On Methods of Geodetic Control Densification

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Metódy zahusťovania geodetickou kontrolou

Newly established CZEPOS network on the territory of Czech Republic enlarges the point positioning possibilities by DGPS and RTK satellite methods. Control densification points can be measured by both the satellite and terrestrial methods now. Methods of common parametric adjustment of satellite and terrestrial quantitiesare emphasized a in selected processing procedures, which give results of a needed quality.

Key word: *CZEPOS network in Czech republic, sequential parametrice adjustment, adjustment into the projection plane, adjustment with fictions quantities, common adjustment of satellite and terrestrial quantities.*

Introduction

Satellite receivers and terrestrial instruments currently become the most frequently used basic instrumentation for the point positioning. With the help of satellite receivers, the baselines ***d*** among selected points, and DGPS and RTK methods directly, the spatial coordinates ${}_DX_T$, ${}_DY_T$, ${}_DZ_T$ of projected points T are obtained, (Nevosád, 2005). The sets of horizontal directions ψ, zenith angles z, slope distances d and height differences h are usually measured with terrestrial instruments. When connecting the terrestrial instruments and satellite receivers, the sets of satellite and terrestrial data are measured. They are used for separate or common parametric adjustment of newly determined control densification points. Procedures of parametric adjustment of satellite baselines and terrestrial quantities are well known, (Nevosád, 2004; Nevosád et al, 2002).

The newly established CZEPOS network in Czech Republic enlarges the measured data sets with spatial coordinates of points obtained by the DGPS and RTK methods. Appropriate computation procedures are divided into two groups:

- Sequential parametric adjustment of satellite and terrestrial measurements.
- Common parametric adjustment of all measured quantities.

This contribution discusses the possibilities of parametric adjustment of horizontal networks with both kinds of measured quantities, with the emphasis on common parametric adjustments.

1. Sequential Parametric Adjustment

The calculation procedures are well known and they are described in (Nevosad, 2005; 2002; Nevosad et al, 2002). Methods of parametric adjustment in 2D and 3D coordinates are usually discerned. In Fig. 1 and Fig. 2 the simplified computation schemes with transfer of the satellite network into the projection plane are given.

1.1 Adjustment of Satellite Network

Satellite measurements are adjusted in several ways. Principal are the epoch and direct adjustment.

Remark: Some of the computation procedures use transfers of measured terrestrial quantities and points only from the projection plane into the given spatial network. In Fig. 1 to Fig. 5 $s = 1, 2, 3 \ldots k$ means the measured sets of satellite and $V = T$, $U \equiv P, Q, R, \ldots Z$ the newly determined satellite and terrestrial points

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1.1.1 Epoch adjustment

a) Connection of measuring data sets (units) into a free network:

$$\boxed{{}_sX, {}_sY, {}_sZ} \rightarrow \boxed{{}_vX, {}_vY, {}_vZ;\ {}_sc'_X, {}_sc'_Y, {}_sc'_Z}$$

measuring sets → coordinates of free network points and relative coordinate shifts of measuring sets

b) Transformation into a given spatial network

$$\boxed{{}_vX, {}_vY, {}_vZ} \rightarrow \boxed{X', Y', Z'} \rightarrow \boxed{v_{X_T}, v_{Y_T}, v_{Z_T}} \rightarrow \boxed{X_T, Y_T, Z_T}$$

free network → conform transformation → coordinate residuals → resulting satellite coordinates

1.1.2 Direct Adjustment

$$\boxed{{}_sX, {}_sY, {}_sZ} \rightarrow \boxed{{}_sv_X, {}_sv_Y, {}_sv_Z} \rightarrow \boxed{\begin{array}{c}\delta X_T, \delta Y_T, \delta Z_T, \\ {}_s\delta c_X, {}_s\delta c_Y, {}_s\delta c_Z\end{array}} \rightarrow \boxed{\begin{array}{c}X_T,\ Y_T,\ Z_T; \\ {}_sc_X,\ {}_sc_Y, {}_sc_Z\end{array}}$$

measuring sets → residuals equations → increments of coordinates and coordinate shifts → resulting coordinates and coordinate shifts

Fig. 1. Adjustment scheme of satellite measurements.

1.2 Adjustment of Terrestrial Quantities

a) Transformation of spatial network into a projection plane

$$\boxed{X,\ Y,\ Z} \rightarrow \boxed{{}_rX',\ {}_rY',\ {}_eH'} \rightarrow \boxed{H'}$$

given spatial network → transformation into projection plane and ellipsoidal heights → conversion to normal heights

b) Adjustment of terrestrial quantities

$$\boxed{{}_rX',\ {}_rY', H'} \rightarrow \boxed{\psi,\ z,\ d,\ h} \rightarrow \boxed{{}_rX'_U,\ {}_rY'_U,\ {}_eH'_U}$$

transformed coordinates → terrestrial quantities → adjusted coordinates in the X', Y' system

c) Transfer into the projection plane and normal heights

$$\boxed{{}_rX'_U,\ {}_rY'_U} \rightarrow \boxed{X'_U,\ {}_rY'_U} \rightarrow \boxed{v_{X_U}, v_{Y_U}, v_{H_U}} \rightarrow \boxed{{}_rX_U, {}_rY_U, H_U}$$

adjusted coordinates → transformation into the geodetic network → coordinate residuals → resulting coordinates

Fig. 2. Adjustment scheme of terrestrial measurements.

2. Common Adjustment of Satellite and Terrestrial Quantities

Basic computation procedures are described (Kratochvil, Fixel, 2003; Nevosad, 2002; Nevosad et. al., 2002). Parametric adjustments in 2D and 3D coordinates are discerned as well. Two selected schemes for the computation of newly determined points in planar coordinates ${}_rX$, ${}_rY$ and normal heights H are in Fig. 3 and Fig. 4.

2.1 Adjustment with Transfer of Satellite Measurements into the Projection Plane

a) Transfer of satellite measuring units into the projection plane

$$\boxed{{}_sX,\ {}_sY,\ {}_sZ} \rightarrow \boxed{{}_sX^{/},\ {}_sY^{/},\ {}_eH^{/}}$$

measuring units — projection plane and vertical system Bpv

b) Parametric adjustment

$$\boxed{\begin{array}{l} {}_s v_{X_V},\ {}_s v_{Y_V},\ {}_s v_{H_V}, \\ v_{\psi_{ij}}, v_{z_{ij}}, v_{d_{ij}}, v_{h_{ij}} \end{array}} \rightarrow \boxed{\begin{array}{l} \delta X_V,\ \delta Y_V,\ \delta H_V, \\ {}_s\delta c_X,\ {}_s\delta c_Y,\ {}_s\delta c_H \\ \delta\alpha_i,\ \delta k_i,\ \delta\mu,\ \delta\omega \end{array}} \rightarrow \boxed{\begin{array}{l} {}_rX_V,\ {}_rY_V,\ H_V, \\ {}_sc_X, \\ {}_sc_Y,\ {}_sc_H \\ \alpha_i,\ k_i,\ \mu,\ \omega \end{array}}$$

residuals of coordinates and terrestrial quantities, — increments of coordinates, coordinate shifts and other unknowns — adjusted coordinate shifts and other unknowns

Fig. 3. Scheme of common adjustment of satellite and terrestrial measurements with a transfer of satellite coordinates into the projecion plane.

Symbols α_i denominate the orientation shifts of measured horizontal directional sets ψ_{ij}, and k_i are the vertical components of refraction angles at points *i*. The symbol μ is the scale of measured slope distances, and ω is the rotation of coordinate system $X^{/}, Y^{/}$.

2.2 Adjustment with Fictitious Quantities in the Projection Plane

a) Computation of fictitious quantities in projection plane

$$\boxed{{}_sX,\ {}_sY,\ {}_sZ} \rightarrow \boxed{\Delta X^{/}_{ij}, \Delta Y^{/}_{ij};\quad \bar{s}_{ij}, \tau_{ij}}$$

measuring sets — two kinds of fictitious quantities

b) Parametric adjustment

$$\boxed{\begin{array}{l} v_{\Delta X^{/}_{ij}}, v_{\Delta Y^{/}_{ij}}, \\ v_{\bar{s}_{ij}}, v_{\tau_{ij}}, \end{array} \ v_{\psi_{ij}}, v_{z_{ij}}, v_{d_{ij}}, v_{h_{ij}}} \rightarrow \boxed{\delta X_V, \delta Y_V, \delta H_V,\ \delta\alpha_{ij}, \delta k_i, \delta\mu,} \rightarrow$$

residuals equations — increments of coordinates and unknowns

$$\rightarrow \boxed{{}_rX_V, {}_rY_V, H_V; \alpha_i, k_i, \mu, \omega}$$

adjusted coordinates and unknowns

Fig. 4. Scheme of common adjustment of satellite and terrestrial measurements with a transfer of satellite baselines into fictitious horizontal quantities.

Symbols $\Delta X^{/}_{ij}, \Delta Y^{/}_{ij}$ and $\bar{s}_{ij}, \tau_{ij}$ are the components of satellite baselines $\bar{\boldsymbol{s}}_{\boldsymbol{ij}}$ in projection plane.

Remark: Some of the computation variants divide the adjustment into two separate parts: adjustment of horizontal coordinates and adjustment of heights.

3. Common Adjustment Including DGPS and RTK Points

The CZEPOS network will substantially affect also the positioning computation procedures for control densification points (Nevosad, 2005). For the integration of an universal measuring unit (total station) and a satellite receiver employing DGPS and RTK are available terrestrial quantities ψ, *z, d, h* as well as the spatial coordinates ${}_DX_T$, ${}_DY_T$, ${}_DZ_T$ of the densification points stemming from the satellite positioning.

Parametric adjustment procedures may be seen from two basic angles. Usually, the coordinates of densification points determined by DGPS a RTK satellite methods are considered to be final. From them and from another control points, the coordinates of remaining densification points are derived with the help of terrestrial measurements – e.g. by the procedure outlined in Fig. 2 (Nevosad et al., 2002). The second and better procedure of parametric adjustment takes account of the GPS coordinates as measured quantities. A scheme of this adjustment is in Fig. 5. It illustrates the variant which divides the computation into two parts – the adjustment of planar coordinates ${}_rX$, ${}_rY$, and the separate adjustment of heights H. Fictitious measuring quantities are created by the planar coordinates ${}_{D\,r}X_T$, ${}_{D\,r}Y_T$ of the newly determined satellite points T, and by horizontal distances s_{ij} computed from respective slope distances d_{ij} and zenith angles z_{ij}.

Scheme of common adjustment of satellite and terrestrial quantities.

a) Transfer of GPS coordinates into the projection plane and into normal heights

$$\boxed{{}_DX_i, {}_DY_i, {}_DZ_i} \rightarrow \boxed{{}_{Dr}X_i^{/}, {}_{Dr}Y_i^{/}, {}_{De}H_i^{/}} \rightarrow \boxed{H_i^{/}}$$

measured GPS-coordinates — transfer to planar coordinates- and ellipsoidal heights — normal heights

b) Parametric adjustment in the projection plane

$$\boxed{v_{X_V^{/}}, v_{Y_V^{/}}\ v_{\psi_{ij}}, v_{s_{ij}}} \rightarrow \boxed{\delta X_V^{/}, \delta Y_V^{/},\ \delta\alpha_{ij}, \delta\mu_s} \rightarrow \boxed{{}_rX_V^{/}, {}_rY_V^{/}}$$

residuals equations — increments of coordinates and unknowns — adjusted coordinates in $X^{/}, Y^{/}$ plane

c) Transformation into given geodetic network

$$\boxed{v_{X_V}, v_{Y_V}} \rightarrow \boxed{\delta X_V, \delta Y_V, \delta\mu, \delta\omega} \rightarrow \boxed{{}_rX_V, {}_rY_V, H_V}$$

residuals equations — increments of coordinates and unknowns — adjusted coordinates

Fig. 5. A computation scheme for the common adjustment of all quantities.

Conclusions

The contribution draws attention to the importance of selection of an appropriate parametric adjustment method for the control densification points when using mixed satellite and terrestrial quantities for the horizontal positioning. Considering the high accuracy of the terrestrial measurements it is possible to achieve better quality of horizontal positions of newly determined points by a common adjustment of all measuring data. To achieve a higher positional accuracy and reliability it is necessary to use sophisticated computation procedures respecting the coordinates obtained by DGPS a RTK satellite methods as measured quantities. The network design and especially the observation scheme are usually also simplified. A common adjustment also provides a significant check of newly determined control densificitation points.

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