Geomechanical problems of an underground storage of spent nuclear fuel and their mathematic modelling.

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Matematické modelovanie geomechanických problémov podzemného uskladňovania vyhoreného nukleárneho paliva
The paper is devoted to the use of mathematical modelling for analysis of the thermo-mechanical (T-M) processes, which are relevant for the assessment of underground repositories of the spent nuclear fuel. Wes shall discuss mathematical formulation, numerical methods and parallel alghorithms, which are capable to solve large-scale complicated and coupled 3D problems. Particularly, we show an application of the described methods and parallel computer simulations for analysis of model problems concerning the Swedish KBS3 concept of underground repository.

***Key word**: mathematic modelling, thermo-mechanical processes, underground deposition*

Introduction

The recent concept of the management of the end of the fuel cycle in Czech nuclear power plants involves putting of fuel elements removed from reactors into water basins in the vicinity of power plants for a period 5 – 7 years and then storing them for about 40 years in an interim storage. After this time and possible recycling, the remaining radioactive waste will be irreversibly putted into a final repository of the spent nuclear fuel (SNF). There are two variants of the interim storage of the SNF in the Czech Republic: a surface storage in buildings in the vicinity of the power plant Dukovany and an underground interim storage at Skalka in the district Žďár n. S for the moment, the former conception of the surface storage is preferred but, nevertheless, the results of the performed investigation and design of the underground interim storage are worthwhile for publishing. The Skalka locality can also serve as a testing example for the mathematical modelling of processes in a rock mass.

The Institute of Geonics systematically deals with problems of the mathematical modelling of coupled processes in the rock mass also in a connection with the assessments of final deep repository projects. In past years, we succeeded in solving the project "Modelling of thermo-mechanical processes connected to the underground deposition of spent nuclear fuel" of the national grant agency (Blaheta et al., 2005). We continue in a collaboration with the TU Liberec and the UPPMAX centre in Uppsala (Blaheta et al. to be published).

Underground interim storage of the spent nuclear fuel

The variant of underground interim storage of the spent nuclear fuel seems to be reasonable from many technical, environmental, safety and safeguards viewpoints. First of all, it makes possible to prolong the interim storage of SNF to 100 and more years without large costs and technical and safety problems. When during 40 years the volume of highly active wastes is reduced by 50 – 60 %, after 100 years it is a 95 – 99 % reduction. Also, the thermal output after 100 years decreases to one third of the initial level. A decrease of the radiation and the thermal output has on one hand an influence on the dimension, construction and costs of the future underground repository. When we directly put the SNF to the final repository we lose a large amount of potential nuclear fuel which can be recovered and so it can represent an important primary source of energy for future. A long-termed interim storage of SNF makes it possible to avoid incorrect decisions for future and to wait for a more information about the technologies of separation and transmutation of SNF. The real waste after the separation would be only a small amount of the fission products. The safety of the interim underground storage in comparison to the surface storage is significantly higher from many viewpoints like natural disasters, plane crash and terrorist attacks. The design of the underground interim storage is based on the dry storing of the SNF in canisters. The cooling medium is air which flows naturally and takes away the heat produced by the nuclear waste. The project of the underground interim storage construction is located into a depth of about 100 m under the ground surface. It consists of a service tunnel and two parallel deposit tunnels of the width 10.9 m, height 12.4 m and the total length 630 m, which serve for the SNF canister storage. The deposit tunnels involve one service strip and two storage strips, where the heat producing canisters can be placed with the spacing 3.6 m.

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The deposit tunnels are equipped with a crane trek for the transport of canisters. A part of the repository is also the ventilation shaft with the depth 93 m, driven during the geologic survey works. The lay-out of the designed storage Skalka is shown in Fig. 1. The cross cut through the deposition tunnel is described in Fig. 2. The detailed geological and geomechanical survey was realised at a preparatory stage. Several underground workings were driven during this stage: the access gallery (450 m), the survey gallery (300 m) and the ventilation shaft (100 m). On the base of the results from in situ and laboratory tests, three models of the rock mass Skalka were prepared: the geological and tectonic model, the hydrogeological model and the final geotechnical model which can serve for a design of driving and supporting methods of the underground structure. The rock mass in the vicinity of the designed workings was divided into three geotechnical types. Their names are after a dominant type of rock: I – serpentinite, II – migmatite, III – gneiss (Tab. 1).

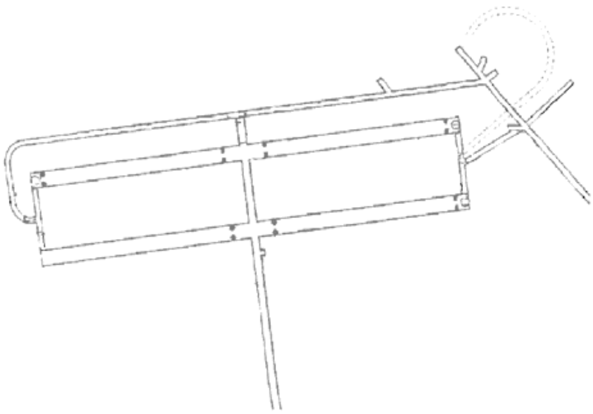


Fig. 1. SKALKA – a plan of the underground construction

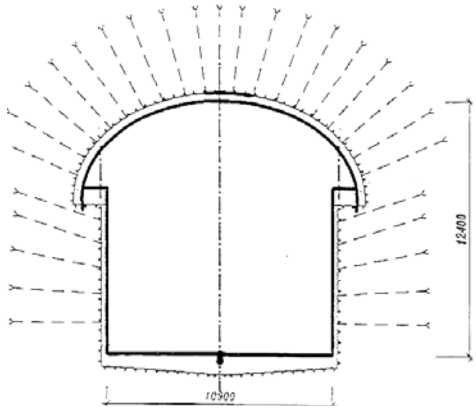


Fig. 2. SKALKA – a cross cut through the deposition tunnel.

Tab. 1. Classification of the rock mass.

Type	RQD	RMR	Q
I.	64	67	10, 7
II.	83	66	41,8
III.	80	73	90

The type I was contacted only in the access gallery. The proposed storage tunnel was designed in the geotechnical setting of types II and III. It is obvious that the rock mass in the locality Skalka represents good rocks with stable conditions.

Thermal influence of the spent nuclear fuel

The stored spent nuclear fuel is an important source of heat. The total heat output of interim storage depending on its volume was calculated to be of the order of MW (5 – 13 MW) (Šik 1999). A thermal influence of radioactive waste on the surrounding rock mass and the support in the case of underground interim storages brings new approaches to the rock mechanics. A thermal influence of the rock mass has two basic aspects :

- change of rock and support material properties due to a temperature increase,
- thermomechanical stress of both the rock mass and the lining construction.

Rocks contain various minerals: each of them has a different coefficient of thermal expansion. When a rock sample is subjected to heat, stresses are developed due to the differences in the thermal expansion coefficient α, which varies with the crystallografic directions. When the stresses exceed a local strength, a microcracking occurs. The relation between a temperature increase and the coefficient of thermal expansion depends on the texture and structure of the rock. An influence of heating of rocks from the storage locality on their mechanical properties was tested on rock samples which were heated in an electric oven to the temperature 400 ° and 800 °C and then tested by an uniaxial compression test. The values of compressive strength and deformation were specified from test records. Typical results of testing samples from gray biotite gneiss are shown in Fig. 3 and 4 (Konečný, 1999).

The laboratory tests show that the temperature increase to 400 °C does not influence the compression strength and the modulus of deformation of tested gneiss rocks substantially. The temperature increase to 800 °C evokes a significant decrease of all measured mechanical properties.

A study of the thermal field in a storing tunnel and its vicinity (Bradáč 1999) as well as a calculation of the cooling ventilation (Taufer 1999) demonstrated that the rock wall temperature in the underground storage should not exceed 85 °C. This temperature practically does not influence mechanical properties of rocks.

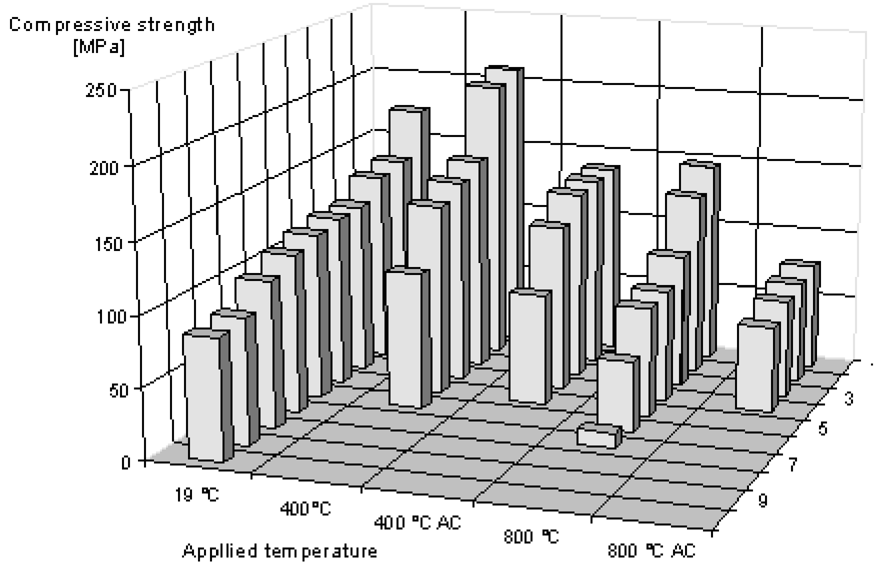


Fig. 3. Influence of heating (applied temperature) on the compressive strength. Light gray-pink biotite migmatite (gray biotite gneiss). AC means after cooling.

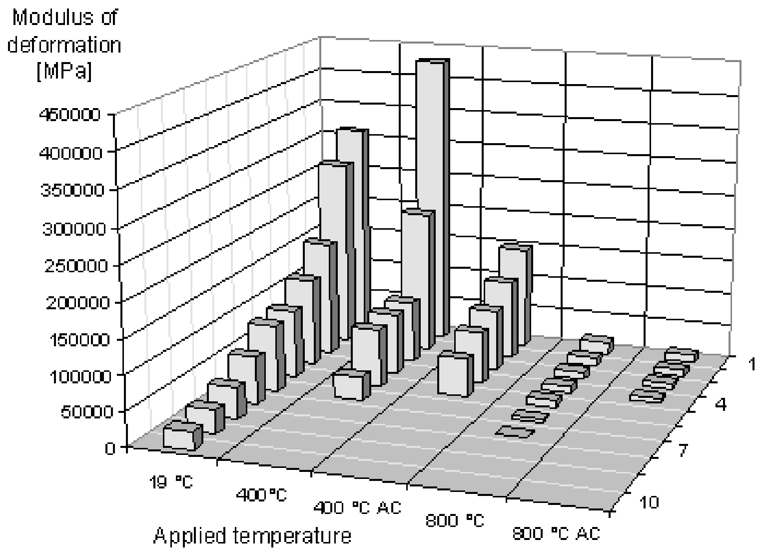


Fig. 4. Influence of heating (applied temperature) on the modulus of elasticity. Light gray-pink biotite migmatite (gray biotite gneiss). AC means after cooling.

Modelling of T-M phenomena from the underground deposition of the spent nuclear fuel

Much more complicated is the second aspect – an investigation of the heat flow and the thermomechanical stress changes of the rock mass and supporting construction. For the assessment of the repository performance, it is fundamental to be able to do *large-scale computer simulations* of various coupled processes as the heat transfer, the mechanical behaviour, water and the gas flow and the chemical processes in rocks and water solutions. Generally, we speak about *T-H-M-C processes* and their modelling. These processes are coupled but only some of the couplings are crucial for a reliable mathematical modelling.

The thermo-mechanical behaviour is fundamental for the stability and the deformation analysis as well as the intermediate and long-term safety assessment. A mathematical model is formulated in a plane cross cut, see Fig. 2.

The rock is assumed to be isotropic with respect to the both elasticity and heat conduction and the expansion. The boundary conditions for the mechanical part consist of the weight of the overburden at the upper side of the model and the zero normal displacements and zero shear stresses on the other sides.

The heat is now transferred not only by the conduction but also by the convection to the ventilation air and the radiation between the canister surface and tunnel walls.

The values of the material parameters for the rock, SNF canister and the concrete lining are given in Table 2. The heat source is given by the formula

$$Q(t) = Q_0 \ e^{-0.056t},$$

where t is the time in years, Q_0 = 1036 W m^{-3}.

Moreover, we assume the constant temperature $\tau = 26.5^\circ C$ of the ventilation air and the adiabatic (zero flux) condition on the outer boundary. The relative emissivity ε for the rock and canister surfaces were estimated, the value used in the model for the canister is somewhat overestimated, strengthening the role of the radiation. The initial condition takes the temperature $\tau = 9^\circ C$ in the rock and $\tau = 94^\circ C$ in the canister.

Tab. 2. Material parameters for the Skalka problem, (Bradáč, 1997).

Constant	Rock	Canister
E [GPa]	60	210
ν	0.23	0.3
ρ [kg/m^3]	2670	5400
k [W/mK]	3.0	3.1
α [1/K]	8 10^{-6}	-
c [Ws/kgK]	760	509
H [W/m^{2}K]	7	7
ε	0.9	0.5

Basic numerical methods

The initial-boundary value problem of the thermo-elasticity is discretized by finite elements in space and the differences in time. Using the linear finite elements and the simplest time discretization leads to a computation of vectors $\underline{\tau}^j$, $\underline{u}^j$ of nodal temperatures and displacements at the different time t_j j = 1,…,N, with the time steps $\Delta t_j = t_j - t_{j-1}$. It gives the following *time stepping algorithm*:

find $\underline{\tau}^0 : M_h \underline{\tau}^0 = \underline{\tau}_0, \quad \underline{u}^0 : A_h \underline{u}^0 = b^0 = b_h\left(\underline{\tau}^0\right),$

for $j = 1,\ldots,N$:

find $\underline{\tau}^j : B_h^{(j)} \underline{\tau}^j = \left[M_h + \theta \Delta t_j K_h\right] \underline{\tau}^j = c^j,$

find $\underline{u}^j : A_h \underline{u}^j = b^j.$

M_h is the capacitance matrix, K_h is the conductivity matrix, A_h is the stiffness matrix, $\theta \in \langle 0,1 \rangle$ is a parameter, $c^j = \left[M_h - (1-\theta)\Delta t_j K_h\right] \underline{\tau}^{j-1} + \theta q_h^j + (1-\theta) q_h^{j-1}$, $b^j = b_h\left(\underline{\tau}^j\right)$ and $\underline{\tau}_0$ is from the initial condition.

Note that q_h represents the heat sources and b_h represents the volume and the surface forces including a thermal expansion term.

We aim at the development of fully robust, stable methods and therefore we restrict our attention to implicit methods with $\theta \in \langle 1/2, 1 \rangle$. Particularly, we shall consider two cases with $\theta = 1/2$ and $\theta = 1$, which correspond to the Crank – Nicolson (CN) and backward Euler (BE) method, respectively. More details can be found e.g. in Quarteroni, Valli (1994).

Non-linear problems with the heat radiation

In the 2D Skalka problem, we need to introduce not only the heat conduction or convection but also more complicated radiation heat transfer between the canisters and walls of the deposit tunnel. For this purpose, we divide the inner boundary to zones Z_k with the length $|Z_k|$ and assume a constant emissivity ε_k and a constant temperature τ_k on Z_k. In our model problem, the zones will be identical with the finite element sides creating the inner boundary.

Now, we can specify the radiation heat transfer from the zone Z_k to the zone Z_l. Firstly, the heat flux $\overline{q}_l$ coming from Z_k to Z_l is

$$\overline{q}_l = |Z_l|^{-1} R_{lk}\, q_k\,, \quad R_{kl} = \frac{1}{2}\int_{Z_l}\int_{Z_k} \frac{\cos\upsilon_k \cos\upsilon_l}{r_{kl}}\, \chi\, ds_k\, ds_l$$

where $q_k = c_{SB}\,\varepsilon_k \tau_k^4$, c_{SB} is the Stefan-Boltzmann constant, r_{kl} (x,y) is the distance between points $x \in Z_k$ and $y \in Z_l$, ϑ_k, ϑ_l are the angles between the segment *xy* and the normals to Z_k, Z_l, respectively. The function $\chi(x,y) = 1$ if *x* is visible from *y*, otherwise $\chi = 0$. Summing up these contributions gets

$$\overline{q} = c_{SB}\, S\, R\, E\, \Theta\,,$$

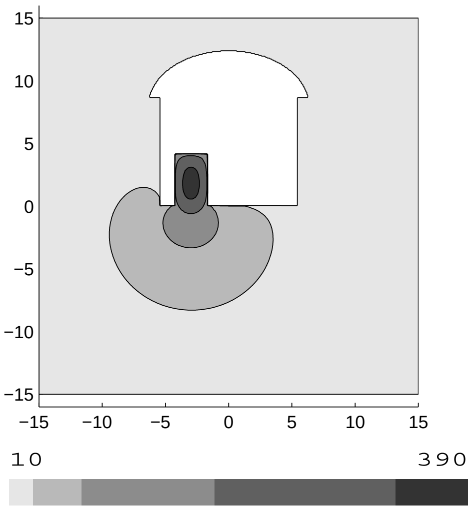
where S, E are the diagonal matrices, $S_{ii} = 1/|Z_i|$, $E_{ii} = \varepsilon_i$ and $R = (R_{ij})$, $\overline{q}$ is the vector of incoming fluxes to the individual zones and $\Theta = \Theta(\tau)$ is the vector of 4th powers of the zone temperatures.

But we must also take into the account that an ε part of the incoming flux is absorbed and the rest is reflected. It leads to the following expression for the vector of radiation fluxes on the zones

$$\overline{q}_r = c_{SB}\, E(I - SR(I-E))^{-1}\, S\, R\, E\, \Theta - c_{SB}\, E\, \Theta\,.$$

This formula is used for updating the time stepping algorithm by a nonlinear term. For $\theta = 1$, it gives $B_h^{(j)} \underline{\tau}^j + N_h(\underline{\tau}^j) = c^j$. This system can be solved by the Newton-type methods, see Byczanski 2000. In our case, it corresponds to two Newton method steps.

The 2D Skalka model is implemented with the aid of MATLAB numerical linear algebra procedures. The emphasis is given on clearing up the effect of radiation heat transfer, testing the numerical methods for the nonlinear radiation heat transfer and enabling an experimentation with model parameters. The exploited triangulation is not structured, using 34 660 triangles, 17 585 nodes and 190 zones on the inner boundary. The time discretization used is very fine, starting from the step of 0.00001 year which is enlarged by a factor 1.01. Totally, the computation requires 1142 time steps for a 100 year period. The computing time is 103 min. on the PC AMD Athlon 1.4GHz computer. The outputs showing a non negligible effect of thermal radiation can be seen in Fig. 5.



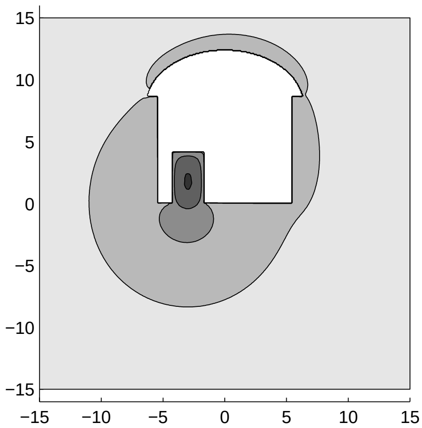


Fig. 5. Temperatures after 1 year after the installation of SNF canisters. Without (above) and with (below) the radiation heat transfer. Nonuniform grey scale.

Concluding remarks

The laboratory tests have proved that temperatures arising in the rock mass in the heated gallery practically do not influence mechanical properties of rocks. Standard tests are sufficient for estimating of rock properties for the input data for a mathematical modelling.

The paper also describes an example of modelling of thermo-mechanical phenomena relevant for the assessment of geological repositories of the SNF. The Skalka model is a nonlinear model including the heat conduction in rocks, the heat convection into cooling air and the heat radiation. The paper includes input data, which were taken from the literature or estimated. We believe that they were sufficiently realistic for the purpose of testing of numerical methods. For the solution of real problems, these data deserve a further discussion. In future, we plan a further development of the solution methods and software including the model extension with groundwater flow in rocks.

The beside the described investigation of mechanical properties of rocks and the mathematical modelling of thermo-elastic behaviour, the Institute of Geonics is also performing a continuous monitoring of stress changes at the Skalka site.

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