Using Sliding Modes in Control Theory

***Renata Wagnerová*[1]**

Použití klouzavých módů v teorii řízení

The paper deals with sliding modes control design. The described control algorithms were applied to position control of the levitating systems in magnetic field. The designed control algorithms were verified by using computer simulations. The results achieved confirm suitable technical means and synthesis by using sliding modes for nonlinear control tasks.

Key words: *nonlinear control, sliding mode, levitation system.*

Introduction

At present time there is no general method for nonlinear control systems synthesis. In addition each method is applicable for certain type of nonlinear control subsystems. The quality of close control systems without knowledge of mathematical model or measurable disturbances can be ensured by using sliding mode control. Sliding mode control means discontinuous control, where according to value switching function the control has marginal value (UTKIN, V. I., 1992). The control is written by equation

$$\boldsymbol{u}^{sl} = \left[u_1^{sl}, u_2^{sl}, \ldots, u_m^{sl}\right]^T, \tag{1}$$

$$u_j^{sl} = \begin{cases} u_j^+ & \text{for} \quad m_j > 0, \\ u_j^- & \text{for} \quad m_j < 0, \end{cases} \tag{2}$$

where: u_j^+, u_j^- - marginal value of control, m_j – element of switching function.

The form of switching function m_j can come out from state variables aggregation method. Then it is describes (ZÍTEK, VÍTEČEK, 1999.)

$$\boldsymbol{u}^{sl} = \boldsymbol{U}^m \mathbf{sgn}(\boldsymbol{m}), \tag{3}$$

$$\boldsymbol{m} = \boldsymbol{D}(\boldsymbol{e} - \boldsymbol{e}_0) + \boldsymbol{T}^{-1}\boldsymbol{D}\int_0^t \boldsymbol{e}\, d\tau, \tag{4}$$

$$\boldsymbol{U}^m = \operatorname{diag}\left[u_1^m, u_2^m, \ldots, u_m^m\right], \tag{5}$$

$$\mathbf{sgn}(\boldsymbol{m}) = \left[\operatorname{sgn}(m_1), \operatorname{sgn}(m_2), \ldots, \operatorname{sgn}(m_m)\right]^T, \tag{6}$$

where: $\boldsymbol{e}$ – error vector dimension n, $\boldsymbol{D}$ – aggregation matrix dimension (m,n) with constant elements, $\boldsymbol{T}$ – time constant matrix dimension (m,m), $\boldsymbol{U}^{\boldsymbol{m}}$ - diagonal matrix, whose elements u_j^m are marginal values of control variables, sgn – signum function.

Sliding mode control is discontinuous, robust and simple, but its disadvantage is control high activities; it means quick switching between marginal values. That is why in practical application is used relay with hysteresis

$$\boldsymbol{u}^{hys} = \boldsymbol{U}^m \mathbf{hys}(\boldsymbol{m}), \tag{7}$$

$$\mathbf{hys}(\boldsymbol{m}) = \left[\operatorname{hys}(m_1), \operatorname{hys}(m_2), \ldots, \operatorname{hys}(m_m)\right]^T, \tag{8}$$

$$\operatorname{hys}(m_j) = \begin{cases} 1 & \text{for} \quad m_j \geq \Delta \\ -1 & \text{for} \quad m_j < \Delta \end{cases} \tag{9}$$

where: Δ – the amount of hysteresis, hys – hysteresis function.

Also this control algorithm causes chattering, which can be removed by using saturation function (continuous approximation of sign function) instead of signum function.

$$\boldsymbol{u}^{sa} = \boldsymbol{U}^m \mathbf{sat}(\Theta^m \boldsymbol{m}), \tag{10}$$

[1] *Renata Wagnerová,* Department of Control Systems and Instrumentation, VSB - Technical University of Ostrava, Ostrava, Czech Republic, renata.wagnerova@vsb.cz
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$$\boldsymbol{\Theta}^m = \mathrm{diag}\left[\theta_1^m, \theta_2^m, \ldots, \theta_m^m\right], \tag{11}$$

$$\mathbf{sat}\left(\boldsymbol{\Theta}^m \boldsymbol{m}\right) = \left[\mathrm{sat}\left(\theta_1^m m_1\right), \ldots, \mathrm{sat}\left(\theta_m^m m_m\right)\right]^T, \tag{12}$$

where: $\boldsymbol{\Theta}^m$ - positive diagonal matrix, sat – saturation function.

When the saturation function is used in (7), relation between the value of switching function m_j and the value of control is linear in interval $-1 \le \theta_j^m m_j \le 1$, this means, we obtained the continuous substitution of (3). But we can also use the smooth continuous substitution of (3), when for example hyperbolic tangent function is used. Then the control is written by equation:

$$\boldsymbol{u}^{tgh} = \boldsymbol{U}^m \mathrm{tgh}\left(\Theta^m \boldsymbol{m}\right), \tag{13}$$

$$\mathrm{tgh}\left(\Theta^m \boldsymbol{m}\right) = \left[\mathrm{tgh}\left(\theta_1^m m_1\right), \mathrm{tgh}\left(\theta_2^m m_2\right), \ldots, \mathrm{tgh}\left(\theta_m^m m_m\right)\right]^T, \tag{14}$$

where tgh is function of hyperbolic tangent.

By using saturation function continuous control with restriction is obtained. The sliding mode control with sign function u_j^{sl} has nonlinear two-state dependence. The sliding mode control with saturation function u_j^{sa} is linear between marginal values with restrictions. Inverse value of θ_j^m specifies slope of curve; the higher value θ_j^m means close course to sign function; the lower value θ_j^m means close course to control with high gain.

Application on levitating system

The sliding modes control was applied to the levitation in magnetic field. The movement of levitating subject has one degree of freedom. The behavior of this kind of system can be written by equations in state space [WAGNEROVÁ, 2004]

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= f_3(\boldsymbol{x}) + g_3(\boldsymbol{x})u,\end{aligned} \tag{15}$$

where: $f_3(x)$, $g_3(x)$ - common nonlinear functions of state variables with a representation,

$$f_3(\mathbf{x}) = \left[\frac{2R}{\dfrac{Q}{X_\infty + x_1} + L_\infty} + \frac{2x_2}{\left(X_\infty + x_1\right)} - \frac{2Qx_2}{\left(X_\infty + x_1\right)^2\left(\dfrac{Q}{X_\infty + x_1} + L_\infty\right)}\right]\left(g - x_3\right),$$

$$g_3(\mathbf{x}) = -\sqrt{\frac{2Q}{m}}\,\frac{\sqrt{g - x_3}}{\left(\dfrac{Q}{X_\infty + x_1} + L_\infty\right)\left(X_\infty + x_1\right)},$$

where: m - a weight of a steel bar [kg], x_1 - distance between magnetic circuit and bar [m], u – control variable, electrical voltage [V], R - electrical resistance [Ω], Q, L_∞, X_∞ - constant values of magnetic stand [H.m, H, m].

According the equation (13) the levitating system is third order that is why the aggregation matrix D and the time constant matrix T have presentation

$$\boldsymbol{D} = \boldsymbol{d} = \begin{bmatrix} \dfrac{1}{T_0^2} & \dfrac{2\xi_0}{T_0} & 1 \end{bmatrix}, \ \mathbf{T} = \mathrm{T}_3,\ \boldsymbol{\Theta}^{\mathbf{m}} = \theta^{\mathrm{m}} \tag{16}$$

where : T_i – time constants choosing according to the required closed-loop system behavior.

Sliding model controls are described:

$$\begin{aligned} u^{sl} &= u^m \,\mathrm{sgn}(m), \\ m &= \left[\frac{1}{T_0^2 T_3}\int_0^t e_1 d\tau + \left(\frac{1}{T_0^2} + \frac{2\xi_0}{T_0 T_3}\right)\left(e_1 - e_{10}\right) + \left(\frac{2\xi_0}{T_0} + \frac{1}{T_3}\right)\left(e_2 - e_{20}\right) + e_3\right],\end{aligned} \tag{17}$$

$$u^{hys} = \boldsymbol{U}^m \mathrm{hys}\left[\Theta^m\left(\frac{2}{T_1}e_1 + \dot{e}_1 + \frac{1}{T_1^2}\int_0^t e_1\, d\tau\right)\right], \tag{18}$$

$$u^{sa} = \boldsymbol{U}^m \mathrm{sat}\left[\Theta^m\left(\frac{2}{T_1}e_1 + \dot{e}_1 + \frac{1}{T_1^2}\int_0^t e_1\, d\tau\right)\right], \qquad (19)$$

$$u^{tgh} = \boldsymbol{U}^m \mathrm{tgh}\left[\Theta^m\left(\frac{2}{T_1}e_1 + \dot{e}_1 + \frac{1}{T_1^2}\int_0^t e_1\, d\tau\right)\right], \qquad (20)$$

where: e_1 – difference between required and real position, $e_2 = \dot{e}_1$, $e_3 = \dot{e}_2$.

The correctness of designed control algorithms were verified by computer simulation with help of program environment Matlab/Simulink. The task was tracking of reference signal (sinusoid course – dash signal). The achieved results are shown in Figure 1, 2, 3.

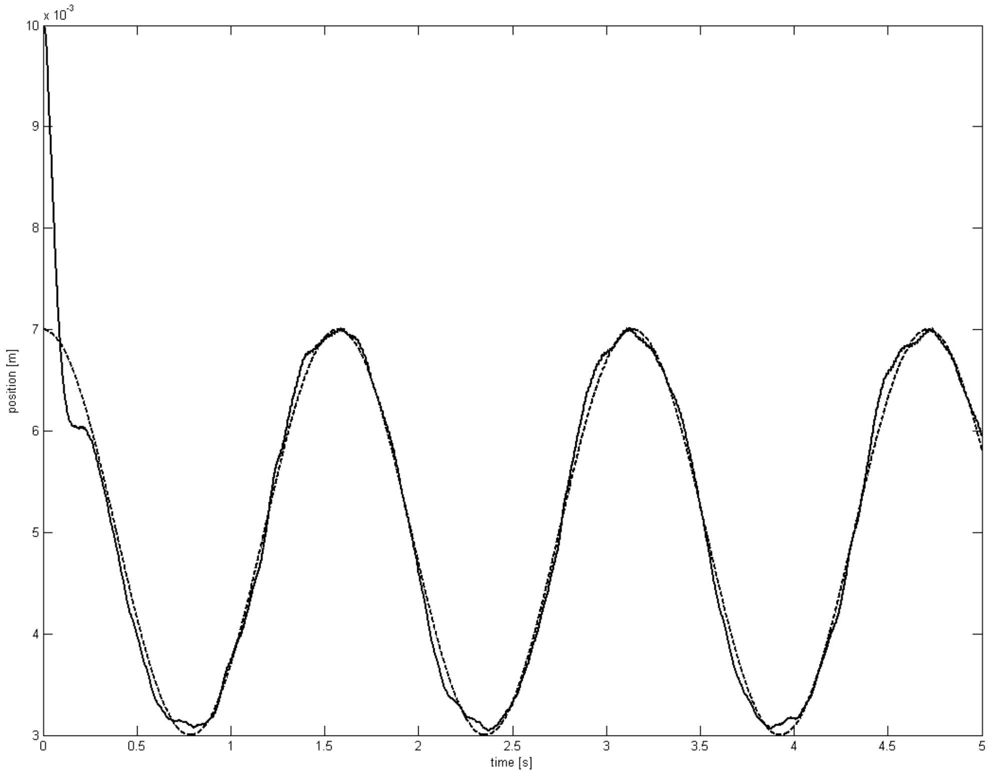

Fig. 1. The course of real and required position for control with hysteresis.

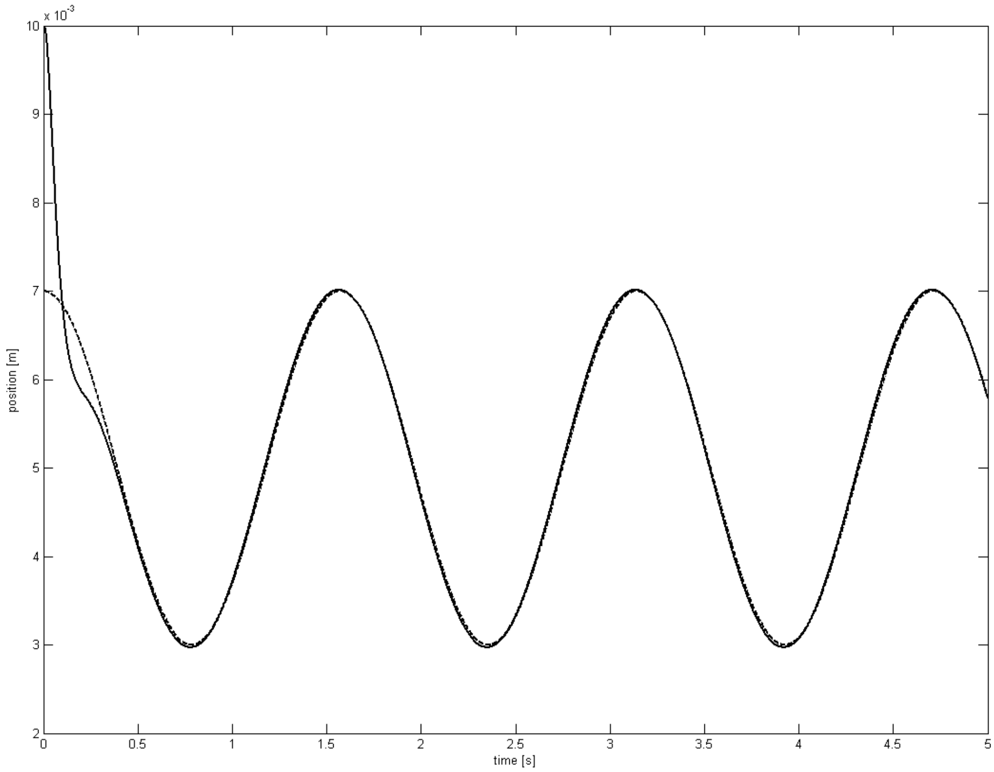

Fig. 2. The course of real and required position for control with saturation.

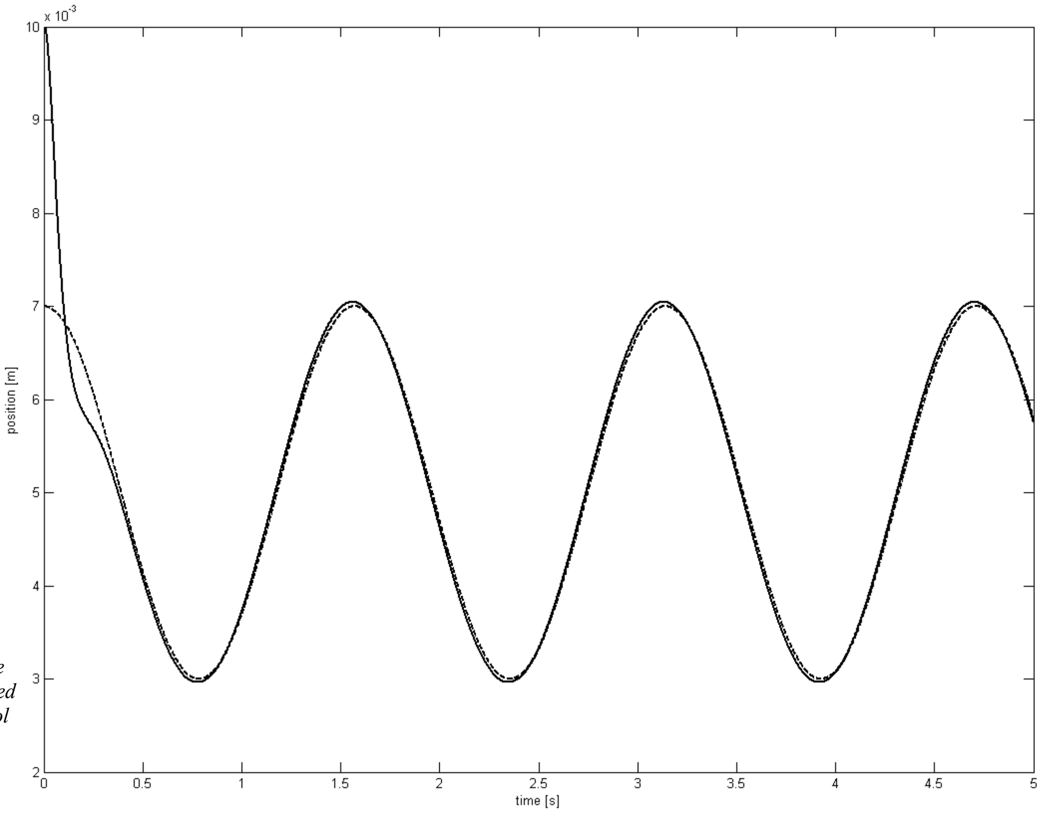

Fig. 3. The course of real and required position for control with hyperbolic tangent function.

Conclusion.

The contribution presents nonlinear control systems synthesis. There are described properties of three types of sliding mode control: discontinuous, continuous and smooth continuous. The controls were used to position control of levitating object in magnetic field. All designed algorithms ensure reaching the required position. The best quality is obtained by using sliding mode control with saturation function.

References

Perruquetti, W., Barbot, J. P.: Sliding Mode Control in Engineering. Control Engineering Series, *11. New York: Marcel Dekker, Inc., 432 pp, 2002. ISBN 0-8247-0671-4*.

Utkin, V.I.: Sliding Modes in Optimization and Control Problems. *New York: Springer Verlag, 1992, 286 pp. ISBN 3-540-53516-0*.

Wagnerová, R.: Robust Control Design for Technological Processes. In 5th International Carpathian Control Conference. Zakopane, Poland: AGH-UST Krakow*, 25–28. 5. 2004, pp. 289-294. ISBN 83-89772-00-0*.

Zítek, P., Víteček: Control design of subsystems with time delay and nonlinearities*. Praha : Vydavatelství ČVUT, 1999. 165 pp. ISBN 80-01-01939-X. (in Czech)*