About the use of spatial interpolation methods to denoising Moroccan resistivity data phosphate "Disturbances" map

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O použití priestorových interpolačných metód na poruchovej mape odhlučnenia Marockých fosfátov údajmi merného odporu

Several methods are currently used to optimize edges and contours of geophysical data maps. A resistivity map was expected to allow the electrical resistivity signal to be imaged in 2D in Moroccan resistivity survey in the phosphate mining domain. Anomalous zones of phosphate deposit "disturbances" correspond to resistivity anomalies. The resistivity measurements were taken at 5151 discrete locations. Much of the geophysical spatial analysis requires a continuous data set and this study is designed to create that surface. This paper identifies the best spatial interpolation method to use for the creation of continuous data for Moroccan resistivity data of phosphate "disturbances" zones. The effectiveness of our approach for successfully reducing noise has been used much success in the analysis of stationary geophysical data as resistivity data. The interpolation filtering approach methods applied to modeling surface phosphate "disturbances" was found to be consistently useful.

Key words*: resistivity, Schlumberger, phosphate, interpolation, denoising, Morocco.*

Introduction

Resistivity data of Moroccan phosphates "disturbances" collected in the survey are contaminated with noise and artifacts coming from various sources. The presence of noise in data resistivity distorts the characteristics of the geophysical signal, resulting in poor quality of any subsequent processing. Consequently, the first step in any processing of such geophysical data is the "cleaning up" of the noise in a way that preserves the signal sharp variations using interpolation methods. Interpolation is a process widely used in earth science. It estimates the value of a parameter at a point from neighbouring. Geophysical data are used routinely in mineral exploration to delineate the geology of an area. Because geophysical attributes are sparsely sampled; interpolation methods are used to grid the individual sets of data. Interpolation is a method or mathematical function that estimates the values at locations where no measured values are available. Interpolation can be as simple as a number line; however, most geographic information science research involves spatial data. Spatial interpolation assumes the attribute data are continuous over space. This allows for the estimation of the attribute at any location within the data boundary. Another assumption is the attribute is spatially dependent, indicating the values closer together are more likely to be similar than the values farther apart. These assumptions allow for the spatial interpolation methods to be formulated. Spatial interpolation is widely used for creating continuous data when data are collected at discrete locations (i.e., at points). The goal of spatial interpolation is to create a surface that is intended to best represent empirical reality thus the method selected must be assessed for accuracy for these larger studies (Montefusco et al., 1989).

It has been shown that there is no single preferred method for data interpolation. Aspects of the algorithm selection criteria need to be based on the actual data, the level of accuracy required, and the time and/or computer resources available. In the absence of criteria for selecting among the available techniques, this paper compares six spatial interpolations - inverse distance weighting, kriging, radial basis functions, minimum curvature, triangulation and Shepard's method – with the goal of determining which method creates the best representation of reality for measured resistivity data of Moroccan phosphates "disturbances". The benefits and limitations of these commonly used interpolation methods are discussed in this paper. Selecting an appropriate spatial interpolation method is the key to surface analysis since different methods of interpolation can result in different surfaces and ultimately different results.

The geophysical context

Resistivity is an excellent parameter and marker for distinguishing between different types and degree of alteration of rocks. Resistivity surveys have long been successfully used by geophysicists and engineering geologists and the procedures are well established. The study area is the Oulad Abdoun phosphate basin which contains the Sidi Chennane deposit. The Sidi Chennane deposit is sedimentary and contains several

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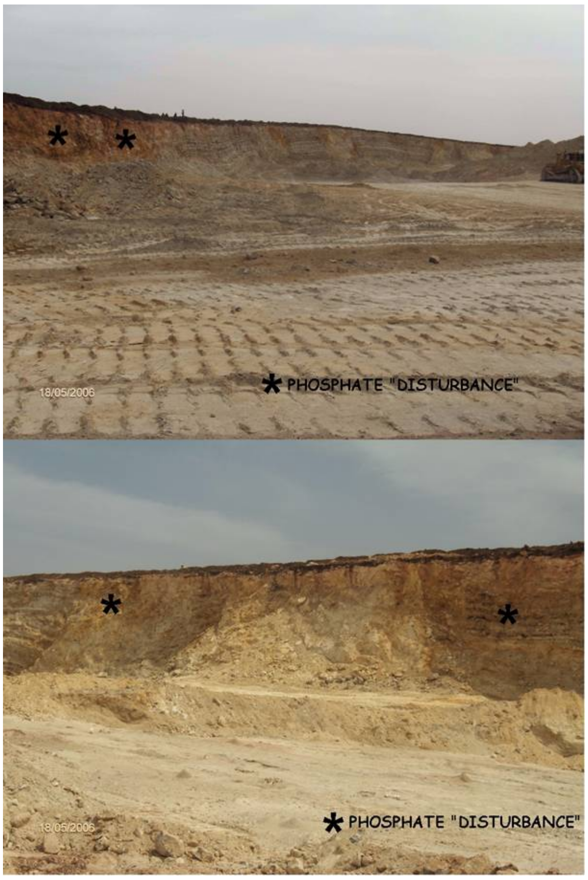

distinct phosphate-bearing layers. These layers are found in contact with alternating layers of calcareous and argillaceous hardpan. However, a new deposit contains many inclusions or lenses of extremely tough hardpan locally known as "*derangements*" or "disturbances" (Fig. 1), found throughout the phosphate-bearing sequence. The hardpan pockets are normally detected only at the time of drilling (Kchikach et al., 2002).

Direct exploration methods such as well logging or surface geology are not particularly effective for estimating phosphate reserves. They interfere with field operations and introduce a severe bias in the estimates of phosphate reserves (Fig. 2).

The study area was selected for its representatives and the apparent resistivity profiles were designed to contain both disturbed and enriched areas (Fig. 3). The sections were also calibrated by using vertical electrical soundings (Fig. 4) (Bakkali et al., 2006a).

Fig. 1. Example of "disturbance" affecting the phosphate stratus.

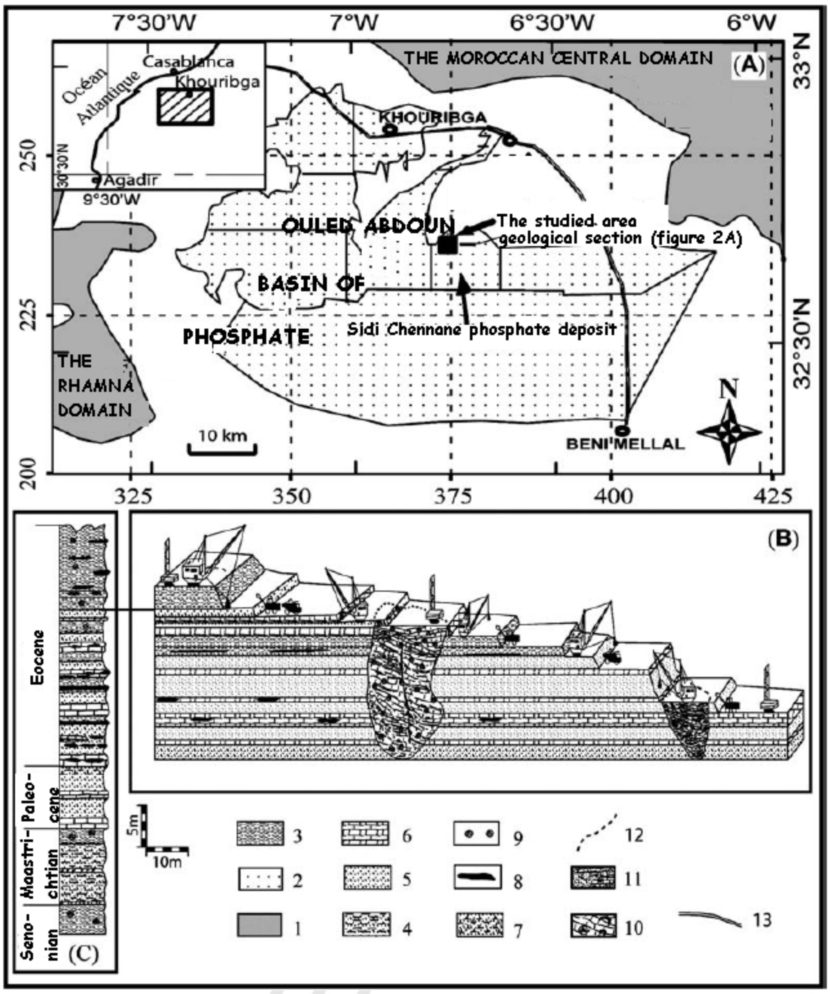

Fig. 2. (A) Location of the studied area in the sedimentary basin of Ouled Abdoun.
(B) Section showing the disruption of the exploitation caused by disturbances.
(C) Stratigraphical log of the phosphatic series of Sidi Chennane: (1) Hercynian massif; (2) phosphatic areas; (3) marls; (4) phosphatic; marls; (5) phosphatic layer; (6) limestones; (7) phosphatic limestone; (8) discontinuous silex bed; (9) silex nodule; (10) "disturbance" formed exclusively of silicified limestone; (11) "disturbance" constituted of a blend of limestone blocks, marls and clays; (12) "disturbance" limit; (13) roads.

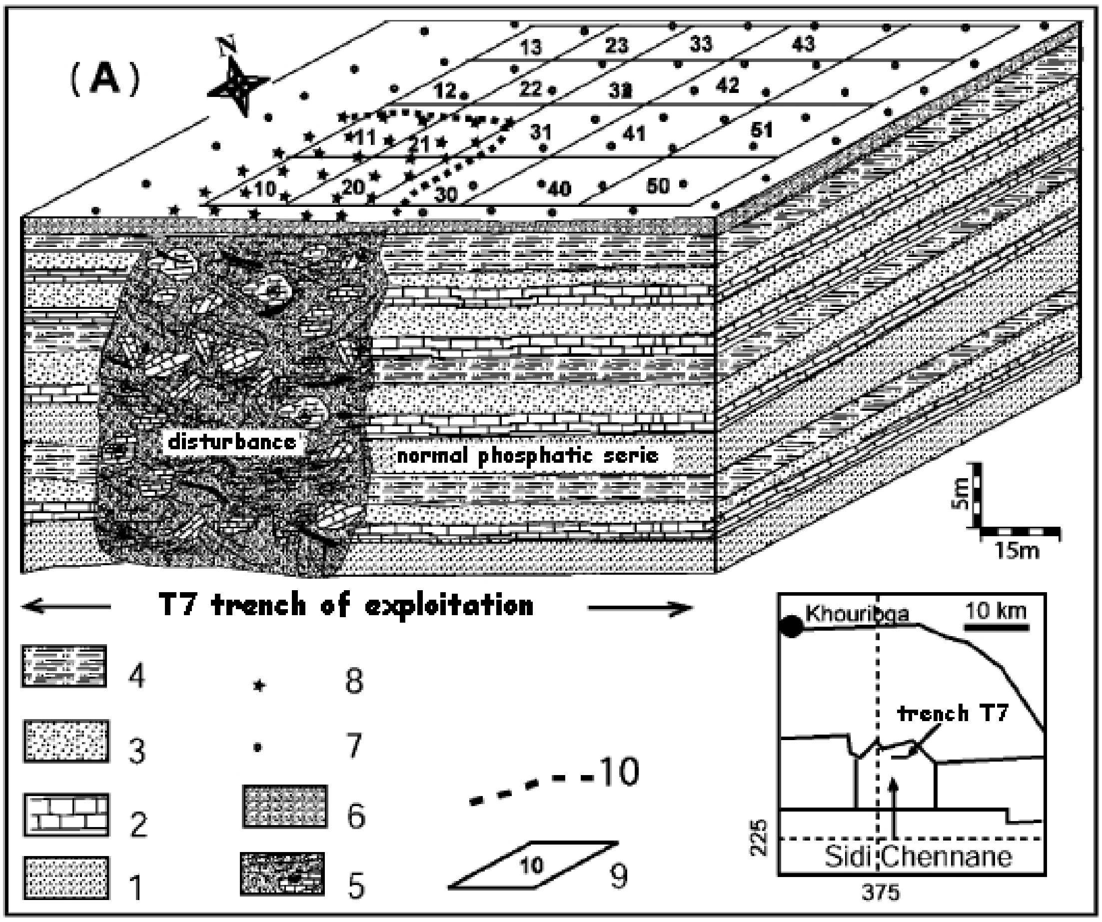

Fig. 3. (A) Geological section of the T7 exploitation trench showing a "disturbance" and position plan of the soundings tests. Apparent resistivity profiles positions while passing from the deranged zone to a normal phosphatic series : (1) phosphatic marls; (2) limestones; (3) phosphatic layer; (4) marls; (5) "disturbance" ; (6) Quaternary cover; (7) borehole crossing a normal phosphatic series; (8) borehole crossing a "disturbance"; (9) measures loop number 10 ; (10) "disturbance" limit.

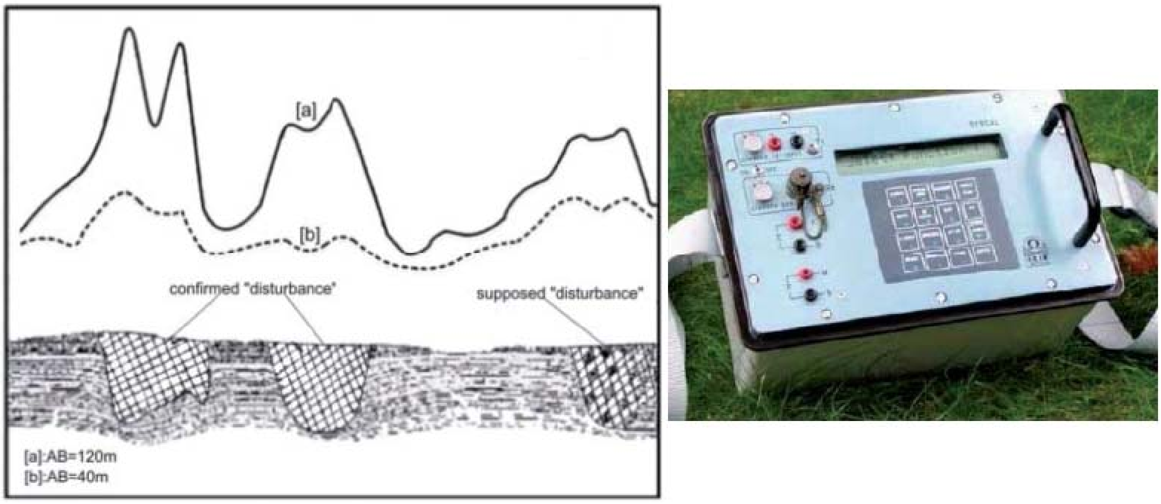

Fig. 4. A synthetic apparent resistivity traverse over three « disturbances » (left side) and the Syscal resistivity meter used in the study (right side).

High values rank of apparent resistivity were encountered due to the presence of near-vertical faulting between areas of contrasting resistivity, and fault zones which may contain more or less highly conducting fault gouge. The gouge may contain gravel pockets or alluvial material in a clay matrix. Such anomalous sections are also classified as *disturbances*. Apparent resistivity values in these profiles locally exceeded 200 $\tilde{\Omega}$.m. The apparent resistivity map (Fig. 5) which one obtains from the survey is actually a map of discrete potentials on the free surface, and any major singularity in the apparent resistivities due to the presence of a perturbation will be due to the crossing from a "normal" into a "perturbed" area or vice versa. In other words, the apparent resistivity map may be considered a map of scalar potential differences assumed to be harmonic everywhere except over the perturbed areas. Interpretation of resistivity anomalies is the process of extracting information on the position and composition of a target mineral body in the ground (Bakkali, 2006). In the present case the targets were essentially the inclusions called *perturbations.* The amplitude of an anomaly may be assumed to be proportional to the volume of a target body and to the resistivity contrast with the mother lode. If the body has the same resistivity as the mother lode no anomaly will be detected. Thus assumed in fact and in first approach that the resistivity anomalies would be representative of the local density contrast between the disturbances and the mother lode. Level disturbance of the anomalous zones is proportionnal to resistivity intensity (Fig. 6).

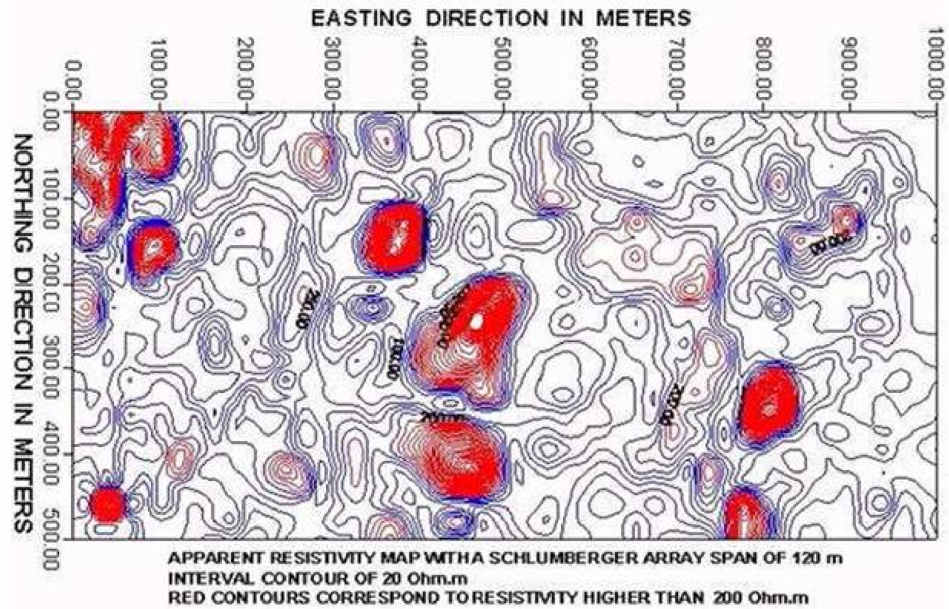

Fig. 5. A map of resistivity anomalies for AB=120 m.

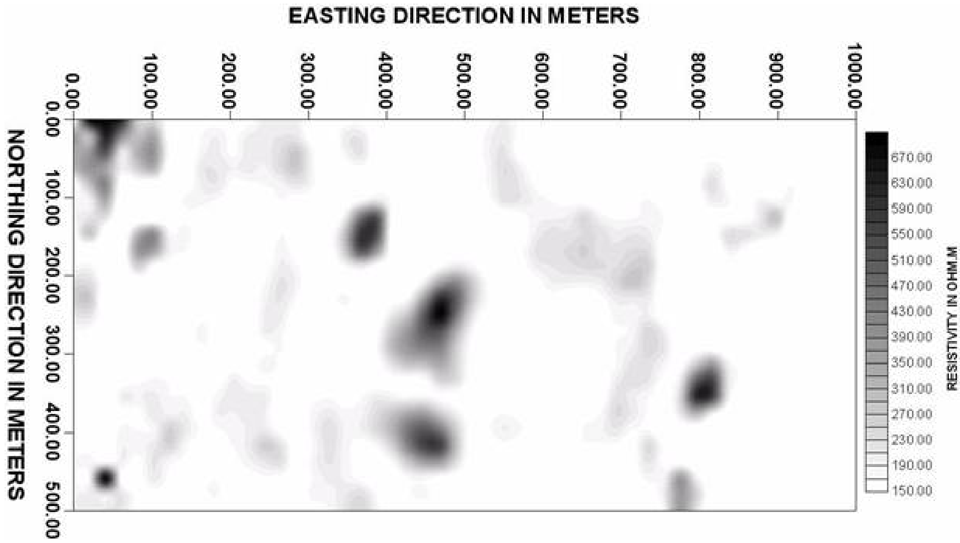

Fig. 6. A map of the disturbed noisy phosphate zones corresponding to Figure 5.

The spatial interpolation methods

Inverse distance

Inverse distance is a weighted average interpolator, which can be either exact or smoothing (Watson et al., 1985). With inverse distance, data are weighted during interpolation, so that the influence of one point, relative to another, declines with distance from the grid node. Weighting is assigned to data through the use of a weighting power, which controls how the weighting factors drop off as distance from the grid node increases. The greater the weighting power, the less effect the points, far removed from the grid node, have during interpolation. As the power increases, the grid node value approaches the value of the nearest point. For a smaller power, the weights are more evenly distributed among the neighboring data points (El Abbas et al., 1990). Normally, inverse distance behaves as an exact interpolator. When calculating a grid node, the weights assigned to the data points are fractions, the sum of all the weights being equal to 1.0. The smoothing parameter is a mechanism for buffering this behavior. One of the characteristics of inverse distance is the generation of "bull's-eyes" surrounding the observation position within the grid area. A smoothing parameter can be assigned during inverse distance to reduce the "bull's-eye" effect by smoothing the interpolated grid (Smith et al., 1990).

Kriging method

Kriging is a geostatistical gridding method that has proven useful and popular in many fields. This method produces visually appealing maps from irregularly spaced data (Cressie, 1991) (Swan et al., 1995). Kriging attempts to express trends suggested in your data, so that, for example, high points might be connected along a ridge rather than isolated by bull's-eye type contours. Kriging is a very flexible gridding method. The Kriging defaults can be accepted to produce an accurate grid of your data (Davis, 1986). Kriging uses a weighting, which assigns more influence to the nearest data points in the interpolation of values for unknown locations. Kriging, however, is not deterministic but extends the proximity weighting approach of inverse distance to include random components where exact point location is not known by the function. Kriging depends on spatial and statistical relationships to calculate the surface (Davis et al., 1975).

Minimum curvature method

This method applies a two-dimensional cubic spline function to fit a smooth surface to the set of input elevation values (Briggs, 1974). The computation requires a number of iterations to adjust the surface so that the final result has a minimum amount of curvature. Minimum Curvature is widely used in the earth sciences. The interpolated surface generated by Minimum Curvature is analogous to a thin, linearly elastic plate passing through each of the data values, with a minimum amount of bending (Lam, 1983). Minimum Curvature generates the smoothest possible surface while attempting to honor your data as closely as possible. Minimum Curvature is not an exact interpolator, however.

Shepard's method

Shepard's method is a well established method for interpolating scattered data (Shepard, 1968). It is also known as the moving average method. The method is based on forming convex combinations of the scattered data values and does not require the solution of any large linear systems (Gordon, 1978). The Shepard's Method uses an inverse distance weighted least squares method. As such, Shepard's Method is similar to the inverse distance interpolator, but the use of local least squares eliminates or reduces the "bull's-eye" appearance of the generated contours. Shepard's Method can be either an exact or a smoothing interpolator (Lancaster et al., 1990).

Radial basis function interpolation method

Radial basis function method is a well established method for interpolating scattered data (Franck et al., 1980). It is based on forming linear combinations of radial functions centred at each of the data sites (Franck, 1982). Radial Basis Function interpolation is a diverse group of data interpolation methods (Fasshauer et al., 1998). In terms of the ability to fit your data and produce a smooth surface, the Multiquadric method is considered by many to be the best (Chen et al., 1996) (Hardy, 1971). All of the Radial Basis Function methods are exact interpolators, so they attempt to honour the data (Goldberg et al., 1996) (Powell, 1992).

Triangulation with linear interpolation method

Triangulation method uses Renka's algorithm (Renka, 1984) to carry out a Delaunay triangulation (Okabe et al., 1992) of the observation points. The purpose is to identify a neighborhood of nearby observation points to be used in the interpolation. This algorithm creates triangles by drawing lines between data points. The original points are connected in such a way that no triangle edges are intersected by other triangles (Brown, 1994). The result is a patchwork of triangular faces over the extent of the grid. This method is an exact interpolator (Watson, 1992). Each triangle defines a plane over the grid nodes lying within the triangle, with the tilt and elevation of the triangle determined by the three original data points defining the triangle. All grid nodes within a given triangle are defined by the triangular surface. Because the original data are used to define the triangles, the data are honored very closely. Triangulation with Linear Interpolation works best when the resistivity data are evenly distributed over the grid area. Data sets containing sparse areas result in distinct triangular facets on the map (Lawson, 1972).

Conclusion

In this paper we compare six different interpolation methods (Fig. 7). The six difference interpolation methods consistently identified *Radial basis function interpolation method* as the best method for interpolating Moroccan phosphate deposit "disturbances" surface. This study has shown that *Radial basis function interpolation method* is most likely to produce the best estimation of a continuous surface of Moroccan phosphate deposit "disturbances". Nevertheless, the research presented here illustrates that regardless of the approach taken these interpolation methods adequately address the real distribution of the phosphate anomalous zones.

The results show a significant suppression of the noise and a very good smoothing and recovery of the resistivity anomalies signal. We have described an analytical procedure to analyze Moroccan phosphate deposit "disturbances" anomalies. The results proved satisfying. Data processing procedure as *Radial basis function interpolation method* response of Moroccan phosphate deposit "disturbances" resistivity data map was found to be consistently useful and the corresponding output map may be used as auxiliary tools for decision making under field conditions.

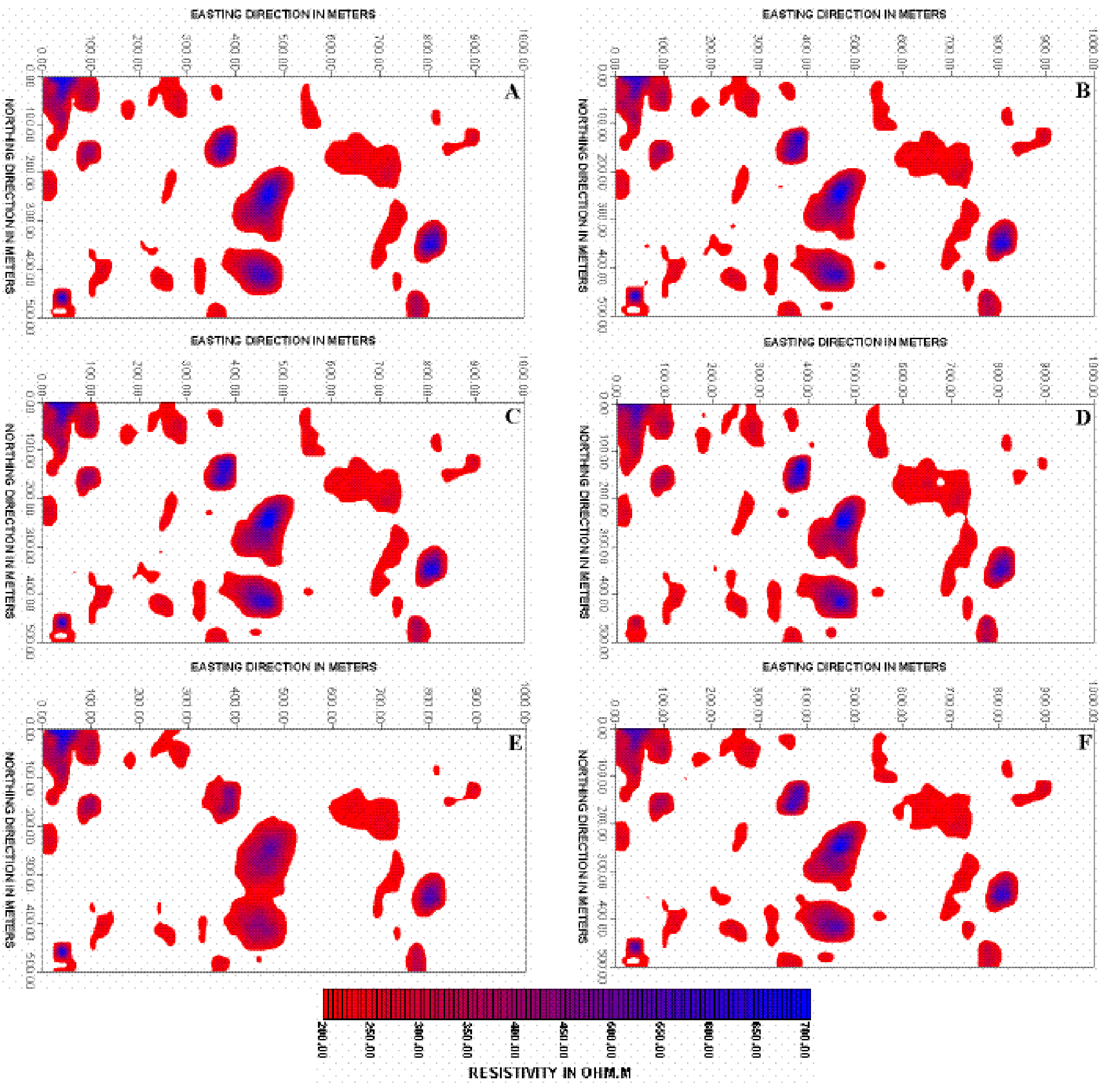

Fig. 7. The output map of the disturbed phosphate zones using different interpolation methods.(A: inverse, B: Triangulation, C:Kriging, D:Minimum Curvature, E:Shepard, F: Radial Basis Functions).

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