Design of the fractional-order PIλDμ controllers based on the optimization with self-organizing migrating algorithm

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Navrh PIλDμ regulatorov necelociselnho radu zalozeny na optimalizacii s vyuzitim samorganizujucich migracnych algoritmov

Design of fractional-order controllers based on optimization methods is one of the intensively developed trends of the present time. There are several quality control criterions to evaluate the controller performance and to design the controller parameters by optimization. All of these objective functions are almost always multimodal in this case - so they have too complex geometric surface with many local extrema. In this context the choice of the optimization method is very important. In this paper we present a synthesis method for the design of fractional-order $PI^{\lambda}D^{\mu}$ controllers based on an intelligent optimization method with so called self-organizing migrating algorithm utilizing the principles of artificial intelligence. Along with the mathematical description we will present also simulation results on illustrative examples to demonstrate the advantages of this method and advantages of the fractional-order $PI^{\lambda}D^{\mu}$ controllers in comparison with traditional PID controllers.

Key words: *fractional-order controller design, fractional calculus, design specifications, optimization, artificial intelligence.*

Introduction

Besides theoretical research of the fractional-order (FO) derivatives and integrals (Oldham, Spanier, 1974; Samko, 1987; Podlubný, 1999 and many others) we can see in last years a growing number of applications of the fractional calculus in very different areas. Considerable development has been achieved in control theory (Manabe,1961; Outstalup e.g., 1995; Axtel, Bise, 1990; Kolojanov, Dimitrova, 1992; Dorčák e.g., 1994; 2006; Podlubný, Dorčák, Koštial, 1997; Chen, 2002, etc.). In the above-mentioned works the first ever generalizations of analysis methods for FO control systems were made (s-plane, frequency response, etc). The main conclusions of this research were that, firstly, it is inadequate to use the integer-order model for the design of the controller parameters for FO controlled object (Dorčák, 1994), secondly - the control system controlled with FO controller is more robust to gain changes than the classical one, thirdly - the possibility to fulfill more design specifications by controller design.

At design time is a very frequently used PID controller. Some design methods for the FO $PI^{\lambda}D^{\mu}$ controllers are based on an extension of the classical PID control theory. Design of fractional-order controllers based on optimization methods is one of the intensively developed trends of the present time. A great contribution to this area have been the works (Chen, 2003; Nonje, 2005; Bettou, 2006; Cao, 2006; etc.), oriented toward the optimization-based design (Zelinka, 2002; Laciak, Kostúr, 2000; etc.). There are several quality control criterions to evaluate the controller performance and to design the controller parameters by optimization. All objective functions have too complex geometric surface with many local extrema. In this context the choice of the optimization method is very important. The methods based on classical deterministic linear or nonlinear function minimization often failed (Dorčák, 2006/a, 2006/b).

Method for the fractional-order $PI^{\lambda}D^{\mu}$ controller design

In this paper we present a synthesis method for the design of the FO $PI^{\lambda}D^{\mu}$ controllers based on an intelligent optimization method with so called self-organizing migrating algorithm (Zelinka, 2002) utilizing the principles of artificial intelligence.

Definition of the system

For the later purposes we consider a simple unity feedback control system with the following transfer functions of the controlled system and controller:

$$F_s(s) = \frac{k_1}{a_2 s^{\alpha} + a_1 s^{\beta} + a_0} \qquad F_r(s) = K_p + \frac{T_I}{s^{\lambda}} + T_D s^{\mu} \qquad (1)$$

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where α, β are orders of derivatives (in general real numbers, α > β), a_i, k_1 are the values of the coefficients of the controlled system, λ and μ are orders of derivation and integration (in general real numbers too), and K_P, T_I, T_D are the values of the coefficients of the controller.

It can be seen that the fractional-order $PI^{\lambda}D^{\mu}$ controller has two parameters more than the conventional PID controller, therefore two more specifications can be met to improve the performance of the control system.

Quality control criterions

There are several quality control criterions to evaluate the controller performance and to design the controller parameters by optimization, which fulfill desired design specifications. We have used the following design specifications: no steady-state error, phase margin (Φ_m) and gain crossover frequency (ω_{cg}) specifications, gain margin (g_m) and phase crossover frequency (ω_{cp}) specifications, robustness in variations in the gain of the plant, robustness in high frequency noise, good output noise suppression, integral square error, overshoot, etc.

The requirement for the no steady-state error can be fulfilled by properly implementing the fractional order integrator in the controller, which provides the steady-state error cancellation. The other requirements constitute the main optimized function and the optimization constrains.

The minimized function

As mentioned before, there are several quality control criterions (J_1, J_2, etc.) to evaluate the controller performance and to design the controller parameters by optimization. The test function, or the minimized function for the tuning of the parameters of fractional-order $PI^{\lambda}D^{\mu}$ controllers in a classical unity feedback control system is often chosen as the integral square error (ISE) with or without restrictions:

$$J_1(K_P,T_I,T_D,\lambda,\mu)=\int_0^{\infty}\left(e(t)\right)^2 dt=\int_0^{\infty}\left(y_d(t)-y(t)\right)^2 dt,\quad J_2=\int_0^{\infty}\left(w_1 e^2(t)+w_2 u^2(t)\right)dt \tag{2}$$

where $y(t)$ is measured or calculated output of the controlled system, $y_d(t)$ is desired value of the output, $e(t)$ is error, $u(t)$ is the plant's input and w_1, w_2 are weighting coefficients. Such criterions can be used only in special cases, but for designing all five parameters it is not convenient, because it often leads to oscillations and the roots of the characteristic equation lie dangerously close to the border of stability. In such cases we ought to use a more complex criterion including e.g. the above-mentioned gain crossover frequency, gain margin, and phase margin (Chen, 2003; Monje, 2004; Dorčák, 2006/a; 2006/b; etc.), control time, overshoot, etc. To fulfill properly specification on gain crossover frequency (ω_{cg}), the following condition must be fulfilled:

$$\left|F_O(i\omega_{cg})\right|_{dB}=\left|F_r(i\omega_{cg})F_s(i\omega_{cg})\right|_{dB}=0\ dB\,, \tag{3}$$

where $F_o(i\omega)$ is the open-loop transfer function of the control system in frequency domain with the Bode diagram depicted in Fig. 1a. We consider the absolute value of the frequency transfer function $|F_o(i\omega)|$ as the main function for minimization (Fig. 1b).

The transfer function of the open-loop system with the controlled system (1) and with the controller (2) has the form:

$$F_O(s)=k_1\frac{K_P s^{\lambda}+T_D s^{\lambda+\mu}+T_I}{a_2 s^{\lambda+\alpha}+a_1 s^{\lambda+\beta}+a_0 s^{\lambda}},\quad F_O(i\omega)=\mathrm{Re}(i\omega)+i\,\mathrm{Im}(i\omega)\,. \tag{4}$$

Then the frequency open-loop transfer function has the following form:

$$\mathrm{Re}(i\omega)=k_1\frac{\mathrm{Re}_1\mathrm{Re}_2+\mathrm{Im}_1\mathrm{Im}_2}{\mathrm{Re}_2^{\,2}+\mathrm{Im}_2^{\,2}},\qquad \mathrm{Im}(i\omega)=k_1\frac{\mathrm{Im}_1\mathrm{Re}_2-\mathrm{Re}_1\mathrm{Im}_2}{\mathrm{Re}_2^{\,2}+\mathrm{Im}_2^{\,2}} \tag{5}$$

$$\begin{aligned}
\mathrm{Re}_1&=K_P\omega^{\lambda}\cos(\lambda\pi/2)+T_D\omega^{\lambda+\mu}\cos((\lambda+\mu)\pi/2)+T_I\\
\mathrm{Im}_1&=K_P\omega^{\lambda}\sin(\lambda\pi/2)+T_D\omega^{\lambda+\mu}\sin((\lambda+\mu)\pi/2)\\
\mathrm{Re}_2&=a_2\omega^{\lambda+\alpha}\cos((\lambda+\alpha)\pi/2)+a_1\omega^{\lambda+\beta}\cos((\lambda+\beta)\pi/2)+a_0\omega^{\lambda}\cos(\lambda\pi/2)\\
\mathrm{Im}_2&=a_2\omega^{\lambda+\alpha}\sin((\lambda+\alpha)\pi/2)+a_1\omega^{\lambda+\beta}\sin((\lambda+\beta)\pi/2)+a_0\omega^{\lambda}\sin(\lambda\pi/2)\,.
\end{aligned} \tag{6}$$

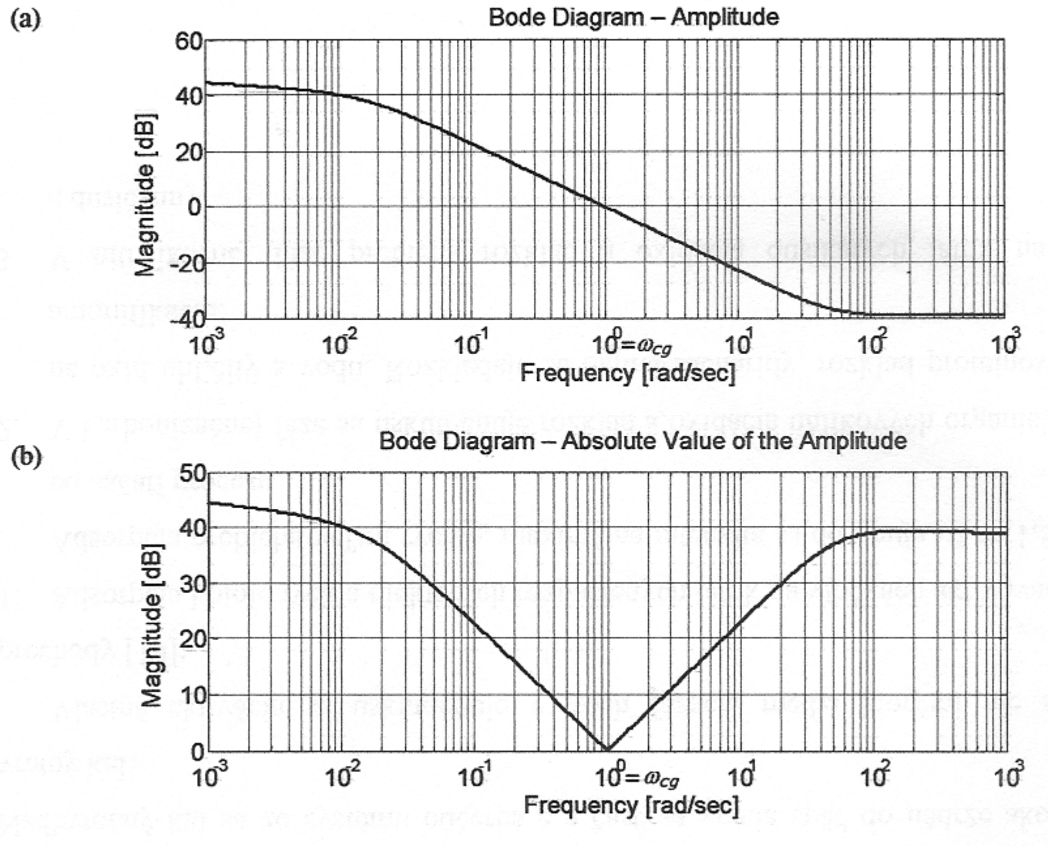


Fig. 1. Bode diagram of the open-loop transfer function (a), Bode diagram – absolute value of the amplitude (b).

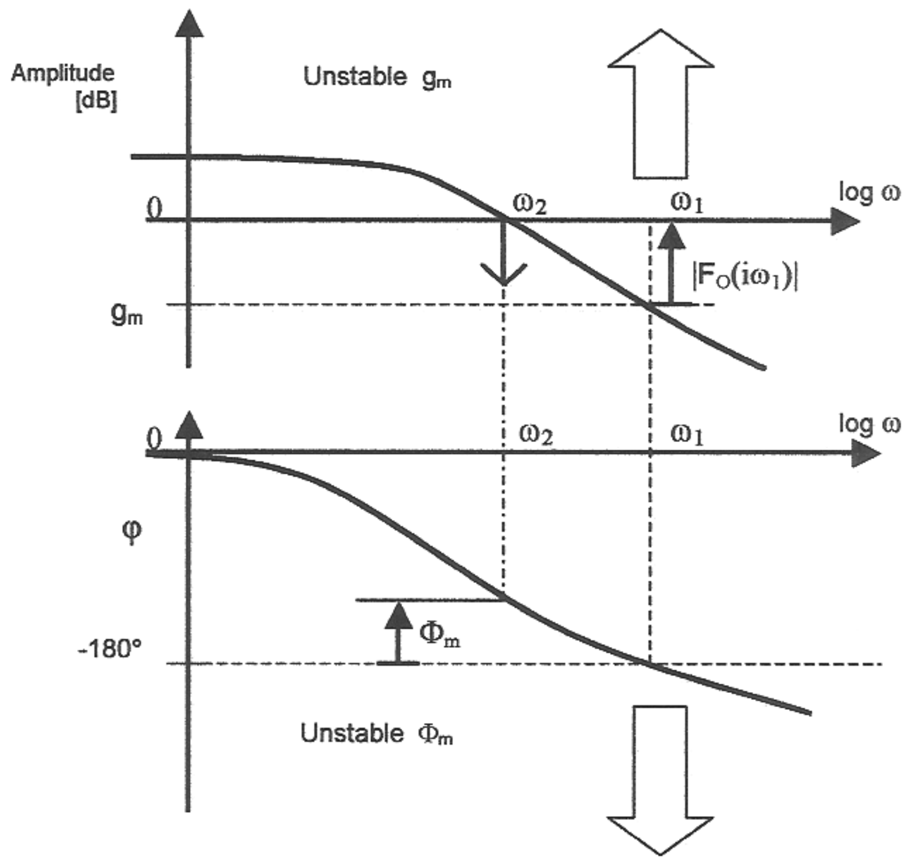


Fig. 2. Phase margin (Φ_m) and gain margin (g_m).

The constraints of the minimization

As the minimization constraints for the controller design we can consider e.g. the following four specifications:

$$\arg\left(F_O(i\omega_{cg})\right) = -180 + \Phi_m\,, \qquad \left(\frac{d\left(\arg\left(F_O(i\omega)\right)\right)}{d\omega}\right)_{\omega=\omega_{cg}} = 0 \tag{7}$$

$$\left|T(i\omega_t)\right| = \left|F_W(i\omega_t)\right| = \left|\frac{F_O(i\omega_t)}{1+F_O(i\omega_t)}\right| \le A\ dB\,, \qquad \left|S(i\omega_s)\right| = \left|\frac{1}{1+F_O(i\omega_s)}\right| \le B\ dB \tag{8}$$

The meaning of the phase margin (Φ_m) at $\omega_{cg}=\omega_2$ and gain margin g_m [Monje 2004] can be seen in Fig. 2. With the condition "Robustness to variations in the gain of the plant" (see [Chen 2003]) the phase is forced to be flat at ω_{cg} and so, to be almost constant within an interval around ω_{cg}. It means that the system is more robust to gain changes and the overshoot of the response is almost constant within the interval. The "Robustness to high and low frequency noise" are illustrated in Fig. 3a ($\forall\ \omega \ge \omega_t \rightarrow T(i\omega) < A_t$) and Fig. 3b ($\forall\ \omega \le \omega_s \rightarrow S(i\omega) < B_s$).

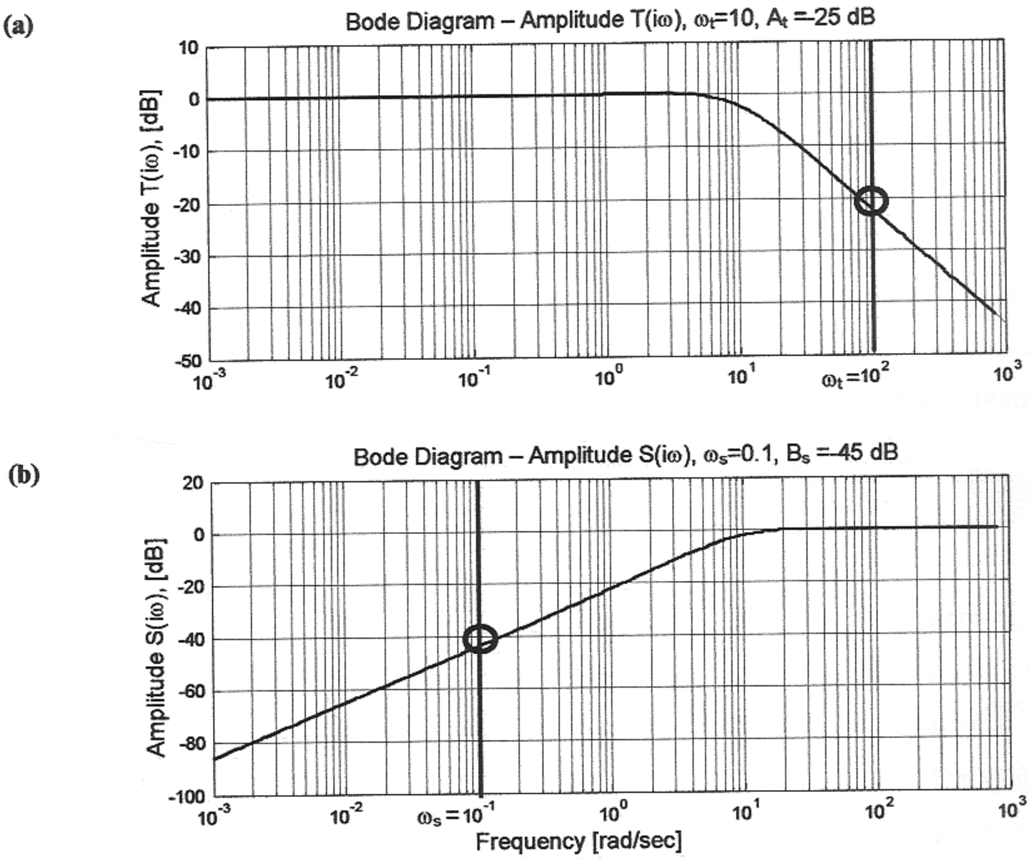


Fig. 3. Bode diagram of T(iω) and S(iω).

The optimization method

All considered objective functions have too complex geometric surface with many local extrema. In this context the choice of the optimization method is very important.

In our earlier works (Dorčák, 2006/a; 2006/b) we have used for tuning of the FO $PI^{\lambda}D^{\mu}$ controller the function FMINCON from the optimization toolbox Matlab, to find the constrained minimum of a function of several variables. This method, based on classical deterministic linear or nonlinear function minimization, is successful only with simpler types of the controlled systems, e.g. first-order system. For higher-order systems, this optimization method often failed.

So in this paper we present a synthesis method for the design of the fractional-order $PI^{\lambda}D^{\mu}$ controllers based on an intelligent optimization method with so called self-organizing migrating algorithm (Zelinka, 2002) utilizing the principles of artificial intelligence. This method was compared in successful cases with the function FMINCON from the optimization toolbox Matlab.

Illustrative examples

In the first example we consider the controlled system with the coefficients values k_1=1, a_0=1, a_1=0.2313, a_2=0.7414, α=2, β=1 (integer-order plant) and integer-order PD controller with the following coefficients: K_P=20.5, T_D=3.7343. The step response of the closed-loop controlled system is depicted in Fig. 4 (dashed line) with 25 % overshoot.

To illustrate the use of so called self-organizing migrating algorithm (SOMA) (Zelinka, 2002) as the simplest one-dimensional optimization method for controller parameters design we solved the task to decrease the value of the overshoot of the mentioned closed-loop controlled system to 15 % by taking into account the controller as the fractional-order PD^{μ} with the same values K_P, T_D and changing only the value μ (Bettou, 2006). Minimizing the appropriate test function (2), specialized to the overshoot, we obtained the optimal value μ=1.162 and from the step response depicted in Fig. 4 (solid line) we can see the decrease of the overshoot to the desired value of 15 %. Simultaneously we have a 16.5 % decrease of the ISE.

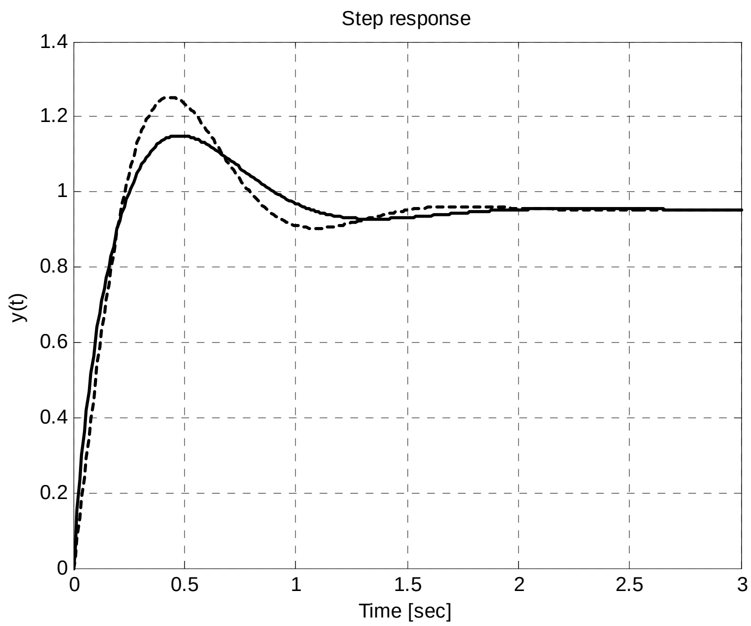


Fig. 4. Comparison of the overshoots of the step responses.

In the second example we consider the controlled system with the coefficient values k_1=0.55, a_0=1, a_1=62, a_2=0.0, α=0, β=1 and specification values ω_{cg}=1, ω_t=10, ω_s=0.01, Φ_m=80, A=-20dB, B=-20 dB. With optimization method we obtained the following $PI^{\lambda}D^{\mu}$ controller: K_P=22.323, T_I=84.79, T_D=7.06, λ=0.176, μ=0.168 (Papajová, 2007). From the step responses depicted in Fig. 5 and from the Table 1 we can see the robustness of the control system by different values of k_1. For such controlled system both optimization methods give satisfactory results, but SOMA is still more safe.

Tab. 1. Comparison of the overshoots and control rate ($Time_{max}$).

	k_1 = 0,3	**k_1 = 0.55**	**k_1 = 0.9**
SOMA – overshoot [%]	1.74	2.4	2.6
SOMA - $Time_{max}$ [sec]	6.95	4.25	2.82
FMINCON - overshoot [%]	1.68	2.38	2.59
FMINCON - $Time_{max}$ [sec]	6.95	4.25	2.82

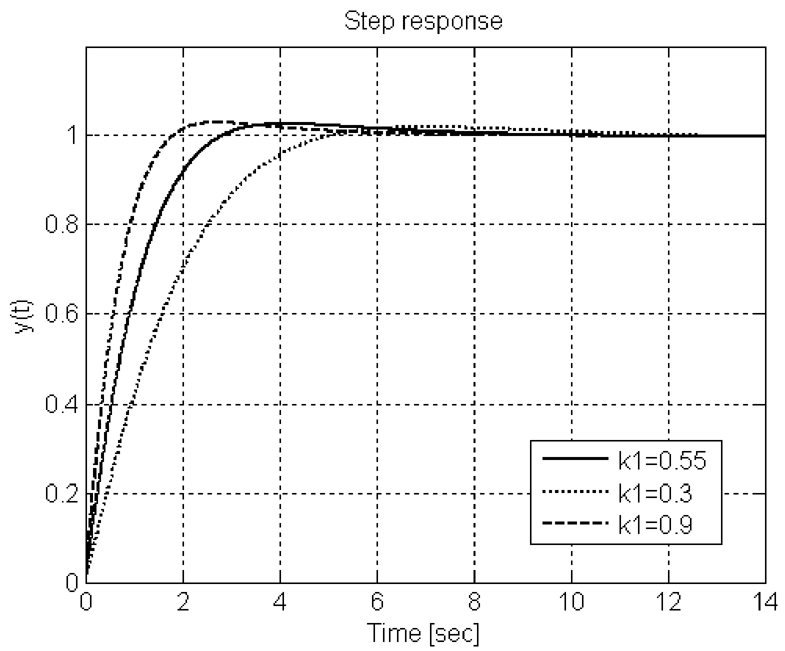


Fig. 5. Robustness to gain k_1 changes.

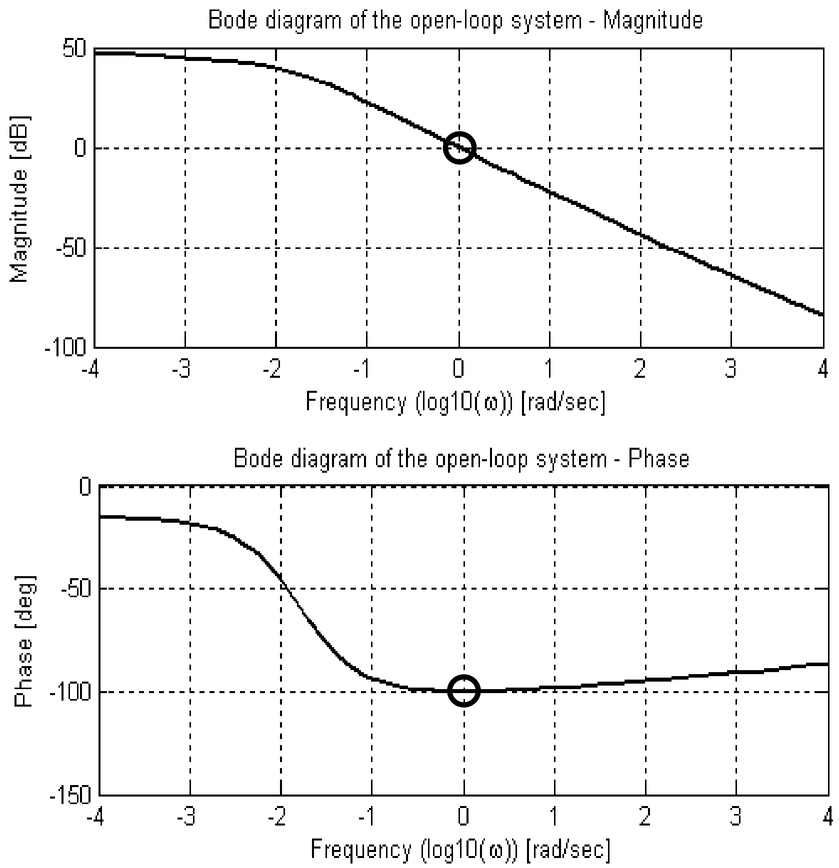


Fig. 6. Phase margin Φm=80 [deg] at s ω_{cg}=1 [rad/sec].

From the bode plots of the open-loop control system depicted in Fig. 6 we can see that the requirements for the phase margin specification (Φ_m=80 deg) at the gain crossover frequency (ω_{cg}=1 rad/sec) and also the robustness to high and low frequency noise (Fig. 7) are satisfied, which means that $\forall\ \omega \geq \omega_t \rightarrow T(i\omega) < A_t$ and $\forall\ \omega \leq \omega_s \rightarrow S(i\omega) < B_s$.

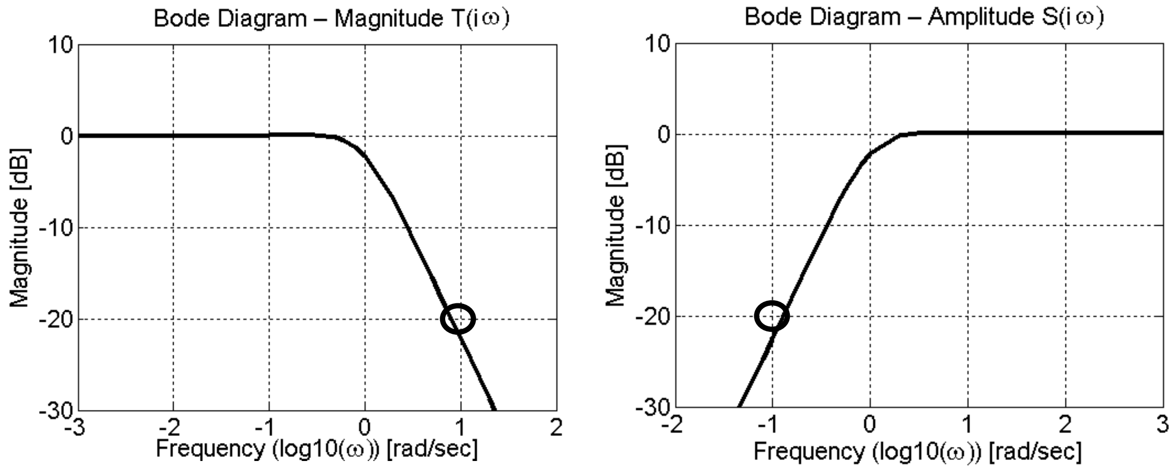
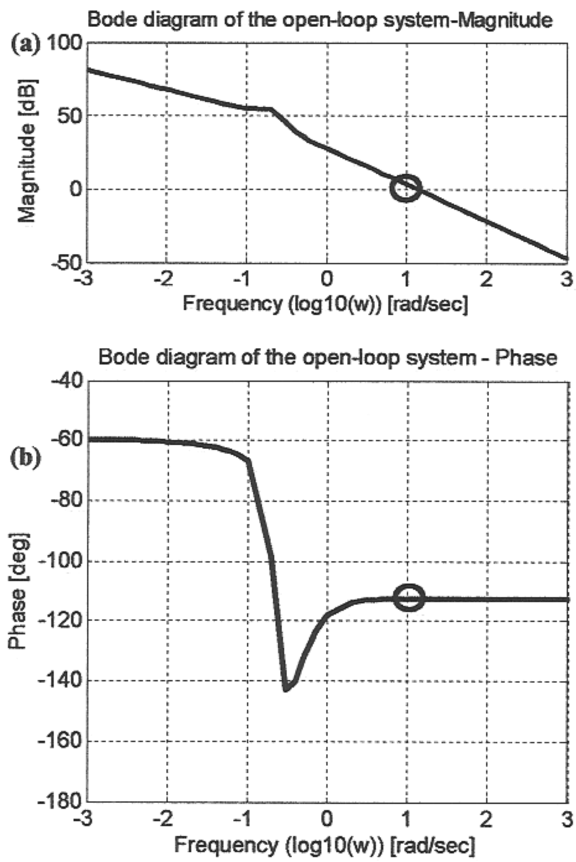


Fig. 7. Robustness to high and low frequency noise.

In the third example we consider the controlled system with the coefficients values $k_1=1$, $a_0=0.4$, $a_1=1$, $a_2=8$, $\alpha=2$, $\beta=1$. Using the dominant roots method with the stability measure $S_t = 2$, damping measure $T_l = 0.4$ and with the value $s_3 = -10$ of the third root we obtained the following parameter values of the integer order controller: $K_P=551.6$, $T_I=2320$, $T_D=111$. With optimization method and with the following specifications values $\omega_{cg}=10$, $\omega_t=100$, $\omega_s=0.01$, $\Phi m=68$, A=-20dB, B=-20 dB we obtained $PI^{\lambda}D^{\mu}$ controller parameters: $K_P=6.218$, $T_I=44.357$, $T_D=203.2$, $\lambda=0.664$, $\mu=0.75$. For such controlled system we must specially modify the optimization function and restrictions (additional frequencies for Φm) for the SOMA optimization method (Papajová, 2007).



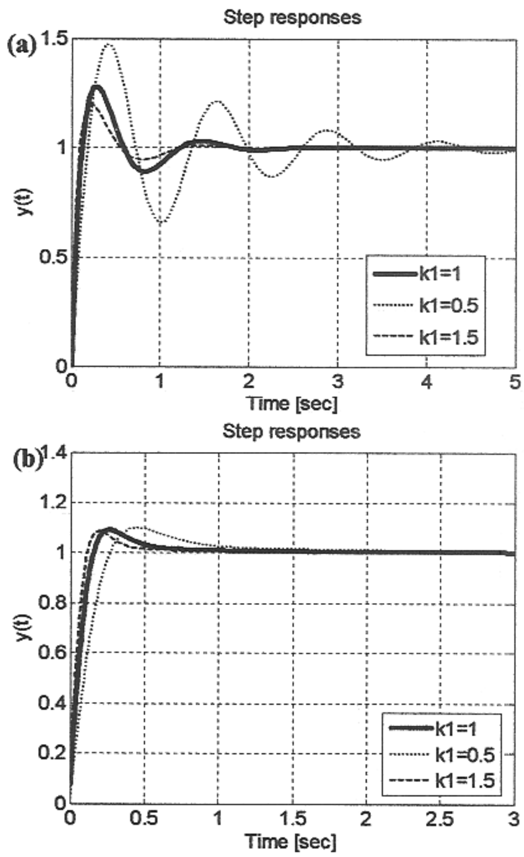


Fig. 8. Phase margin $\Phi m=68$ [deg] at s $\omega_{cg}=10$ [rad/sec]

Fig. 9. Step responses of the FO and IO systems.

From the bode plots of the open loop control system depicted in Fig.8 we can see that the requirements for the phase margin specification (Φ_m=68 deg, Fig. 8b) at the gain crossover frequency (ω_{cg}=10 rad/sec, Fig. 8a) are satisfied. In Fig. 9 are compared step responses of the control system with fractional-order controller (Fig. 9b) and integer-order controller (Fig. 9a) for different values of k_1. From the step responses depicted in Fig. 9 and from the Table 2 we can see the robustness of the control system with fractional-order $PI^{\lambda}D^{\mu}$ controller opposite to the integer-order PID controller by different values of k_1.

Tab. 2. Comparison of the overshoots and control rate ($Time_{max}$) for the PID and $PI^{\lambda}D^{\mu}$ controllers.

	PID		**$PI^{\lambda}D^{\mu}$**	
k_1	**Overshoot** [%]	**$Time_{max}$** [sec]	**Overshoot** [%]	**$Time_{max}$** [sec]
0.5	44.61	0.4	9.39	0.455
1	26.11	0.27	8.95	0.26
1.5	18.58	0.22	8.55	0.19

In the fourth example we consider the controlled system with the coefficients values k_1=1, a_0=1.0, a_1=0.5, a_2=0.8, α=2.2, β=0.9 (fractional-order plant). With optimization method and with the following specifications values ω_{cg}=1, ω_t=100, ω_s=0.1, Φ_m=80, A=-20dB, B=-20 dB (and additional frequencies for Φ_m, ω_1=1.1, ω_2=1.5, ω_3=10) we obtained $PI^{\lambda}D^{\mu}$ controller parameters: K_P=5.0442, T_I=14.6784, T_D=10.095, λ=1.046, μ=1.0964. For such controlled system we must specially modify the optimization function and restrictions for the SOMA optimization method (Papajová, 2007).

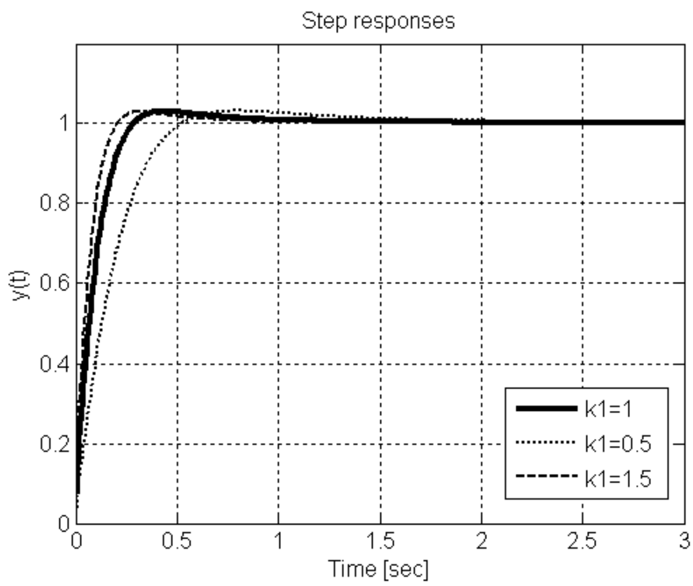


Fig. 10. Robustness to gain k_1 changes.

In Fig. 10 are depicted step responses of the control system with FO plant and FO controller for different values of k_1. From the step responses depicted in Fig. 10 we can see also the robustness of the control system with fractional-order $PI^{\lambda}D^{\mu}$ controller by different values of k_1 of the controlled plant.

Conclusions

The obtained results by using the SOMA algorithm - based on an intelligent optimization with so called self-organizing migrating algorithm (Zelinka, 2002) utilizing the principles of artificial intelligence - are very stimulating mainly for simple type of the controlled plant and showed the possibility to use this method for the FO controller parameters design. This method is more successful in the majority of cases in comparison with the method based on classical deterministic linear or nonlinear function minimization.

From the above illustrative examples can also be seen the advantage of the FO $PI^{\lambda}D^{\mu}$ regulator as compared with the classical integer-order PID regulator mainly as regards the properties of the robustness of the regulation system.

The main disadvantage of this algorithm is computational time and new problems arise in the design of parameters of FO regulators with this method mainly with more complex types of regulated systems such as for example the plant in the fourth example. In such a case it will be necessary to pay more attention to the choice of optimization constraints and the form of the optimization function.

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