \tag{6b}$$

The random variable $x^{1/2}$, according to the same theorem, will have the following distribution:

a) $y_1 = x^{1/2}$ $\qquad x = y_1^2 = x(y_1)$

$$f_2(y_1) = f_1[x(y_1)]\,2y_1 \tag{7a}$$

$$f_2(y_1) = f_1\left(x = y_1^2\right)2y_1 \tag{7b}$$

Analogically, the random variable d occurs in Eq.(2) to 0.5 power. If $g(d)$ is the frequency function of variable d, the random variable $y_2 = d^{1/2}$ will have the following distribution:

$$f_3(y_2) = g[d(y_2)]\,2y_2 \tag{8a}$$

$$f_3(y_2) = g\left(d = y_2^2\right)2y_2 \tag{8b}$$

$$d = y_2^2 = d(y_2) \tag{9}$$

According to the above transformations, the particle settling velocity, as the random variable V_s, will be expressed by the following formula:

$$V_s = 4{,}43 y_1 y_2 \tag{10}$$

Denoting:

$$W = 4{,}43 y_1 \tag{11}$$

formula (10) will take the form:

$$V_s = W y_2 \tag{12}$$

and the distribution of the random variable W is:

$$f_4(w) = f_2[y_1(w)]\frac{1}{4{,}43} \tag{13a}$$

$$f_4(w) = f_2\left(y_1 = \frac{w}{4{,}43}\right)\frac{1}{4{,}43} \tag{13b}$$

$$y_1(w) = \frac{w}{4{,}43} \tag{14}$$

As it can be seen from Eq.(12), settling velocity is the product of two random variables. The frequency function of the random variable, which is the product of two independent random variables $S = T\,U$, is expressed by the following formula (Gerstenkorn et al. 1972):

$$h(s) = \int f_1(t) f_2\left(\frac{s}{t}\right)\frac{1}{t}dt \tag{15}$$

where: f_1 and f_2 are the frequency functions of random variables T and U, respectively. According to Eq.(15), the frequency function of settling velocity is:

$$h(v_s) = \int_{w_{\min}}^{w_{\max}} f_4(w)\, f_3\left(\frac{v_s}{w}\right)\frac{1}{w}dw \tag{16a}$$

$$h(v_s) = \int_{w_{min}}^{w_{max}} f_4(w)\, f_3\left(y_2 = \frac{v_s}{w} \right) \frac{1}{w} dw \tag{16b}$$

Distribution of settling velocity of spherical particles for linear distributions of particle density and size

As an example, we calculated the distribution of settling velocity for two combinations of linear frequency functions of particle size and density.

1. The sample contains mostly fine particles of low density for the ranges of particle size and density given below:

$$g(d) = (-5{,}54d + 0{,}11085)\cdot 10^3 \qquad \text{for} \quad d \in [0{,}001;0{,}02] \tag{17}$$

and

$$f(\rho) = -8{,}89\cdot 10^{-7}\rho + 2{,}445\cdot 10^{-3} \qquad \text{for} \quad \rho \in [1250;2750] \tag{18}$$

where d is expressed in [m] while ρ in [kg/m^3]. Both functions are normalized to 1, i.e.:

$$\int_{0{,}001}^{0{,}02} g(d)dd = 1 \quad \text{and} \quad \int_{1250}^{2750} f(\rho)d\rho = 1 .$$

The cumulative distribution functions of particle size and particle density are presented in Figs 1. and 2.

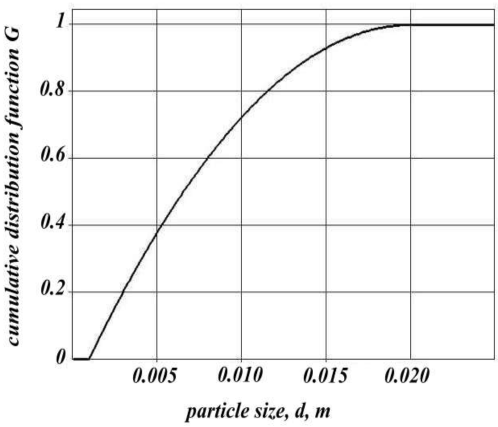

Fig. 1. Cumulative distribution function of particle size

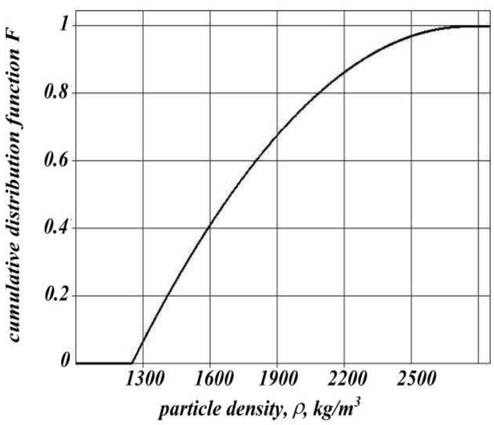

Fig. 2. Cumulative distribution function of particle density

In order to calculate the distribution of settling velocity for Newton's range (Eq.16), the distributions $f_1(x)$, $f_2(y_1)$, $f_3(y_2)$, $f_4(w)$ and $f_3\left(\frac{v_s}{w}\right)$ should be calculated. The function $f_1(x)$, according to Eqs (6b) and (18), is equal:

$$f_1(x) = -0{,}889x + 1{,}556 \qquad x \in [0{,}25;1{,}75] \tag{19}$$

Function $f_2(y_1)$, according to Eq.(7b), is as follows:

$$f_2(y_1) = -1{,}778y_1^3 + 3{,}112y_1 \qquad y_1 \in [0{,}5;1{,}32] \tag{20}$$

Function $f_3(y_2)$, according to Eq.(8b), is:

$$f_3(y_2) = (-11{,}08y_2^3 + 0{,}2217y_2)\cdot 10^3 \quad y_1 \in [0{,}03;0{,}14] \tag{21}$$

Function $f_4(w)$, according to the formula (13b), is equal to:

$$f_4(w) = -4{,}62\cdot 10^{-3}w^3 + 0{,}16w \qquad w \in [2{,}215;5{,}848] \tag{22}$$

Substituting distributions $f_4(w)$ and $f_3\left(\frac{v_s}{w}\right)$ into Eq.(16b), after integration and normalization to 1, the following formulas for the frequency of settling velocity and cumulative distribution functions are obtained:

$$h(v_s) = -117{,}12v_s^3 + 22{,}23v_s \qquad v_s \in [0{,}07;0{,}44] \ [m/s] \tag{23}$$

$$H(v_s) = \int_{0,07}^{v} h(v_s)dv_s = -29,28v_s^4 + 11,113v_s^2 - 0,054 \qquad (24)$$

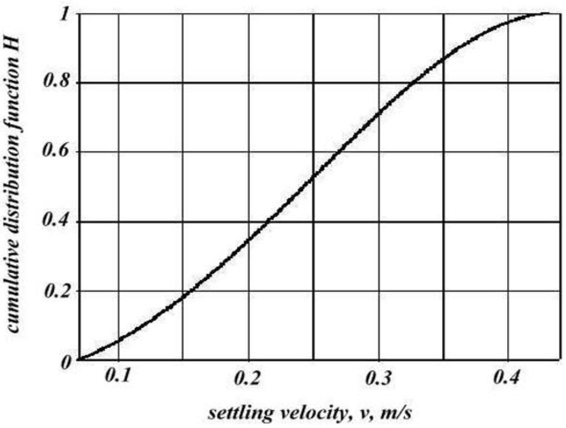

Fig. 3. presents the cumulative distribution function of settling velocity. As it can be seen in Fig.3, the largest fraction is constituted by the particles whose settling velocity is placed in the middle of the range of obtained values.

Fig. 3. Cumulative distribution function of spherical particle settling velocity according to Eq.(24)

2. The sample contains mostly large particles of high densities. The frequency functions of particle size and density are given by:

$$g(d)=5,54\cdot 10^3 d - 5,54 \qquad \text{for} \quad d \in [0,001;0,02] \qquad (25)$$

$$f(\rho) = 8,89\cdot 10^{-7}\rho - 1,11\cdot 10^{-3} \qquad \text{for} \quad \rho \in [1250;2750] \qquad (26)$$

The cumulative distribution functions are presented in Figs 4. and 5.

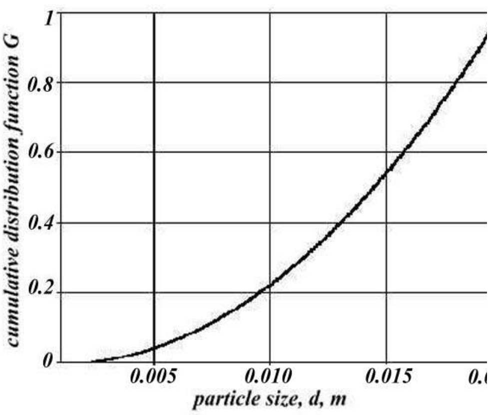

Fig. 4. Cumulative distribution function of particle size

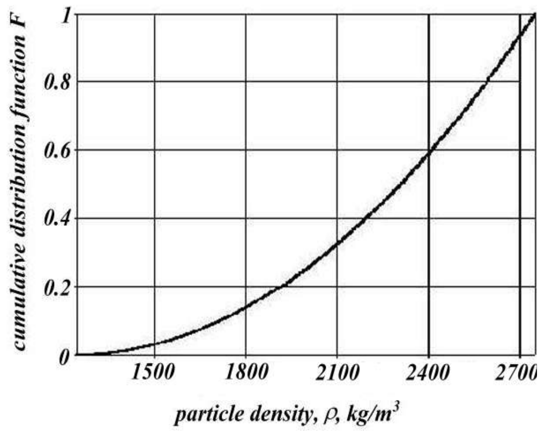

Fig. 5. Cumulative distribution function of particle density

The calculated frequency and the cumulative distribution functions of settling velocity are as follows:

$$h(v_s) = 26,563\, v_s^3 - 0,483\, v_s \qquad v_s \in [0,07\,;0,636] \;\; [m/s] \qquad (27)$$

$$H(v_s) = 6,641\, v_s^4 - 0,2415\, v_s^2 \qquad (28)$$

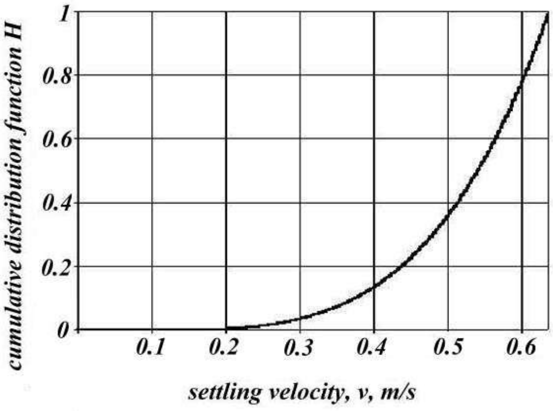

Fig. 6. presents the cumulative distribution function of settling velocity according to Eq.(28).

Fig. 6. The cumulative distribution function of spherical settling velocity according to Eq.(28).

Distribution of settling velocity of non-spherical particles

As it results from the formulas (2) and (3), the settling velocity of a non-spherical particle v_N can be expressed by the following formula:

$$v_n = z^{1/2} v_s = \left(\frac{K_1}{K_2}\right)^{1/2} v_s \qquad (29)$$

Therefore the settling velocity is the function of three random variables K_1, K_2, V_S. The random variable Z is the product of two random variables K_1 and K_2. Let the functions of distribution density be $f_1(k_1)$ and $f_2(k_2)$, respectively. Consequently, the function of distribution density of the random variable Z will be equal (Gerstenkorn et al., 1972);

$$u_1(z) = \int_0^{\infty} k_2 f_2(k_2) f_1(zk_2) dk_2 \qquad (30)$$

The following random variable $Z^{1/2} = P$, in agreement with the algorithm presented in formulas (7) and (8) will have the distribution:

$$u_2(p) = u_1(z = p^2)2p \qquad (31)$$

Consequently, the falling velocity of the non-spherical particle will be the product of two random variables

$V_S * P = V_n$ of the following composition:

$$h_n(v_n) = \int_{V_{s\min}}^{V_{s\max}} h(v_s) u_2\left(\frac{v_n}{v_s}\right)\frac{1}{v_s} dv_s \qquad (32)$$

The expression (32) presents a general formula for the function of distribution density of settling velocity of non-spherical particle in the conditions of turbulent motion. To calculate this distribution it is necessary to know the distribution of settling velocity of a spherical particle, screen diameter and distributions of shape coefficients.

Concluding

The presented algorithm of determining the distribution of settling velocity of particles concerns the random variables which are stochastically independent. The investigations performed for coal fines contribute to a hypothesis of independence of random variables representing the density and particle size (Brożek, 1993; Tumidajski, 1997). Similar investigations should be performed for all the random variables which occur in the formulas of settling velocity.

In order to determine the distribution of settling velocity of irregular particles it is necessary to consider their shape. The investigations indicate that the distributions of shape coefficients of coal particles are of the so-called gamma type (Brożek et al., 2004; Hodenberg, 1998). Also the distributions of particle size and density in case of fine coal particles (Brożek et al., 2004) are independent.

The distribution of settling velocities is the main parameter of hydraulic classification applied for fine coal separation in which the motion of particles is of turbulent character. The presented simulation of distribution of the settling velocity as a function of the distribution of particle size and density describes the velocity distribution changes due to sample characteristics. The distribution of settling velocity affects the separation efficiency measured by the probable error, (determined by means of the densimetric analysis).

This work was performed as a part of the project No.10.10.100.955 of the University of Science and Technology Research Program.

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