Decentralized robust control design using LMI

***Anna Filasová*[1] *and* Dušan Krokavec**

Návrh robustného decentralizovaného riadenia pomocou LMI

The paper deals with application of decentralized controllers for large-scale systems with subsystems interaction and system matrices uncertainties. The desired stability of the whole system is guaranteed while at the same time the tolerable bounds in the uncertainties due to structural changes are maximized. The design approach is based on the linear matrix inequalities (LMI) techniques adaptation for stabilizing controller design.

Key words: *Decentralized control, state feedback, parameter uncertainties, linear matrix inequalities*

Introduction

Recently, a number of efforts have been made to extend the application of robust control techniques to decentralized systems using convex optimization involving LMI (Linear Matrix Inequalities). The paper presents some extensions and modifications of the problems concerning the system robust stability in the presence of interconnection among subsystems, as well as parameter uncertainties in the subsystem state transition matrices, and was motivated by the technique presented in (Befekadu & Erlich, 2005), which is generalized here for uncertain large-scale systems.

System description

Generally, a large-scale interconected system composed of N subsystems can be modelled by differential equations of the form

$$\dot{\mathbf{q}}_i(t) = (\mathbf{A}_i + \Delta\mathbf{A}_i(t))\mathbf{q}_i(t) + \mathbf{B}_i\mathbf{u}_i(t) + \sum_{h=1,h\neq i}^{N} (\mathbf{A}_{ih} + \Delta\mathbf{A}_{ih}(t))\mathbf{q}_h(t), \qquad (1)$$

$$\mathbf{y}_i(t) = \mathbf{C}_i\mathbf{q}_i(t), \qquad (2)$$

with given subsystem matrix $\mathbf{A}_i$, subsystem input matrix $\mathbf{B}_i$ and subsystem output matrix $\mathbf{C}_i$ of appropriate dimension, where i = 1,2, ..., N, $\mathbf{q}_i(t)$ is the subsystem state vector, $\mathbf{u}_i(t)$ is the subsystem control input vector and $\mathbf{y}_i(t)$ is the subsystem measurement vector. It is assumed the matrices $\mathbf{B}_i$, $\mathbf{C}_i$ to be of full rank, all the subsystem states can be observed or measured, pairs $(\mathbf{A}_i,\mathbf{B}_i)$ and $(\mathbf{A}_i,\mathbf{C}_i)$ are stabilizable and observable, respectively, and the subsystem (1), (2) is controlled by local state feedback control law

$$\mathbf{u}_i(\mathbf{q}_i(t)) = \mathbf{K}_i\mathbf{q}_i(t), \qquad (3)$$

i.e. each subsystem is controlled by local control law, where $\mathbf{K}_i$ is a $m_i \, x \, n_i$ constant matrix.

It is supposed that the interconnection uncertainty terms in (1) can be written as

$$\sum_{h=1,h\neq i}^{N} (\mathbf{A}_{ih} + \Delta\mathbf{A}_{ih}(t))\mathbf{q}_h(t) = \mathbf{G}_i\mathbf{g}_i(\mathbf{q}(t)) \leq \mathbf{G}_i \sum_{h=1,h\neq i}^{N} \varepsilon_{ih}\mathbf{H}_{ih}\mathbf{q}_h(t) \leq \mathbf{G}_i\varepsilon_i\mathbf{H}_i\mathbf{q}(t), \qquad (4)$$

where

$$\mathbf{H}_i = [\mathbf{H}_{i1}, \cdots \mathbf{H}_{iN}], \quad \mathbf{H}_{ii} = \mathbf{0}, \qquad \varepsilon_i = \max_h \varepsilon_{ih}, \qquad \mathbf{q}(t) = \begin{bmatrix} \mathbf{q}_1^T(t) & \mathbf{q}_2^T(t) & \cdots & \mathbf{q}_N^T(t) \end{bmatrix}^T, \qquad (5)$$

$$\mathbf{g}_i^T(\mathbf{q}(t))\mathbf{g}_i(\mathbf{q}(t)) \leq \varepsilon_i^2\mathbf{q}^T(t)\mathbf{H}_i^T\mathbf{H}_i\mathbf{q}(t), \qquad (6)$$

$\varepsilon_i > 0$ is a parameter related to interconnection uncertainties in the subsystem, and $\mathbf{H}_i$ and $\mathbf{G}_i$ are constant matrices of appropriate dimensions.

It is assumed that considered uncertainty matrices $\Delta\mathbf{A}_i(t)$, i = 1,2, ..., N, are norm bounded and can be described as (Krokavec & Filasová, 2002)

[1] *Anna Filasová, Dušan Krokavec,* Department of Cybernetics and Artificial Intelligence, Technical University of Košice, Košice, Slovak Republic. anna.filasova@tuke.sk, dusan.krokavec@tuke.sk
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$$\Delta\mathbf{A}_i(t) = \mathbf{M}_i\mathbf{L}_i(t)\mathbf{N}_i\,, \tag{7}$$

$$\mathbf{L}_i^T(t)\mathbf{L}_i(t) < \mathbf{I}_i\,, \tag{8}$$

where $\mathbf{M}_i$ is a $n_i\, x\, l_i$ constant matrix, $\mathbf{N}_i$ is a $h_i\, x\, n_i$ constant matrix, and $\mathbf{I}_i$ is $h_i\, x\, h_i$ identitity matrix. Known matrices $\mathbf{M}_i$, $\mathbf{N}_i$ define the structure of the state transition matrix uncertainties of the i^{th} subsystem and the parameter uncertainty matrices $\mathbf{L}_i(t)$ belong to the set

$$\mathrm{L}_i = \{\mathbf{L}_i(t) \in \mathrm{R}^{l_i \times h_i} : \mathbf{L}_i^T\mathbf{L}_i < \mathbf{I}_i\},\quad i = 1, 2, \ldots N\,, \tag{9}$$

Using the overall system state variable vector $\mathbf{q}(t)$ defined in (6), the interconnected system model can be compactly written as follows

$$\dot{\mathbf{q}}(t) = (\mathbf{A} + \Delta\mathbf{A}(t))\mathbf{q}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{G}\mathbf{g}(\mathbf{q}(t))\,, \tag{10}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{q}(t)\,, \tag{11}$$

$$\mathbf{u}(t) = \begin{bmatrix}\mathbf{u}_1^T(t) & \mathbf{u}_2^T(t) & \cdots & \mathbf{u}_N^T(t)\end{bmatrix}^T,\quad \mathbf{y}(t) = \begin{bmatrix}\mathbf{y}_1^T(t) & \mathbf{y}_2^T(t) & \cdots & \mathbf{y}_N^T(t)\end{bmatrix}^T, \tag{12}$$

$$\mathbf{A} = \mathrm{diag}\begin{bmatrix}\mathbf{A}_1 \cdots \mathbf{A}_N\end{bmatrix},\qquad \Delta\mathbf{A}(t) = \mathrm{diag}\begin{bmatrix}\Delta\mathbf{A}_1(t) \cdots \Delta\mathbf{A}_N(t)\end{bmatrix}, \tag{13}$$

$$\mathbf{B} = \mathrm{diag}\begin{bmatrix}\mathbf{B}_1 \cdots \mathbf{B}_N\end{bmatrix},\qquad \mathbf{C} = \mathrm{diag}\begin{bmatrix}\mathbf{C}_1 \cdots \mathbf{C}_N\end{bmatrix},\qquad \mathbf{G} = \mathrm{diag}\begin{bmatrix}\mathbf{G}_1 \cdots \mathbf{G}_N\end{bmatrix}, \tag{14}$$

with the uncertainties upper-bounds (9), and interconections upper-bounds

$$\mathbf{g}^T(\mathbf{q}(t))\mathbf{g}(\mathbf{q}(t)) = \sum_{i=1}^{N}\mathbf{g}_i^T(\mathbf{q}(t))\mathbf{g}_i(\mathbf{q}(t)) \le \mathbf{q}^T(t)\left[\sum_{i=1}^{N}\varepsilon_i^2\mathbf{H}_i^T\mathbf{H}_i\right]\mathbf{q}(t)\,. \tag{15}$$

Thus, the control law for the overall system will be

$$\mathbf{u}(t) = \mathbf{K}\mathbf{q}(t)\,,\quad \mathbf{K} = \mathrm{diag}\begin{bmatrix}\mathbf{K}_1 & \mathbf{K}_2 & \cdots & \mathbf{K}_N\end{bmatrix}. \tag{16}$$

Lyapunov function

Since overall system (10), (11) is linear in $\mathbf{q}(t)$, the Lyapunov function candidate $\mathrm{v}(\mathbf{q}(t))$ can be of the form

$$\mathrm{v}(\mathbf{q}(t)) = \mathbf{q}^T(t)\mathbf{P}(t)\mathbf{q}(t)\,. \tag{17}$$

Here $\mathrm{v}(\mathbf{q}(t))$ is a quadratic positive definite function with symmetric positive definite weighting matrix $\mathbf{P}(t)$. If a steady-state solution $\mathbf{P}$ of $\mathbf{P}(t)$ under the boudary condition $\mathbf{P}(\infty) = \mathbf{0}$ exists, the evaluating derivative of (17) for steady-state solution $\mathbf{P}$ gives

$$\tfrac{\mathrm{dv}(\mathbf{q}(t))}{\mathrm{d}t} = \mathbf{q}^T(t)(\mathbf{A}^T\mathbf{P} + \Delta\mathbf{A}^T(t)\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{P}\Delta\mathbf{A}(t) + \mathbf{K}^T\mathbf{B}^T\mathbf{P} + \mathbf{P}\mathbf{B}\mathbf{K} + \mathbf{g}^T(\mathbf{q}(t))\mathbf{G}^T\mathbf{P} + \mathbf{P}\mathbf{G}\mathbf{g}(\mathbf{q}(\mathbf{t})))\mathbf{q}(t) < 0\,, \tag{18}$$

It now follows for (18), using identity (e.g. Krokavec & Filasová, 2006a)

$$\mathbf{U}\mathbf{V}^T + \mathbf{V}\mathbf{U}^T \le \mathbf{U}\mathbf{U}^T + \mathbf{V}\mathbf{V}^T\,, \tag{19}$$

that

$$\begin{aligned}\tfrac{\mathrm{dv}(\mathbf{q}(t))}{\mathrm{d}t} &\le \mathbf{q}^T(t)(\mathbf{A}^T\mathbf{P} + \Delta\mathbf{A}^T(t)\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{P}\Delta\mathbf{A}(t) + \mathbf{K}^T\mathbf{B}^T\mathbf{P} + \mathbf{P}\mathbf{B}\mathbf{K} + \mathbf{g}^T(\mathbf{q}(t))\mathbf{g}(\mathbf{q}(\mathbf{t})) + \mathbf{P}\mathbf{G}\mathbf{G}^T\mathbf{P})\mathbf{q}(t) \le \\ &\le \mathbf{q}^T(t)(\mathbf{A}^T\mathbf{P} + \Delta\mathbf{A}^T(t)\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{P}\Delta\mathbf{A}(t) + \mathbf{K}^T\mathbf{B}^T\mathbf{P} + \mathbf{P}\mathbf{B}\mathbf{K} + \mathbf{P}\mathbf{G}\mathbf{G}^T\mathbf{P} + \sum_{i=1}^{N}\varepsilon_i^2\mathbf{H}_i^T\mathbf{H}_i)\mathbf{q}(t) < 0\,,\end{aligned} \tag{20}$$

$$\mathbf{A}^T\mathbf{P} + \Delta\mathbf{A}^T(t)\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{P}\Delta\mathbf{A}(t) + \mathbf{K}^T\mathbf{B}^T\mathbf{P} + \mathbf{P}\mathbf{B}\mathbf{K} + \mathbf{P}\mathbf{G}\mathbf{G}^T\mathbf{P} + \sum_{i=1}^{N}\varepsilon_i^2\mathbf{H}_i^T\mathbf{H}_i < 0\,, \tag{21}$$

respectively, where (21) is the matrix Lyapunov equation.

Using (7), (8) the sum $\mathbf{P}\Delta\mathbf{A}(t)$ **+** $\Delta\mathbf{A}^T(t)\mathbf{P}$ in (21) can be rewritten as

$$\mathbf{P}\Delta\mathbf{A}(t)+\Delta\mathbf{A}^T(t)\mathbf{P}=\mathbf{P}\mathrm{diag}\left[\frac{\nu_1}{\nu_1}\mathbf{M}_1\mathbf{L}_1(t)\mathbf{N}_1\cdots\frac{\nu_N}{\nu_N}\mathbf{M}_N\mathbf{L}_N(t)\mathbf{N}_N\right]+\mathrm{diag}\left[\frac{\nu_1}{\nu_1}\mathbf{N}_1^T\mathbf{L}_1^T(t)\mathbf{M}_1^T\cdots\frac{\nu_N}{\nu_N}\mathbf{N}_N^T\mathbf{L}_N^T(t)\mathbf{M}_N^T\right]\mathbf{P}\,, \tag{22}$$

where $\upsilon_i > 0$, $i = 1,2, ..., N$, are design parameters. Also, using (19), an upper bound of (22) is

$$\begin{aligned}\mathbf{P}\Delta\mathbf{A}(t)+\Delta\mathbf{A}^T(t)\mathbf{P}&\le\mathbf{P}\mathrm{diag}\left[\nu_1^2\mathbf{M}_1\mathbf{M}_1^T\cdots\nu_N^2\mathbf{M}_N\mathbf{M}_N^T\right]\mathbf{P}+\mathrm{diag}\left[\frac{1}{\nu_1^2}\mathbf{N}_1^T\mathbf{L}_1^T(t)\mathbf{L}_1(t)\mathbf{N}_1\cdots\frac{1}{\nu_N^2}\mathbf{N}_N^T\mathbf{L}_N^T(t)\mathbf{L}_N(t)\mathbf{N}_N\right]\le\\ &\le\mathbf{P}\mathrm{diag}\left[\nu_1^2\mathbf{M}_1\mathbf{M}_1^T\cdots\nu_N^2\mathbf{M}_N\mathbf{M}_N^T\right]\mathbf{P}+\mathrm{diag}\left[\frac{1}{\nu_1^2}\mathbf{N}_1^T\mathbf{N}_1\cdots\frac{1}{\nu_N^2}\mathbf{N}_N^T\mathbf{N}_N\right].\end{aligned} \tag{23}$$

Denoting

$$\mathbf{M}_i^\bullet=\left[\mathbf{0}\cdots\mathbf{0}\,\mathbf{M}_i^T\,\mathbf{0}\cdots\mathbf{0}\right]^T,\qquad \mathbf{N}_i^\bullet=\left[\mathbf{0}\cdots\mathbf{0}\,\mathbf{N}_i\,\mathbf{0}\cdots\mathbf{0}\right], \tag{24}$$

inequality (23) takes on the form as follows

$$\mathbf{P}\Delta\mathbf{A}(t)+\Delta\mathbf{A}^T\mathbf{P}(t)\le\sum_{i=1}^{N}\left(\nu_i^2\mathbf{P}\mathbf{M}_i^\bullet\mathbf{M}_i^{\bullet T}\mathbf{P}+\frac{1}{\nu_i^2}\mathbf{N}_i^{\bullet T}\mathbf{N}_i^\bullet\right), \tag{25}$$

and, in adition, the matrix Lyapunov equation (21) has an expression of the form

$$\mathbf{A}^T\mathbf{P}+\mathbf{PA}+\mathbf{K}^T\mathbf{B}^T\mathbf{P}+\mathbf{PBK}+\mathbf{PGG}^T\mathbf{P}+\sum_{i=1}^{N}\left(\varepsilon_i^2\mathbf{H}_i^T\mathbf{H}_i+\nu_i^2\mathbf{P}\mathbf{M}_i^\bullet\mathbf{M}_i^{\bullet T}\mathbf{P}+\frac{1}{\nu_i^2}\mathbf{N}_i^{\bullet T}\mathbf{N}_i^\bullet\right)<0\,. \tag{26}$$

That ensures the negative definiteness of the quadratic function derivative under the constraints (9) and (15) for all trajectories of the closed-loop interconnected system (10), (11) under the control.

Schur complement

Let the linear matrix inequality be given as

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & -\mathbf{R}\end{bmatrix}<0,\quad \mathbf{Q}=\mathbf{Q}^T,\quad \mathbf{R}=\mathbf{R}^T\,. \tag{27}$$

Then Gauss elimination yields (Boyd *at al.*, 1994)

$$\begin{bmatrix}\mathbf{I} & \mathbf{SR}^{-1}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & -\mathbf{R}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{R}^{-1}\mathbf{S}^T & \mathbf{I}\end{bmatrix}=\begin{bmatrix}\mathbf{Q}+\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T & \mathbf{0}\\ \mathbf{0} & -\mathbf{R}\end{bmatrix},\qquad \det\begin{bmatrix}\mathbf{I} & \mathbf{SR}^{-1}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}=1\,, \tag{28}$$

and (28) implies

$$\begin{bmatrix}\mathbf{Q} & \mathbf{S}\\ \mathbf{S}^T & -\mathbf{R}\end{bmatrix}<0\ \Leftrightarrow\ \begin{bmatrix}\mathbf{Q}+\mathbf{SR}^{-1}\mathbf{S}^T & \mathbf{0}\\ \mathbf{0} & -\mathbf{R}\end{bmatrix}<0\ \Rightarrow\ \mathbf{Q}+\mathbf{SR}^{-1}\mathbf{S}^T<0,\quad \mathbf{R}>\mathbf{0}\,. \tag{29}$$

LMI form of design conditions

Inequality (26) can be converted into LMI form using a technique based on Schur complements (29). Note that matrix inequality (26) is not convex in **P** and **K,** but with any linear fractional transformation can be transformed to those form. Thus, pre-multiplying (26) from left and right hand side by matrix $\mathbf{P}^{-1}$ (matrix **P** is a positive definite matrix and then matrix $\mathbf{P}^{-1}$ is positive definite, too) gives

$$\mathbf{P}^{-1}\mathbf{A}^T+\mathbf{AP}^{-1}+\mathbf{P}^{-1}\mathbf{K}^T\mathbf{B}^T+\mathbf{BKP}^{-1}+\mathbf{GG}^T+\sum_{i=1}^{N}\left(\varepsilon_i^2\mathbf{P}^{-1}\mathbf{H}_i^T\mathbf{H}_i\mathbf{P}^{-1}+\nu_i^2\mathbf{M}_i^\bullet\mathbf{M}_i^{\bullet T}+\frac{1}{\nu_i^2}\mathbf{P}^{-1}\mathbf{N}_i^{\bullet T}\mathbf{N}_i^\bullet\mathbf{P}^{-1}\right)<0\,. \tag{30}$$

Using substitutions

$$\mathbf{X}=\mathbf{X}^T=\mathbf{P}^{-1}>0,\qquad \mu_i^{-1}=\varepsilon^2>0,\quad \eta_i=\nu_i^2>0,\quad i=1,2,\ldots N,\qquad \mathbf{Y}=\mathbf{KP}^{-1}=\mathbf{KX}\,, \tag{31}$$

inequality (30) takes on the following form

$$\mathbf{XA}^T+\mathbf{AX}+\mathbf{Y}^T\mathbf{B}^T+\mathbf{BY}+\mathbf{GG}^T+\sum_{i=1}^{N}\left(\mu_i^{-1}\mathbf{XH}_i^T\mathbf{H}_i\mathbf{X}+\eta_i\mathbf{M}_i^\bullet\mathbf{M}_i^{\bullet T}+\eta_i^{-1}\mathbf{XN}_i^{\bullet T}\mathbf{N}_i^\bullet\mathbf{X}\right)<0\,, \tag{32}$$

which is convex in **X** and **Y**. If this LMI problem in **X** and **Y** has a solution, then the Lyapunov function $v(\mathbf{q}(t)) = \mathbf{q}^T(t)\mathbf{X}^{-1}\mathbf{q}(t)$ proves the quadratic stability of the closed-loop system with state feedback $\mathbf{u}(t) = \mathbf{Y}\mathbf{X}^{-1}\mathbf{q}(t) = \mathbf{K}\,\mathbf{q}(t)$.

Using LMI variables (31), the constraint (32) can be represented as the LMI

$$\begin{bmatrix} \mathbf{R} & \mathbf{G} & \mathbf{X}\mathbf{H}_1^T & \cdots & \mathbf{X}\mathbf{H}_N^T & \mathbf{X}\mathbf{N}_1^{\bullet T} & \cdots & \mathbf{X}\mathbf{N}_N^{\bullet T} \\ \mathbf{G}^T & -\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{H}_1\mathbf{X} & \mathbf{0} & -\mu_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{H}_N\mathbf{X} & \mathbf{0} & \mathbf{0} & \cdots & -\mu_N\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{N}_1^{\bullet}\mathbf{X} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\eta_1\mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{N}_N^{\bullet}\mathbf{X} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & -\eta_N\mathbf{I} \end{bmatrix} < 0, \qquad (33)$$

where

$$\mathbf{R} = \mathbf{X}\mathbf{A}^T + \mathbf{A}\mathbf{X} + \mathbf{Y}^T\mathbf{B}^T + \mathbf{B}\mathbf{Y} + \sum_{i=1}^{N}\eta_i\mathbf{M}_i^{\bullet}\mathbf{M}_i^{\bullet T}\,. \qquad (34)$$

Note that it can also be considered an LMI in **X**, and μ_i, η_i, $i = 1,2, \ldots, N$, defined in (31).

The solution of this problem can be expressed as a state feedback (3) with state feedback gain matrix

$$\mathbf{K} = \mathbf{Y}\mathbf{X}^{-1} = \mathbf{Y}\mathbf{P}\,, \qquad (35)$$

X, **Y** is the unique solution of (33), **X** is a symmetric block-diagonal positive definite matrix, and **Y**, **K** are block diagonal matrices, respectively, generally nonsymmetric.

Illustrative example

To demonstrate the algorithm properties the multiarea model of a power system was used (Veselý *et al.* 2002, Krokavec & Filasová, 2006b). Given model structures satisfy (10) - (11), where

$$\mathbf{A}_1 = \mathbf{A}_1 = \begin{bmatrix} -10.05 & 0 & -5.21 & 0 \\ 1.33 & -1.33 & 0 & 0 \\ 0 & -6 & -0.05 & 6 \\ 0.55 & 0 & 0 & 0 \end{bmatrix},\quad \mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 12.5 \\ 0 \\ 0 \\ 0 \end{bmatrix},\quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 \end{bmatrix},\quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 \end{bmatrix},$$

$$\mathbf{G}_1 = \mathbf{G}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},\quad \mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 \end{bmatrix},\quad \mathbf{H}_1 = \begin{bmatrix} 0\;0\;0\;0\;0\;0 -0.55\;0 \end{bmatrix},\quad \mathbf{H}_2 = \begin{bmatrix} 0\;0 -0.55\;0\;0\;0\;0\;0 \end{bmatrix},$$

$$\Delta\mathbf{A}_1(t) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & a_{32}(t) & 0 & a_{34}(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Delta\mathbf{A}_2(t) = 1.75\Delta\mathbf{A}_1(t), \mathbf{M}_1 = \mathbf{M}_2 = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \mathbf{N}_1^T = \begin{bmatrix} 0 \\ 0.5 \\ 0 \\ -0.5 \end{bmatrix}, \mathbf{N}_2^T = 1.75\mathbf{N}_1^T,$$

$$\mathbf{M}_1^{\bullet} = \begin{bmatrix} \mathbf{M}_1 \\ \mathbf{0} \end{bmatrix},\quad \mathbf{M}_2^{\bullet} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}_2 \end{bmatrix},\quad \mathbf{N}_1^{\bullet} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{0} \end{bmatrix},\quad \mathbf{N}_2^{\bullet} = \begin{bmatrix} 0 & \mathbf{N}_2 \end{bmatrix}$$

The goals were to design equal feedback gain matrices for both subsystems, independent on different subsystems parameter uncertainties.

The problem was solved using the Self-Dual-Minimization (SeDuMi) package for MATLAB (Peaucelle *et al.*, 2002). This package is constructed based upon the principles that one can describe the system to be analyzed using a MATLAB standard function and LMI variables, inequality constraints as well as LMI solution to be specified within the SeDuMi interface add-on for MATLAB. The LMI problem was feasible in **X** and **Y** and results in

$$\mathbf{X} = \begin{bmatrix} 8.0866 & 2.4763 & -4.7279 & 2.6705 & 0 & 0 & 0 & 0 \\ 2.4763 & 3.4233 & -3.3195 & 2.2104 & 0 & 0 & 0 & 0 \\ -4.7279 & -3.3195 & 7.1296 & -1.9731 & 0 & 0 & 0 & 0 \\ 2.6705 & 2.2104 & -1.9731 & 2.8338 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.0866 & 2.4763 & -4.7279 & 2.6705 \\ 0 & 0 & 0 & 0 & 2.4763 & 3.4233 & -3.3195 & 2.2104 \\ 0 & 0 & 0 & 0 & -4.7279 & -3.3195 & 7.1296 & -1.9731 \\ 0 & 0 & 0 & 0 & 2.6705 & 2.2104 & -1.9731 & 2.8338 \end{bmatrix},$$

$$\mathbf{Y} = \begin{bmatrix} 5.8493 & -0.4473 & -1.5758 & 2.0209 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.8493 & -0.4473 & -1.5758 & 2.0209 \end{bmatrix},$$

$\mu_1 = 7.3757, \quad \mu_2 = 7.2517, \quad \eta_1 = 1.2714, \quad \eta_2 = 1.3058$.

According to (35), the gain matrices were obtained as follows

$$\mathbf{K}_1 = \mathbf{K}_2 = \begin{bmatrix} 0.7555 & -1.4826 & -0.1114 & 1.0800 \end{bmatrix}.$$

One can easily verify, that closed loop is stable with the same eigenvalues spectrum of the subsystem transition matrix $\rho(\mathbf{A}_1) = \rho(\mathbf{A}_2) = \{- 1.1692 \pm j\ 0.0988,\ - 2.0488 \pm j\ 7.7686\}$.

Concluding remarks

The decentralized robust controller design is formulated as an optimization problem involving linear matrix inequalities and solved by LMI programming method, where the controller structures take into account the interactions among subsystems and the subsystem state transition matrices uncertainties. The most important practical application is that there are effective and powerful algorithms for these by LMI reformulated problems, that is, algorithms that compute the global optimum with non-heuristic stopping criteria (the global optimum is obtained to within some pre-specified accuracy). Presented decomposition principle gives enough flexibility to allow the inclusion of more general interaction structures and parametric uncertainties within all matrices of linear systems, given by the state-space description.

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