Description of behavior of national economies in state space

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Popis spravania narodnych ekonomik v stavovom priestore

This paper deals with analysis of behavior of national economies by using the total least squares method. National economies are described in state space of the following variables: gross domestic product, inflation, and unemployment. Their behavior is investigated by fitting their phase trajectories to plane. A possible method of description of the dynamical properties of national economies by differential equations is proposed. In conclusion the use of the investigated mathematical model for prediction and evaluation of the performance of national economies is discussed. A MATLAB routine for fitting 3D data to lines and planes in 3D is provided.

Key words*: total least squares, national economies, state space*

Introduction

In this article we consider the different versions of the least squares method – so called orthogonal regression, which has been studied by several authors (see References). The main reason is the fact that orthogonal regression is a suitable tool for fitting lines and surfaces in multidimensional space, while even the use of classical regression in 2D is not so natural. We apply the orthogonal regression method to describing the national economies of the "well-developed" countries group in the state space of three variables: gross domestic product (GDP), inflation, and unemployment. The development of the national economies under study is described by their trajectories in the chosen state space. We discovered that for each particular country the trajectory of its national economy lies approximately in one plane, so the normal vector of this plane and one of its points (e.g. the one coinciding with the data centroid) can be associated with a particular economy.

Each particular economy can be described by three differential equations contain of three variables where the solution of these equations is the phase trajectory in state space which is created from the same variables (GDP, inflation, and unemployment).

State space description of national economies

The expression "state of economy" can be given an exact meaning by adopting the tools that are available in the theory of dynamical systems and in automatic control. Namely, we will use the technique called state space description and phase trajectories.

Suppose we have a dynamical process (that is, the process developing in time), which is characterized by three quantities $x(t)$, $y(t)$, $z(t)$ (of course, number or dimensions can be different, not necessarily 3). Choosing x, y, and z as the three coordinates, we can assign a point $(x(t), y(t), z(t))$ to each value of t – that is, we assign a point in a 3D space to a state of the considered process at time t. The variables x, y, z are called the state variables. The line formed by the points $(x(t), y(t), z(t))$ when t takes on the values from a given interval (usually $[0,T]$ for some finite T, or $[0,\infty)$) is called the phase trajectory of the process.

As state variables for this study, we selected those which are standard: gross domestic product (GDP), inflation, and unemployment. Obviously, these economic indicators cannot be readily divided into clear groups of independent and dependent variables. In the subsequent sections, we demonstrate the advantages of the state space description of national economies and possible ways for further developments and scientific investigations.

The case of the "well – developed" countries

The traditional approach to visualization of the change of GDP, inflation, and unemployment uses 2D plots – see Fig. 1 and Fig. 3. Although such visualization provides partial information about each particular

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aspect of a particular economy, and even allows comparisons of a given particular indicator for several economies, it does not provide any information about the economy as a whole.

This is why we plot the data for each country (USA and UK) – in the state space with three coordinates: GDP, inflation, and unemployment. We use the data for the period since 1980 till 2006 from the EconStats web page report.

One can see that, in contrast with the traditional 2D plots, the phase trajectories shown in Fig. 2 and Fig.4 (left) nicely show how the national economies developed in time.

The most interesting observation regarding these phase trajectories is that for each particular country its phase trajectory lies approximately in one plane – see Fig.2 and Fig.4 (right). This observation indicates that we can associated such planes in state space with particular economies, and that the global properties of each particular national economy as a whole are described by the associated plane (or, in other word, by its normal vector and the data centroid, which is a point belonging to the plane). The values of normal vectors and centroids of the planes describing national economies of the countries (USA and UK) are listed here (normal vector; centroid; approximation error):

Tab. 1. Normal vectors, centroids, and errors for the planes corresponding to the countries (USA, UK).

Country	Normal vector	Centroid [x, y, z]	Error
USA	[-0.206, 0.207, -0.956]	[3.04, 3.81, 6.17]	26.07
UK	[0.887, 0.348, -0.302]	[2.33, 4.16,7.61]	19.02

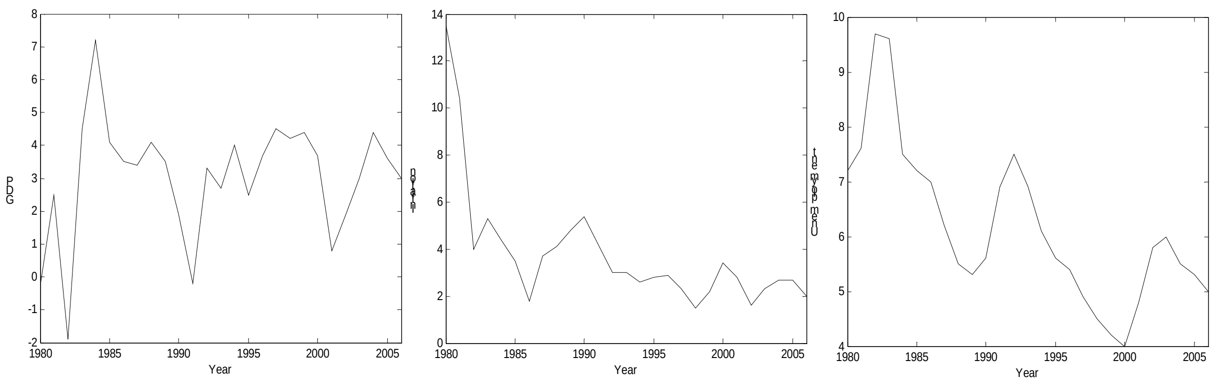

Fig. 1. 2D visualization of economic indicators (GDP, Inflation, Unemployment) of USA.

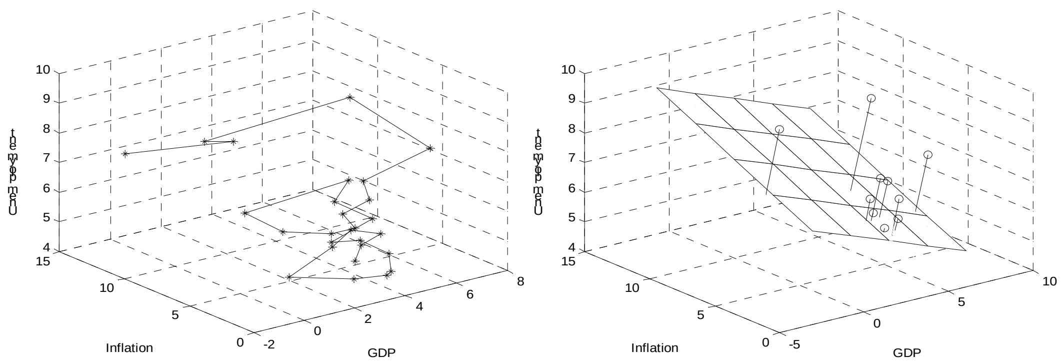

Fig. 2. Phase trajectory and plane of the national economy of the USA in 3D state space.

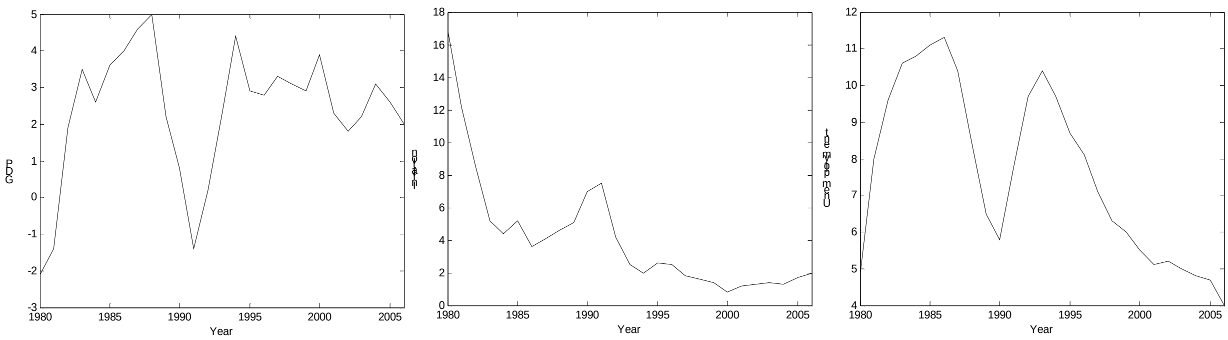

Fig. 3. 2D visualization of economic indicators (GDP, Inflation, Unemployment) of UK.

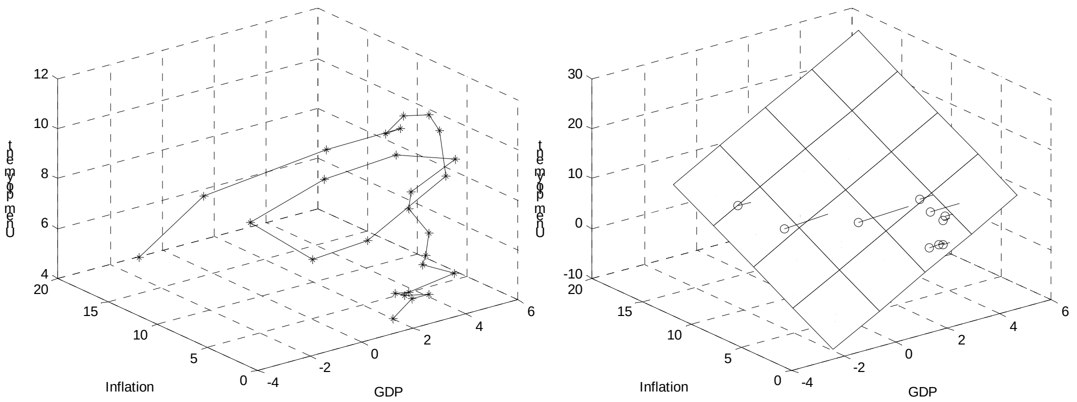

Fig. 4. Phase trajectory and plane of the national economy of the UK in 3D state space.

As is well known from the analytic geometry, each plane is uniquely described by these data. The values of the total error of approximation are given as well. Similar approach was used for the V4 countries in the paper (Petráš, Podlubný, 2007) and for the Scandinavian countries in the paper (Podlubný et al., 2007).

Conclusion

In this article we have presented a new approach to description of national economies. This approach is based on the state space viewpoint, which is used mostly in the theory of dynamical systems and in control theory. Gross domestic product, inflation, and unemployment rates were taken as state variables. We demonstrated that for the considered period of time the phase trajectory of each of the well-developed country lies approximately in one plane, so that the economic development of each country can be associated with a corresponding plane in the state space.

The suggested approach opens a way to a new set of economic indicators. Among possible indicators of economies we can mention, for example, normal vectors of national economies, various plane slopes, 2D angles between the planes corresponding to different economies, etc.

The tool used for computations is orthogonal regression (alias orthogonal distance regression, alias total least squares method).

Appendix

We created a MATLAB routine called **fit_3D_data** for performing computations used in our article. Since our routine can be used in many other investigations utilizing the orthogonal regression (total least squares) method, we published it at MATLAB File Exchange.

```
function [Err,N,P] = fit_3D_data ( XData, YData, ZData, geometry,
                                   visualization, sod )
%
%  [Err, N, P] = fit_3D_data(XData, YData, ZData,
%  geometry, visualization, sod)
%
%  Orthogonal Linear Regression in 3D-space
%  by using Principal Components Analysis
%
%  Input parameters:
% - XData: input data block -- x: axis
% - YData: input data block -- y: axis
% - ZData: input data block -- z: axis
% - geometry: type of approximation (’line’,’plane’)
% - visualization: figure (’on’,’off’) -- default is ’on’
% -sod: show orthogonal distances (’on’,’off’) --
% default is ’on’
```

```
%
%  Return parameters:
% - Err: error of approximation - sum of orthogonal
% distances
% - N: normal vector for plane, direction vector for
%   line
% - P: point on plane or line in 3D space
% Note: Written for Matlab 7.0 (R14) with Statistics %  Toolbox
% We sincerely thank Peter Perkins, the author of the % demo,
% and John D'Errico for their comments.
%
% Ivo Petras, Igor Podlubny, May 2006.
```

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