Reliability and continuous regeneration model

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Spoľahlivosť a spojitý model obnovy

The failure-free function of an object is very important for the service. This leads to the interest in the determination of the object reliability and failure intensity.

The reliability of an element is defined by the theory of probability.The element durability T is a continuous random variate with the probability density f. The failure intensity $\lambda(t)$ *is a very important reliability characteristics of the element. Often it is an increasing function, which corresponds to the element ageing.*

We disposed of the data about a belt conveyor failures recorded during the period of 90 months. The given ses behaves according to the normal distribution. By using a mathematical analysis and matematical statistics, we found the failure intensity function $\lambda(t)$. *The function* $\lambda(t)$ *increases almost linearly.*

Key words: *corrective procedure, reliability, failure, failure intensity.*

Introduction

The theory of reliability is closely connected with regeneration models. We issue from the failure-free function probabilty, based on an appropriate number of objects that are monitored until they reach the moment of function *t*, which is the base for determining the object (element) reliability. Then, based on the reliability of elements, it is possible to deduce the system reliability.

Supposing the object failure can occur at any moment, and, that the corrective procedure is carried out immediately after the failure, we will obtain the continuous process of regeneration. The models of continuous regeneration are frequently used in simulating certain technical procedures.

Reliability of elements

Quantitative characteristics of reliability are based on the reliability understood as the probability of the fact that the object will be failure-free at the time *t*.

Definition: ***The Reliability*** of the element at the interval $(0,t)$ is the probability that the durability of the element is longer than *t*, it means the element will be working at the interval $(0,t)$. We note that

$$P(t) = P(T > t) \text{ at } t > 0 , \tag{1}$$

where *T* is the element´s durability. It is a continuous random variate with the probability density *f*, for which the following is valid

$$f(t) > 0 \text{ at } t \in (0, \infty) , \tag{2}$$
$$f(t) = 0 \text{ otherweise.}$$

The random variate *T* obtains values from the interval $(0,\infty)$. The distribution function *F* of the random variate *T* is determined by the relation

$$F(t) = \int_0^t f(x)dx \quad \text{at } t \in (0,\infty) \tag{3}$$

and means the probability that the random variate obtains values which are lower than or equal to *t*, so that:

$$F(t) = P(T \leq t). \tag{4}$$

Then the element reliability is

$$P(t) = 1 - F(t). \tag{5}$$

The failure intensity is defined by the relation

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$$\lambda(t) = \frac{f(t)}{P(t)} = \frac{f(t)}{1-F(t)} \quad , \tag{6}$$

The function $\lambda(t)$ is one of important reliability characterstics of the element. Very often, it is an increasing function, which corresponds to the element ageing That is why, with the time *t,* the probability of failure, becomes higher at the interval $(t, t+\Delta t)$.

The functions *f*, *F*, *P* a λ are determining characteristics of the random variate *T* and if we know one of them, it is easy to determine the others.

Reliability and transport belts

A belt failure can occur at any moment and the corrective procedure (repair) is carried out immediately after the failure. Technical objects behave according to some continuous contributions. So we can suppose that a belt failure is also the subject of one of them.

We disposed of the data about a belt conveyor from the Ostrava brown coal field, failures of which had been recorded during the period of 90 months (chart no. 1).

z_i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	19	21	24	29
n_i	6	5	6	10	6	5	10	4	4	9	2	5	3	4	2	4	2	1	1	1

where z_i is the number of failures in one month and n_i is the frequency.

By using Pearson´s test, we found out that the given set behaves according to the normal distribution $N(\mu, \sigma^2) = N(7{,}333333;\ 31{,}4)$. The value $\frac{(np_i - n_i)^2}{np_i}$ = 10,218092 and $\chi^2_{0,95}(6) = 12{,}6$. Since the variable *t* is the time, which is always positive, the set behaves according to the normal truncated distribution. *The probability density* of this distribution is

$$f(t) = \frac{1}{a\sigma\sqrt{2\pi}} . e^{\frac{-1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \quad \text{at } t \geq 0, \tag{7}$$

$f(t) = 0$ otherwise

If we mark

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad , \tag{8}$$

then

$$f(t) = \frac{1}{a\sigma} \varphi\left(\frac{t-\mu}{\sigma}\right) \quad . \tag{9}$$

If it is postulated that $\int_0^\infty f(x)dx = 1$, then $a = \frac{1}{\sigma\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt$.

The distribution function is determined by the formula

$$F(t) = \frac{1}{a}\int_0^t f(x)dx = \frac{1}{a\sigma}\int_0^t \varphi\left(\frac{x-\mu}{\sigma}\right)dx \quad \text{at } t \geq 0 \tag{10}$$

and the element´s *reliability* is determined by the relation

$$P(t) = 1 - F(t) \ . \tag{11}$$

For *the failure intensity,* the following is true:

$$\lambda(t) = \frac{f(t)}{P(t)} = \frac{\frac{1}{\sigma}\varphi\left(\frac{t-\mu}{\sigma}\right)}{1-F(t)} \quad . \tag{12}$$

The parameters μ and σ^2 in the above relations represent the expected value and the dispersion of the random variate, the normal distribution of which is non-truncated.

After having calculated the first derivation of the failure intensity function $\lambda'(t)$, it is possible to show that $\lambda'(t) > 0$, which means that the failure intensity is for normal distribution an increasing function within the time *t*. This function has at $t \to \infty$ an asymptote with the angular coefficient $\lambda_1(t) = kt + q$. The following is true:

$$k = \lim_{t\to\infty} \frac{f(t)}{t(1-F(t))} = \lim_{t\to\infty} \frac{1}{t} \cdot \frac{f(t)}{1-F(t)},$$

where $\frac{f(t)}{1-F(t)} \to \frac{0}{0}$ for $t \to \infty$. Using L' Hospital´s, we attain $k = \frac{1}{\sigma^2}$. Similarly, the following is true:

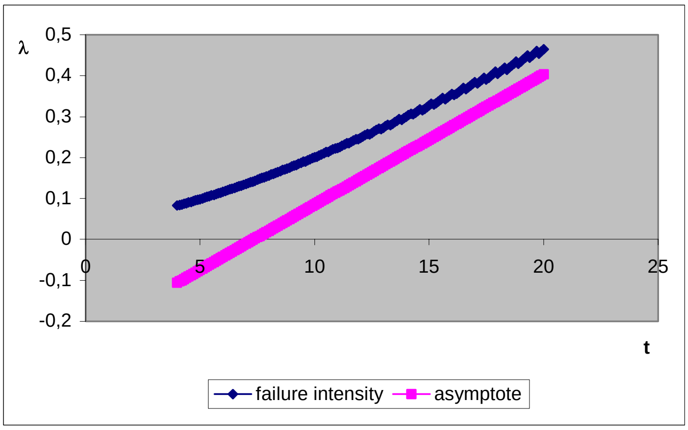
$$q = \lim_{t\to\infty}\left(\frac{f(t)}{1-F(t)} - \frac{t}{\sigma^2}\right) = -\frac{\mu}{\sigma^2}.$$

The asymptote equation is:

$$\lambda_1(t) = \frac{t-\mu}{\sigma^2}. \quad (13)$$

At $t \to \infty$ the function $\lambda(t)$ approaches the linear function $\lambda_1(t) = \frac{t-\mu}{\sigma^2}$, so the following is true:

$$\lim_{t\to\infty}\left(\lambda(t) - \frac{t-\mu}{\sigma^2}\right) = 0.$$



At higher values, the function $\lambda(t)$ increases approximately linearly. For the set given in the chart no 1, where $\mu = 7{,}333333$ and $\sigma = 5{,}603570$, the graphs of both functions are in Fig. 1.

Fig. 1.

Conclusion

To study belt conveyors in more details and to find the most appropriate models of the operational analysis and their regeneration theory allowing us to make the best simulation possible, higher quality recordings and service data would be needed. This can become the subject of further research.

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