Modelling and robust control of a flexible beam Quanser experiment

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In this paper, the robust optimal model matching control design method is applied to a flexible beam Quanser experiment. The neglected nonlinearities of the real experiment in the controller design step lead to important differences between the simulation and experimental results, including vibrations of the real beam and/or stationary errors due to insensibility zone. In order to eliminate these tracking errors and reduce the vibrations of the beam we proposed an improved control law by using some blocks with variable structure. Because some of the model's parameters are imprecisely known, an identification procedure is needed. The paper presents a subspace identification method, which is a relatively new approach used for the state space model identification. Finally, the performance of the proposed controller is illustrated by some experimental results.

Key words: *Robust control, model matching, flexible beam, subspace based identification.*

Introduction

The flexible beams dynamic is described by partial differential equations that are obtained based on Euler-Lagrange theory. The mathematical model of a flexible beam can be obtained, using modal decomposition, as an infinity sum of second order dynamics (Balas, 1978). This model is approximated, for control and simulation purposes, by a finite dimensional one. The neglected dynamics lead to the so-called "spillover-effect" that represents the robust stability problem against the model uncertainty.

The real mathematical model of the system is very complicated, so for control purpose a simplified model is typically used. In general, the model of a rotating flexible beam is derived using Lagrange's energy equations, and consequently generalized dynamic equations are obtained. In order to obtain useful models for control design, approximations of this model can be derived (represented by nonlinear ordinary differential equations). The model parameters are identified using the subspace based methods. The paper presents a relatively new subspace identification procedure. Subspace-based methods have attracted a great deal of attention in the field of system identification (Van der Veer et al., 1993), (Van Overschee et al., 1996), (Oshumi et al., 2002). Most of the researches were concentrated on the modelling of discrete-time linear systems (Oshumi et al., 2002). Subspace identification algorithms are based on concepts from system theory, linear algebra and statistics. In this approach, certain subspace matrices of the process are, at the first step, calculated through data projection. These matrices include extended observability matrix, lower triangular block-Toeplitz matrix and state sequences. These subspace matrices are directly calculated from the input/output data Hankel matrices without any iteration compared to the iterative schemes used in the prediction error methods. At the second step, the system state space matrices are recovered from the extended observability matrix and lower triangular block-Toeplitz matrix. One obtains a state space linear model.

The main conceptual novelties in subspace identification algorithms are:

- the state of a dynamical system is emphasised in the context of system identification, whereas classical approaches are based on an input-output framework. This relatively recent introduction of the state into the identification area may come as a surprise since in control theory and the analysis of dynamical systems, the importance of the concept of state has been appreciated for quite some time now;
- the subspace system identification approach makes full use concepts and algorithms from numerical linear algebra. While classical methods are basically inspired by least squares, this approach uses modern algorithms such as the QR-decomposition, the singular value decomposition, and angles between subspaces;
- the subspace system identification approach provides a geometric framework, in which seemingly different models are treated in a unified manner.

When they are implemented correctly, subspace identification algorithms are fast, despite the fact that they are using QR and singular decomposition. As a matter of fact, they are faster then the classical identification methods, such as Prediction Error Methods, because they are not iterative. Hence there are also

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no convergence problems. Moreover, numerical robustness is guaranteed precisely because of these well-understood algorithms from numerical linear algebra. As a consequence, the user will never be confronted with problems such as lack of convergence, slow convergence or numerical instability.

Since the main interest lies in using the models in a computer aided control system design environment and because, when using linear theories, the complexity of the controller is proportional to the order of the system, one is inclined to obtain models with order as low as possible. In subspace identification, the reduced model can be obtain directly, without having to compute first the high order model, and this directly from input-output data. In the paper, using this subspace identification procedure, a linear model of the flexible beam is obtained.

In this paper, the two-degree of freedom controller technique (Chen and Yang, 1993; Popescu et al, 1995; Popescu and Bobasu, 2001) is used. The neglected nonlinearities of the real experiment in the controller design step lead to important differences between the simulation and experimental results, including vibrations of the real beam and/or stationary errors due to insensibility zone. In order to eliminate these tracking errors and reduce the vibrations of the beam we proposed an improved control law by using some blocks with variable structure. Finally, the performance of the proposed controller is illustrated by some experimental results obtained by using a flexible beam Quanser experiment**.**

The mathematical model of a flexible beam

The flexible link experiment consists of a mechanical and an electrical subsystem. The Quanser experimental set-up contains the following components (Quanser Consulting Inc., 1998): Quanser Universal Power Module UPM 2405/1503; Quanser MultiQ PCI data acquisition board; Quanser Flexgage – Rotary Flexible Link Module; Quanser SRV02-E servo-plant; Quanser cables and accessories; computer equipped with Matlab/Simulink and WinCon software.

The modelling of the mechanical subsystem consists in describing the tip deflection and the base rotation dynamics. The electrical subsystem involves modelling a DC servomotor that dynamically relates voltage to torque.

The Flexible Link module consists of a flat flexible arm at the end of which is a hinged potentiometer (Fig. 1). The flexible arm is mounted to the hinge. Measurement of the flexible arm deflection is obtained using a strain gage. The equations of motion involving a rotary flexible link, involves modelling the rotational base and the flexible link as rigid bodies.

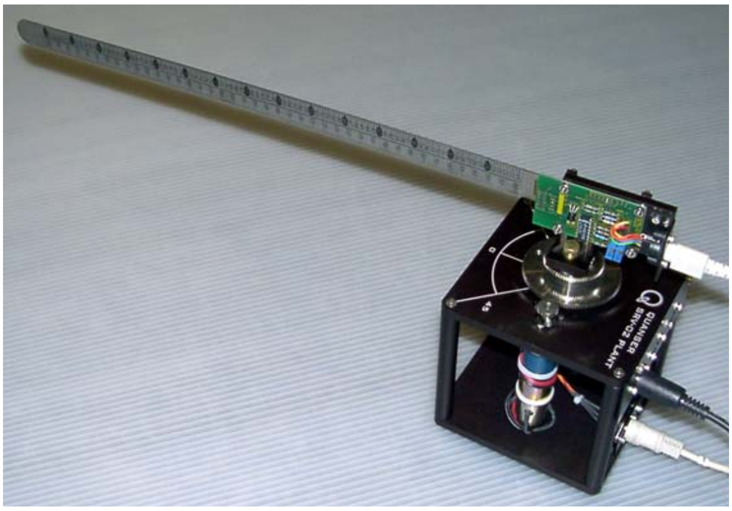


Fig. 1. Quanser flexible beam.

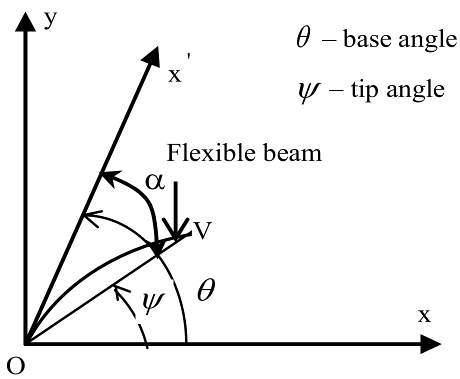


***Fig. 2.** Base configuration of a flexible beam.*

Let consider a flexible beam having the base configuration as represented in Figure 2. The Ox axis represents the initial direction of the beam, and Ox' axis corresponds to the movement of the beam rigid end. If we choose the output as the angle (ψ) of the beam tip (V), we obtain a configuration in which the sensor and actuator are non-collocated. The flexible beam dynamic is described by partial differential equations that are obtained based on Euler-Lagrange theory.

As a simplification to the partial differential equation describing the motion of a flexible link, a lumped single degree of freedom approximation is used.

The Lagrange's energy equations:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\theta}}\right)-\frac{\partial T}{\partial \theta}+\frac{\partial P}{\partial \theta}=Q_{\theta}\quad , \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{\alpha}}\right)-\frac{\partial T}{\partial \alpha}+\frac{\partial P}{\partial \alpha}=Q_{\alpha}\quad , \tag{2}$$

can be used and the generalised dynamic equations for the tip and base dynamics can be obtained, in terms of a set of generalised variables α and θ, where α is the angle of tip deflection and θ is the angle of base rotation. After some straightforward calculations, tacking into consideration the formulas of the kinetic and potential energies, from the generalised dynamic equations we can obtain a linear model expressed by ordinary differential equations (Ionete, 2003):

$$\begin{aligned}\ddot{\theta}&=-\frac{K_{stiff}}{J_{base}}\alpha+\frac{1}{J_{base}}\tau\\ \ddot{\alpha}&=-K_{stiff}\left(\frac{1}{J_{link}}+\frac{1}{J_{base}}\right)\alpha+\frac{1}{J_{base}}\tau\end{aligned} \tag{3}$$

where τ is the torque applied to the rotational base, K_{stiff} is an equivalent torsion spring constant, J_{base} and J_{link} are rotational inertia respectively.

Next, rewriting equations (1) into a state space form gives

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\dfrac{K_{stiff}}{J_{base}} & 0 & 0\\ 0 & -K_{stiff}\left(\dfrac{1}{J_{link}}+\dfrac{1}{J_{base}}\right) & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \dfrac{1}{J_{base}}\\ \dfrac{1}{J_{base}}\end{bmatrix}\tau \tag{4}$$

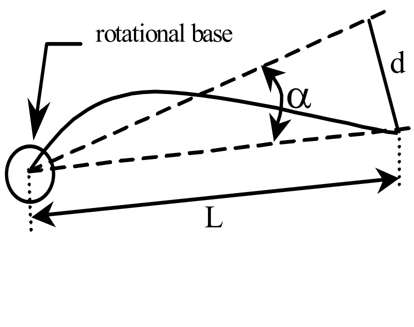
Since the control input into the mechanical model of equation (2) is a torque τ, an electrical dynamic equation relating voltage to torque is needed. First, the torque applied to the rotational base, on the right hand side of equation (2), is converted to the torque applied to the gear train by the DC servomotor by means of a gear ratio K_g given as $\tau = K_g \tau_m$, where τ_m is the torque applied by the servomotor.

The DC servomotor is an electromechanical device that relates torque to current through a proportionality gain K_T. Applying Kirchoff's voltage law to the DC circuitry of the motor, and after some calculations, we obtain a state space model of (2), rewritten to utilize an electrical control voltage as input (Ionete, 2003):

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\dfrac{K_{stiff}}{J_{base}} & -\dfrac{K_T K_b K_g^2}{J_{base}R} & 0\\ 0 & -K_{stiff}\left(\dfrac{1}{J_{link}}+\dfrac{1}{J_{base}}\right) & -\dfrac{K_T K_b K_g^2}{J_{base}R} & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix}+\underbrace{\begin{bmatrix}0\\ 0\\ \dfrac{K_T K_g}{J_{base}R}\\ -\dfrac{K_T K_g}{J_{base}R}\end{bmatrix}}_{b}V_{PC} \tag{5}$$

where K_b is a proportional constant between angular velocity of the motor and the voltage applied by the motor shaft, R is the resistance of the resistor of DC circuitry and V_{PC} is the voltage supplied by the data acquisition board.

Next, a transformation between relative angular position and relative displacement about a neutral axis is used within the state space model.



The relative angular position and the velocity with respect to the rotating base are proportional to the relative displacement and to the velocity of the flexible link tip (i.e. $\sin(\alpha) \approx \alpha$, for α small) respectively: $d = \alpha \cdot L$, $\dot{d} = \dot{\alpha} \cdot L$, where d is the relative displacement and L is the length of the flexible link. The Fig. 3 shows the relationship of these three parameters.

Fig. 3. The relationship between relative angular position and relative angular displacement

Substituting the above equations into the state space dynamics previously obtained gives the following state space equation

$$\begin{bmatrix} \dot{\theta} \\ \dot{d} \\ \ddot{\theta} \\ \ddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{K_{stiff}}{J_{base}L} & -\dfrac{K_T K_b K_g^2}{J_{base}R} & 0 \\ 0 & -\dfrac{K_{stiff}}{L}\left(\dfrac{1}{J_{link}}+\dfrac{1}{J_{base}}\right) & -\dfrac{K_T K_b K_g^2}{J_{base}R} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ d \\ \dot{\theta} \\ \dot{d} \end{bmatrix} + bV_{PC} \tag{6}$$

The purpose of the system is to design a controller that reduces (eliminates) the undamped oscillations of the tip while commanding the tip to a desired position.

Subspace based identification of flexible rotary link

Deterministic subspace identification algorithms compute state space models from given input-output data. The unknown system is represented in Fig. 4. In our case we suppose that there are neither measurement nor process noise (v_k=w_k=0). The identification problem consists in the following:

"Given s measurements of the input $u_k \in \Re^m$ and the output $y_k \in \Re^l$ generated by the unknown deterministic system of order n:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k + Du_k \end{cases} \tag{7}$$

determine:

- the order n of the unknown system;
- the system matrices $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{l\times n}$, $D \in R^{l\times m}$ (up to within a similarity transformation)."

Block Hankel matrices play an important role in subspace identification algorithms. These matrices can be easily constructed from the given input output data. Since there are no external inputs for purely stochastic systems, we will not use any input block Hankel matrices. Output block Hankel matrices are defined as in subspace based identification theory (see for example (Van Overschee et al., 1996)):

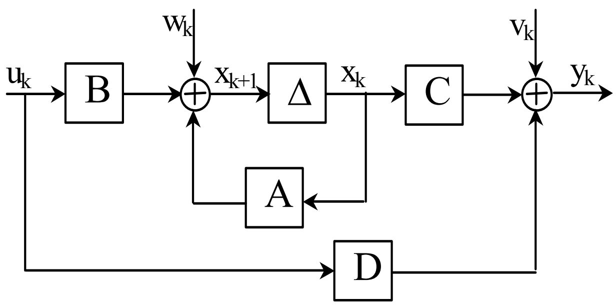


Fig. 4. A linear time-invariant system.

$$Y_{0,2i-1} = \left(\begin{array}{ccccc} y_0 & y_1 & y_2 & \cdots & y_{j-1} \\ y_1 & y_2 & y_3 & \cdots & y_j \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{i-1} & y_i & y_{i+1} & \cdots & y_{i+j-2} \\ \hline y_i & y_{i+1} & y_{i+2} & \cdots & y_{i+j-1} \\ y_{i+1} & y_{i+2} & y_{i+3} & \cdots & y_{i+j} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{2i-1} & y_{2i} & y_{2i+1} & \cdots & y_{2i+j-2} \end{array} \right) = \left(\frac{Y_{0,i-1}}{Y_{i,2i-1}} \right) = \left(\frac{Y_p}{Y_f} \right) = \left(\frac{Y_{0,i}}{Y_{i+1,2i-1}} \right) = \left(\frac{Y_p^+}{Y_f^-} \right) \tag{8}$$

The input block Hankel matrices $U_{0,2i-1}$, U_p and U_f are defined in a similar way. We define the block Hankel matrices consisting of inputs and outputs as:

$$W_{0,i-1}=\begin{pmatrix}U_{0,i-1}\\ Y_{0,i-1}\end{pmatrix}=\begin{pmatrix}U_p\\ Y_p\end{pmatrix}=W_p \tag{9}$$

State sequences play an important role in the derivation and interpretation of subspace identification algorithms. The state sequence X_i is defined as:

$$X_i=\begin{pmatrix}x_i & x_{i+1}\dots x_{i+j-2} & x_{i+j-1}\end{pmatrix}\in\Re^{n\times j} \tag{10}$$

where subscript *i* denotes the subscript of the first element of the state sequence. Analogous to the past inputs and outputs, we denote the past state sequence by X_p and the future state sequence X_f:

$$X_p=X_0\,,\ X_f=X_i \tag{11}$$

Subspace identification algorithms make use of observability and controllability matrices and their structure. The extended ($i > n$) observability matrix Γ_i is defined as:

$$\Gamma_i=\begin{pmatrix}C\\ CA\\ CA^2\\ \dots\\ CA^{i-1}\end{pmatrix}\in\Re^{li\times n} \tag{12}$$

If the pair $\{A, C\}$ is observable, the rank of Γ_i is equal to *n*. The following algorithm is used in flexible rotary link identification:

Step 1. Calculate the oblique projections:

$$O_i=Y_f\,/_{U_f}\,W_p\,,\ O_{i-1}=Y_f^-\,/_{U_f^-}\,W_p^+ \tag{13}$$

Step 2. Calculate the singular value decomposition (SVD) of the weighted projection:

$$W_1O_iW_2=USV^T \tag{14}$$

Step 3. Determine the order by inspecting the singular values in *S* and the partition of the SVD accordingly to obtain U_1 and S_1.

Step 4. Determine Γ_i and Γ_{i-1} as:

$$\Gamma_i=W_1^{-1}U_1S_1^{1/2}\,,\ \Gamma_{i-1}=\underline{\Gamma_i} \tag{15}$$

where $\underline{\Gamma_i}$ denotes the matrix Γ_i without the last *l* rows.

Step 5. Determine X_i and X_{i+1} as:

$$X_i=\Gamma_i^*O_i\,,\ X_{i+1}=\Gamma_{i-1}^*O_{i-1} \tag{16}$$

Step 6. Solve the set of linear equations for *A*, *B*, *C* and *D*:

$$\begin{pmatrix}X_{i+1}\\ Y_{i/i}\end{pmatrix}=\begin{pmatrix}A & B\\ C & D\end{pmatrix}\begin{pmatrix}X_i\\ U_{i/i}\end{pmatrix} \tag{17}$$

The data set used for identification is presented in Figure 5 and Figure 6. The process is excited using a zero mean white Gaussian noise sequence as inputs. The input of the model is the voltage (supplied by the data acquisition board) and the output is the tip displacement (the deflection).

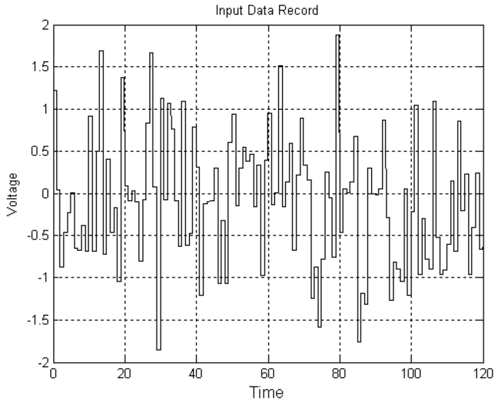


Fig. 5. Input data record

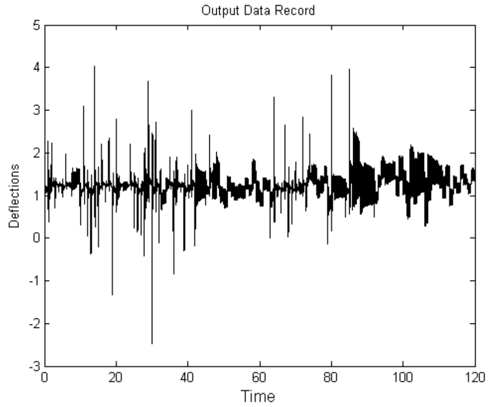


Fig. 6. Output data record

As a result of identification procedure, estimated state matrices are obtained, with a good precision with respect to the real model. The estimated state matrices are the following:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1060.8 & -57.7 & 0 \\ 0 & -1465.2 & 57.7 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 107.39 \\ -107.39 \end{bmatrix}, \quad (18)$$
$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \; D = 0$$

Two-degree of freedom controller

The robust stabilizing controller design method with robust stability-degree assignment (Kimura et al., 1991) allows faster suppressing of the flexible structure vibrations but, the simulation results showed that the input step responses have big overshoots. In (Chen and Yang, 1993) the two-degree of freedom controller technique is used to design a robust controller with optimal model matching. Here, these two techniques are combined to design a two-degree of freedom controller.

Now, consider the overall control system represented by the configuration of Figure 7, with a two-degree of freedom controller (*R*, *S*, *T*). The design objective is to specify the two-degree of freedom controller to achieve the following two aims:

a) The controller robustly stabilizes the nominal model $G_0(s)$, with robust stability-degree assignment, against the neglected dynamics $\Delta G(s)$, by specifying $R(s)$ and $S(s)$.

b) The transfer function from v to y is as close to the desired model $M(s)$ as possible via an adequately chosen $T(s)$.

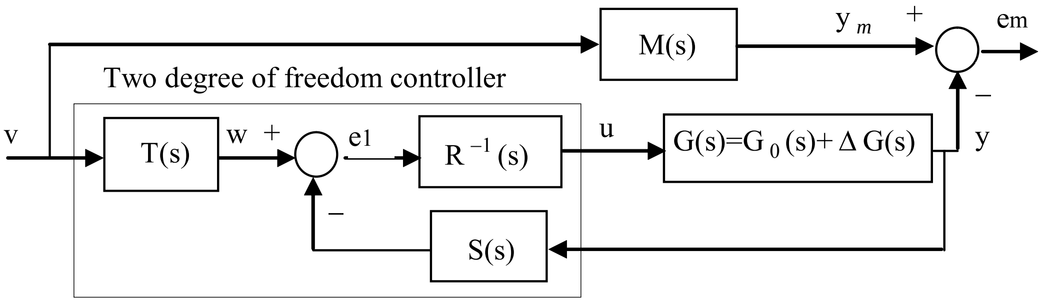


Fig. 7. The overall control system.

Robust Stability Degree Assignment

We consider o stable function, with minimum phase, $\alpha(s)$, so that:

$$\left|\Delta G(j\omega-\beta)\right| < \left|\alpha(j\omega)\right|, \quad (\forall)\omega \geq 0 \quad (19)$$

In order to robustly stabilize the nominal model $\bar{G}_0(s) = G_0(s-\beta)$ against the unmodeled dynamics ΔG that satisfies (19), we use a coprime factorization of $\bar{G}_0$ (Francis, 1997):

$$\bar{G}_0 = ND^{-1} = \tilde{D}^{-1}\tilde{N}$$
$$\begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & Y \\ N & X \end{bmatrix} = I \quad (20)$$

If the following compensators are considered

$$\bar{R}(s) = \tilde{X}(s) - Q(s)\tilde{N}(s)$$
$$\bar{S}(s) = -\tilde{Y}(s) + Q(s)\tilde{D}(s) \quad (21)$$

where $Q(s)$ is any stable proper rational function, the system from w to y, in the left moved plane, is internally stable. The sufficient condition for robust stabilization of $\bar{G}_0(s)$ against $\Delta G(s)$ that satisfies (19), can be obtained as follows:

$$\left|\alpha(j\omega)D(j\omega)[-\tilde{Y}(j\omega) + Q(j\omega)\tilde{D}(j\omega)\right| \leq 1, \; (\forall)\omega \geq 0 \quad (22)$$

Now, using the inverse transformation ($s \to s+\beta$), $S(s) = \bar{S}(s+\beta)$ and $R(s) = \bar{R}(s+\beta)$ stabilize the nominal model $G_0(s)$, against $\Delta G(s)$ that satisfies (19), with robust stability degree β.

Optimal Model Matching

Based on rise time, overshoot, settling time, and so on, can be selected a desired model $M(s)$. Consider the performance of nominal system, i.e., for $\Delta G(s) = 0$. For optimal model matching the following integration error

$$J(T) = \int_0^\infty e^2(t)dt \tag{23}$$

must be minimized.

Here, the following two cases are analyzed (Francis, 1997):

1. The nominal transfer function $G_0(s)$ is minimum phase. Then,

$$T_{opt}(s) = N^{-1}(s)M(s) \tag{24}$$

achieves the perfect model matching, i.e., $J(T) = 0$.

2. The nominal transfer function is non-minimum phase. Then, $N(s)$ can be represented as

$$N(s) = N_-(s)N_+(s) \tag{25}$$

where $N_-(s)$ is a Blaschke product and $N_+(s)$ is free of poles and zeros in $\mathrm{Re}[s] \ge 0$. If the desired model $M(s)$ is selected as the same zeros as $N(s)$ in $\mathrm{Re}[s] \ge 0$, $M(s)$ can be represented as

$$M(s) = N_-(s)M_1(s) \tag{26}$$

where $M_1(s)$ is stable. Then, the perfect model matching is achieved by

$$T_{opt}(s) = N_+^{-1}(s)M_1(s) \tag{27}$$

Experimental results

In Fig. 8 is presented the Simulink model for the real-time experiment. Using RTW (Real-Time Workshop) provided with MATLAB/Simulink, the WinCon software generate from this Simulink model the real-time code used for the flexible link control. For real-time control purposes, the above model generates the reference for the flexible link position using a *Signal Generator*. Thus the commands are stepped. The PC running WinCon Server must have a compatible version of The MathWorks' MATLAB installed, in addition to Simulink, and the Real-Time Workshop toolbox. WinCon™ is a real-time Windows 98/NT/2000/XP application. It allows you to run code generated from a Simulink diagram in real-time on the same PC (also known as local PC) or on a remote PC. Data from the real-time running code may be plotted on-line in WinCon Scopes and model parameters may be changed on the fly through WinCon Control Panels as well as Simulink. The automatically generated real-time code constitutes a stand-alone controller (i.e. independent from Simulink) and can be saved in WinCon Projects together with its corresponding user-configured scopes and control panels.

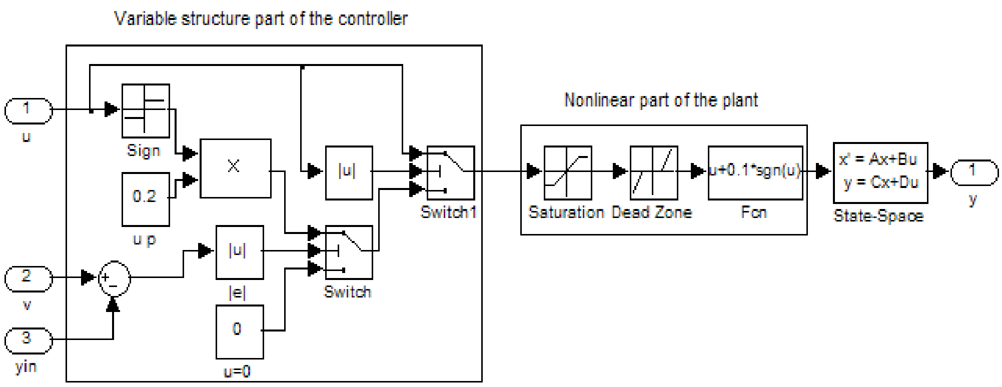


Fig. 8. The variable structure part of the controller and the plant nonlinearities.

Using the numerical values for the parameters identified in section 3, the two-degree of freedom controller (R, S, T) presented in section 4 is designed. The neglected nonlinearities (saturation, dead zone, Coulomb & viscous friction) in the controller design step lead to important differences between the simulation and experimental results, including vibrations of the real beam and/or stationary errors. In order to eliminate these tracking errors and reduce the vibrations of the beam we propose an improved control law by using some blocks with variable structure (Fig. 8).

The performance of the proposed controller is illustrated by some experimental results obtained by using a flexible beam Quanser experiment (Figure 9 and 10). We note that in Figure 10 it is represented the control law u(t) generated by (R, S, T) controller, at the input of the "variable structure part". The tuning parameters that influence the performances of this controller are: the thresholds of the two switches (one for the stationary error and the other to compensate the dead zone of the plant) and the value *up*. Also, we used some corresponding (low pass, anti-aliasing) filters to the inputs from the acquisition board and the step input is replaced by a smooth one to minimize the spillover effect.

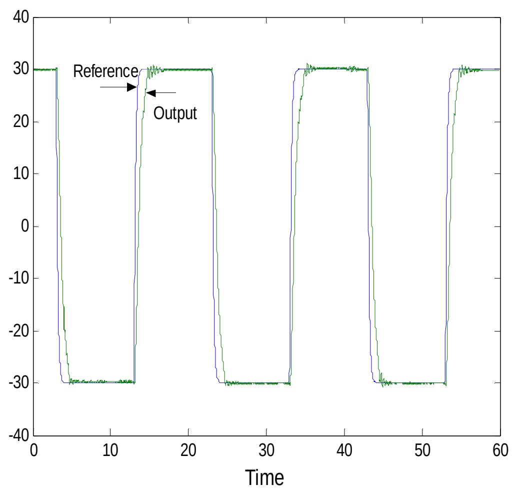


Fig. 9. Experimental output.

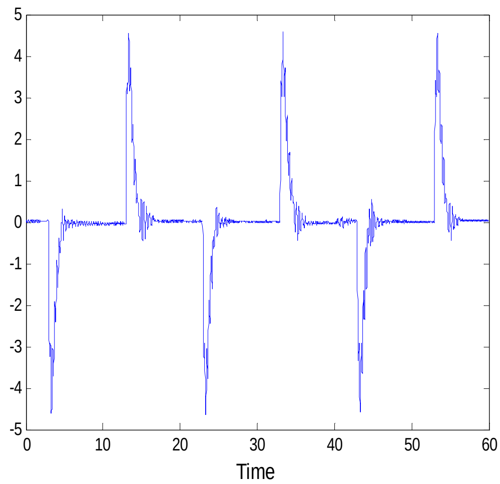


Fig. 10. Control input u(t).

Concluding remarks

The paper presents some aspects regarding modelling, identification and control of a rotating Quanser flexible beam. Using the formulas of the kinetic and potential energies, from the generalized dynamic equations we obtained a linear model expressed by ordinary differential equations. The paper also presents a fast subspace identification method, which use QR-decomposition, the singular value decomposition, and angles between subspaces. Subspace identification algorithms are based on concepts from system theory, linear algebra and statistics. In this approach, certain subspace matrices of the process are, at the first step, calculated through data projection. These matrices include extended observability matrix, lower triangular block-Toeplitz matrix and state sequences. These subspace matrices are directly calculated from the input/output data Hankel matrices without any iteration compared to the iterative schemes used in the prediction error methods. At the second step, the system state space matrices are recovered from the extended observability matrix and lower triangular block-Toeplitz matrix. One obtains a state space linear model.As a result of this identification procedure, estimated state matrices of the flexible beam model are obtained, with a good precision with respect to the real model.

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