Establishment of Local Geodetic Nets

***Gabriel Weiss*[1], *Slavomír Labant*[1], *Erik Weiss*[2], *Ladislav Mixtaj*[2] *and* *Heidy Schwarczová*[3]**

Establishment of Local Geodetic Nets (LGN) is closely related to the compatibility of geodetic points and to their importance. It also involved with the processes of setting up 2D LGNs and methods of verifying the datum points´ compatibility in the net. The assessment of points´ compatibility is performed using: a) distances among the coordinate points´ positions, b) distances among the physical points´ positions, c) differences among the „coordinate" and „measured" distances as well as incompatible datum points identification.

Keywords: *establishment of LGN, compatibility of 2D geodetic points, coordinate discrepancies, compatibility indicators, distance differences, coordinate differences, testing true point movement.*

Introduction

In many countries, appart from their obligatory geodetic foundations with corresponding coordinate systems, in various localities and for various local needs there are also established independent nets tailored to extent – local geodetic nets (LGNs). These LGNs are established either on the basis of independent local coordinate systems or on obligatory coordinate systems of the given country. The networks offer wide areas of both current and prospective applications as they can be established as:

- 1D height LGNs, to determine the absolute, reps. relative heights H of points in a height system, both official or purposeful (local),
- 2D position LGNs, to determine the X, Y coordinates for points of the obligatory state or certain locally established 2D position system,
- 3D area LGNs, to determine the X, Y, Z coordinates of points of the ETRS-89 system, or transforming them, to determine the X, Y, H coordinates in suitable positional and height sytems, e.g. S-JTSK and Bpv.

In all cases measurements can be performed as terrestrial, satellite or combined ones in accordance with the situation and equipment availability for measurements. LGNs are used (with much wider future applications) to generate documents for a multiple of purposes, such as:

- periodic detection of 1D, 2D, 3D deformations on objects and area units,
- measurement of the actual status of areal units for projections, building, rebuilding and reconstructions,
- checks and monitorings of adherence to the projected parameters in the course of building objects,
- generation of the geodetic documentations of settlements for managing and controlling the processes of building as well as for documenting the current status of buildup,
- ex-post measurements of objects and areas in the construction area.
- pointing out and control activities in the course of building, construction, and activities related to the area of interest.

and for further various purposes requiring geodetic assistance.

Of the wide areas of coverage as above, the next part is devoted to the generation of the needed properties required for the 2D LGNs only.

Establishing 2D LGN

When establishing a 2D LGN, it is necessary to solve and choose various conditions of the characteristics, capabilities and properties of the LGN for purpose of its use, such as:

- defining the suitable area for the LGN,
- datum ensurance of the LGN using mostly such points of the surrounding field of points situated in the LGN area, or in its vicinity and which, by their properties, are of use as datum points, DB. Datum of 2D LGN is usually given by coordinates of the selected points DB_1, DB_2, ... (datum points) for a certain

[1] *prof. Ing. Gabriel Weiss, PhD., Ing. Slavomír Labant, PhD.,* Institute of Geodesy, Cartography and Geographic Information Systems, Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice, Letná 9, 042 00 Košice, gabriel.weiss@tuke.sk , slavomir.labant@tuke.sk

[2] *Ing. Erik Weiss, PhD., Ing. Ladislav Mixtaj, PhD.,* Institute of Geotourism, Faculty of Mining, Ecology, Process Control and Geotechnologies, Technical University of Košice, Letná 9, 042 00 Košice, erik.weiss@tuke.sk , ladislav.mixtaj@tuke.sk

[3] *Ing. Heidy Schwarczová*, University of Central Europe in Skalica, Kráľovská 386/11, 909 01 Skalica, riaditel@sevs.sk

(Review and revised version 4. 2. 2010)

LGN. Coordinates of DB define the concrete realisation $\varphi_{\mathrm{DB}}(\mathrm{DB}_1, \mathrm{DB}_2, ...)$ of the theoretical coordinate frame for the coordinate system $\varphi \equiv S - JTSK$ used for LGN,

- verifying the required compatibility of the points used as DB, as their mutual, suitable compatibility is the necesssary condition for harmony between the DB coordinates and other measurable quantities,
- selecting the suitable measuring technology in the LGN, particularly in terms of the required accuracy for the UB points to be determined in the LGN developed,
- determining a-priory the accuracy limits for measurements in view of the UB position and the accuracy required for determining the functions of the UB coordinates,

as well as further requirements set for the characteristics of the existing field of points, necessary for the newly established LGN to really fulfill its mission (Weiss, G., Jakub, Weiss, E. 2006; Jakub, 2001; Sütti, 1996; Gašinec, Gašincová, Rajniak, 2000).

The 2D LGN being established is mostly realised by a field of points in its prospective area using:

- points of the State Trigonometric Network (ŠTS) located in the area with coordinates defined and laid down in state documentation,
- points of various LGSs set up during the previous periods of time for the given area, or in its vicinity and take over by the ŠTS having assigned ther relevent data of evedence,
- new UB points, with coordinates and other relevant parameters to be determined for the the prospective LGN.

UB points for the new LGN are localized in view of its prospective application, i.e. in line with the purposes and needs applicable for the use of UBs as well as the localization of the DBs used.

When establishing an LGN, mostly two kinds of LGNs are used:

- bounded LGN, (connected) to the state field of points, i.e. using coordinates for LGN points from the official 2D State Coordinate System (ŠTS), consequently from S-JTSK,
- independent LGN, with its own, local coordinate system (LCS), defined by a minimum points (two points) of its field of points.

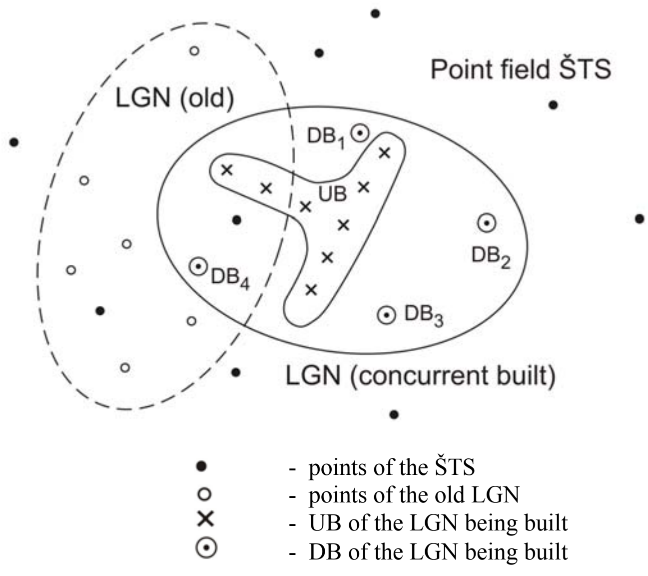

Fig. 1. LGN in the field of points.

The selected points of the existing field of points for the function of DB in the LGN being build in the past. They were established in different periods of time as densifying points of the ŠTS, as points of different LGNs of their time etc., located in the neighbourhood or in the area overlapping the LGN being built (fig. 1). It can be really assumed that some of the points of these surrounding structures selected for the „functions" of DB, will not prove for compatible for the establishment of the current LGN. As a result, the determination of the UB coordinates in the LGN under construction will be dependent on the DBs used. Their coordinates will generally be incompatible not only mutually but also with the coordinates of the rest of DBs available in the vicinity (the UB coordinates determined e.g. from DB_1, DB_2 may significantly differ from their values determined from different points, for example from DB_3, DB_4, ... and the like).

It is known, if DB in the LGN proves incompatible, this will result in the MLS processing (Method of Least Squares): in the residual vector v with unacceptable values v_i of some measured quantities, in unacceptable positional accuracy of UB and also, as a rule, it can come to rejection of the null hypothesis $H_0 : s_0^2 = \sigma_0^2$.

Therefore, the compatibility of the points selected from the surrounding field of points to function as DB points, which along with the determined UB points in the given area will make up the new LGN, are to be verified so as to avoid generation of various, mutually differing coordinate fixes (values) of UBs varyiing according to which DB is used for this purpose of MNS processing. If any of the DB proves incompatible, i.e. its coordinate position does not coincide acceptably with its physical position mark, it cannot be used as a datum point (DB) when establishing the LGN.

DB compatibility can be verified by various procedures, for example by way of suitable transformations, probability analyses and statistical assessments of the differences between the marking positions and the RB coordinates as well as by combination of the procedures mentioned (Weiss, G., Jakub, Weiss, E., 2006; Jakub, 2001; Sütti, 1996; Stichler, 1985; and others).

Verification of the compatibility of points selected from the area of the LGN, in terms of their suitability for the DB function, and exclusion of those unsuitable for further use is the basic condition of establishing a proper LGN for the purposes set.

Verification of the DB compatibility in the LGN

Appart from the known procedures of verifying compatibility of geodetic points, compatibility of the position of DB points in the LGN can also be assessed also on the basis of the compatibility of two differently generated distances between each pair of DB_i, DB_j points.

Between each pair of DB_i, DB_j points of the DB set, one distance is represented by the measured value of d_m between the physical markers of the points and the second distance by the value of d_s, determined from the given (known) coordinates of the DB_i, DB_j points. If both distances are compatible, i.e. their coordinate points and those created by their physical markers would create a stochastic identiy, both distances between the points (coordinate and measured ones) would differ only by a small value. In such case, both distances could be considered as stochastically identical, i.e. compatible.

If only one of the points, e.g. DB_i from the pair of DB_i, DB_j is incompatible, i.e. its coordinate position will not be stochastically identical to its physical position (point marker)*), the distance between the coordinate and physical markers of both points will make up a significant difference, too.

The status and characteristics givern for the DBi, DBj point pairs within the selected set of DBs for the LGN enable us to determine through appropriate testing the significant and non-significant pairs of distances (obtained by measurement and from coordinates).

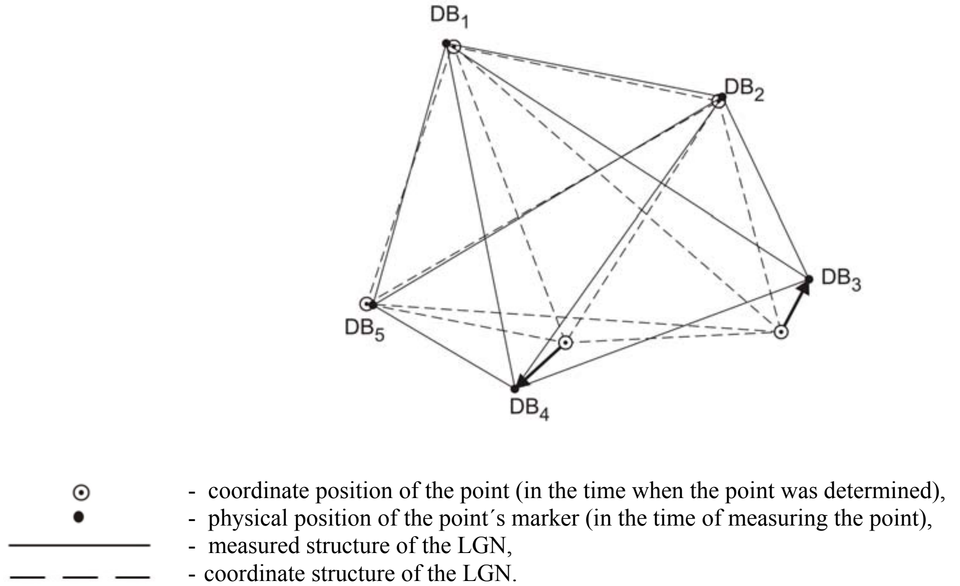

Fig. 2. Coordinate and physical structure of the LGN with two incompatible points: DB3 and DB4.

* the physical marker of the point can be shifted by the time as a result of various powers into its present position, which is not identical to its coordinate postition determined in the time when the point was established.

Based on this reality, we are able to determine all those DB points that are causes to incompatibility of the field of DB points in the LGN.

On weeding out the incompatible DB points from the LGN, the rest of the DB points will of use as compatible ones, i.e. suitable for mesasuring and developing the LGN and they are not expected to degrade determination of the coordinates of UB points.

In cases when for any of the DB_i, DB_j pairs we obtain different length values as a results of measurings and those of the coordinates, it becomes evident that one of the points of this DB pair is not compatible with its either physical or coordinate position. Such a point will have significantly different horizontal positions between the point´s physical marker and that of the coordinate. This situation will be actual also in cases when both final physical points prove not compatible with the appropriate coordinate position of points.

The comparative approach of assessing the quality of selected DBs for the LGN, on the basis of the values of physical lengths and coordinate lengths (between the DB markers and the coordinates respectively), is demonstrated by the LGN situation in fig. 2, i.e. for 5 points selected from the surrounding field of points to „function" as datum points DB_1, ..., DB_5 for the LGN being built.

It is supposed that not all the DB points selected for the new LGN will be mutually compatible. Therefore, it is necessary to choose from these points only the ones suitable and acceptably compatible on the basis of the suggested procedure (Chapter 4). In situation as illustrated in fig. 2 the positions of DB_1, DB_2, DB_5 points are considered compatible, whereas positions of points DB_3, DB_4 as incompatible, i.e. by their physical markers shifted against their original postions and therefore they are of no use in view of extablishing the LGN.

Assessing the DB points´ compatibility for the LGN

Compatibility or flaws in the compatibility of the physical and coordinate markers of selected DB points to be used in the LGN under development are asssessed on the basis of statistical verification of the coordinate and physical (measured) distances between DBs.

Distances between the coordinate positions of DB points.

DB points are determined by their coordinates (for example in the S-JTSK) in the appropriate cartographic plain. For DB_i, i=1,2,...,5 are known the official coordinates $C_{DBi} = [XY]_{DBi}$ from the state database and often their standard deviations $s_{\hat{C}[DBi]} = [s_{\hat{X}} s_{\hat{Y}}]_{DBi}$ as well. Provided that the given document „Data on the geodetic point", makes no mention of the $s_{\hat{X}}, s_{\hat{Y}}$ values, it is appropriate to find out their approximate estimate values at least, using the accuracy data of points of the neighbouring field of points.

Using the coordinates C_{DBi} one can determine the „coordinate distances" between the points by

$$d_{sij} = \left((X_j - X_i)^2 + (Y_j - Y_i)^2 \right)^{\frac{1}{2}} = \left(\Delta X_{ij}^{\ 2} + \Delta Y_{ij}^{\ 2} \right)^{\frac{1}{2}} \quad (1)$$

i.e. their numerical values:

1-	2-	3-	4-
d_{S12}			
d_{S13}	d_{S23}		
d_{S14}	d_{S24}	d_{S34}	
d_{S15}	d_{S25}	d_{S35}	d_{S45}

(2)

Also determined can be their accuracies – standard deviations of the d_s quantities by

$$s_{ds_{ij}} = \left\{ \left(\frac{\Delta X_{ij}}{d_{sij}} \right)^2 \left(s_{Xi}^2 + s_{Yi}^2 \right) + \left(\frac{\Delta Y_{ij}}{d_{sij}} \right)^2 \left(s_{Xi}^2 + s_{Yi}^2 \right) \right\}^{\frac{1}{2}} \quad (3)$$

consequently, the values of the coordinate distances among the 5 DB points and their accuracies will be determined by expressions (1),(3).

The distances between the physical positions of DB points

Within the complete measurement of the LGN with its DB and UB points, i.e. within the measuring of all the quantities necessary for the processing of the LGN, one is to measure also the d_{mij} distances between its DB points and determine the necessary reductions into the cartographic plane.

Each length between the DB points is independently measured p-times, which is carried out in line with the by the project of the LGN development (measuring distances between DBs forms part of measuring all the quantities necessary for the LGN to be established).

The d_{mij} distances are measured by suitable EOD with a known (and verified) equation from the manufacturer, applicable to standard deviations (apriory) measured d_{mij} distances, e.g.

$$s_{dmij} = a + b\overline{d}_{mij} \tag{4}$$

where: *a,b* are values supplied by the manufacturer,

$\overline{d}_{mij}$ is the adjusted value of the distanc d_{mij}, i.e. the average from its individually measured d_{mij1}, d_{mij2} ,..... d_{mijP} values with equal or various weights (cofactors).

The values of $\overline{d}_{mij}$, by their arrangement as (2), will also form a similar overview on the basis of the distances obtained through their measuring between DB_i

1-	2-	3-	4-
$\overline{d}_{m12}$			
$\overline{d}_{m13}$	$\overline{d}_{m23}$		
$\overline{d}_{m14}$	$\overline{d}_{m24}$	$\overline{d}_{m34}$	
$\overline{d}_{m15}$	$\overline{d}_{m25}$	$\overline{d}_{m35}$	$\overline{d}_{m45}$

(5)

The differences between the „coordinate" and „measured" distances within the LGN

From both surveys, (2), (5), i.e. from the values „coordinate distances" d_{sij} and „measured distances" $\overline{d}_{mij}$, the diferences are determined as

$$dif(d_{ij}) = d_{sij} = \overline{d}_{mij}\,, \tag{6}$$

for all the measured distances between DB in the LGN, the numerical magnitude of which will serve as a basis for the assessment of the compatibility of the point, or points, corresponding to each value of $dif(d_{ij})$.

Their standard deviatons will be as follows:

$$s_{dif(d_{ij})} = \left(s_{dsij}^2 + s_{dmij}^2\right)^{1/2}. \tag{7}$$

The magnitude of $dif(d_{ij})$ of the individual connecting lines will be decisive, whether the appropriate d_{ij} distance with its final points (DB_i, DB_j) from both the coordinates and measurements will be stochastically (practically) identical or they will significantly differ, i.e. whether the DB_i, DB_j final points of the corresponding distance d_{ij} are compatible or not from standpoint of the coordinate and physical identity markers.

Identification of incompatible differences among DB points

The magnitude of $dif(d_{ij})$ can be at each distance be can assessed using suitable statistical tests and in the given case even a simple test can do (Reissmann, 1980; Böhm et al. 1990; Weiss, G., Jakub, Weiss E., 2006; and others) to verify the null hypothesis:

$$H_0 : d_{sij} = \overline{d}_{mij}$$

against the alternative hypothesis $H_a : d_{sij} \neq \overline{d}_{mij}$ applying the testing statistics to each connecting line between two DB points

$$T_{ij} = \frac{dif(d_{ij})}{s_{dif(d_{ij})}} \sim N(\alpha) \tag{8}$$

with selecting the significance level (usually $\alpha = 0,05$) i.e. for $N(\alpha) = 1,645$.

If $T_{ij} \leq N(\alpha)$,

the H_0 is accepted, the distance between the two DB_i and DB_j points determined by the coordinates and by the measurement are stochastically identical, i.e. the points DB_i and DB_j are compatible and usable for developing the LGN.

If $T_{ij} > N(\alpha)$,

the H_0 is to be rejected at the significance level α, as the coordinate and measured distances length between points DB_i and DB_j will be significantly different, indicating incompatibility of at least one of the points, DB_i or DB_j, or both of the points of the corresponding distance.

The values as in (2) and (5) give rise to differences between the distances determined by coordinates and measurements

$$dif(d) = d_s - \overline{d}_m = \begin{bmatrix} dif(d_{12}) & & & \\ dif(d_{13}) & dif(d_{23}) & & \\ dif(d_{14}) & dif(d_{24}) & dif(d_{34}) & \\ dif(d_{15}) & dif(d_{25}) & dif(d_{35}) & dif(d_{45}) \end{bmatrix} \tag{9}$$

from which, using tests (8), one is able to find out which $dif(d)$ values are statistically important or unimportant, i.e. which differences between the coordinate and measured distances are of significance and which are not.

Determining the incompatible DB points

Let for $dif(d)$ between 5 points of the LGN (fig. 2) there arises a concrete situation:

$$dif(d) = \begin{Bmatrix} 12 & & & \\ (13) & (23) & & \\ (14) & (24) & (34) & \\ 15 & 25 & (35) & (45) \end{Bmatrix} \tag{10}$$

where the statistically significant differences between d_s and $\overline{d}_m$ are in parenthesis. Then, from the state (10) using logical considerations, for the individual points DB of LGN follows that:

- the compatible points in the LGN are DB_1, DB_2, DB_5 (making up the distances of 12, 15, 25), for which the appropriate tests revealed no significant differences between the coordinate and physical points, i.e. the following 3 points, namely the DB_1, DB_2, DB_5, are points compatible within the LGN,
- the points incompatible within the LGN are namely the points DB_3, DB_4 (making up the end points for sides 13, 14, 23, 24, 34, 35, 45), for which significant differences between their coordinate and physical distances have been proved by the appropriate tests.

The situation based on the conclusios as above is illustrated in fig. 2.

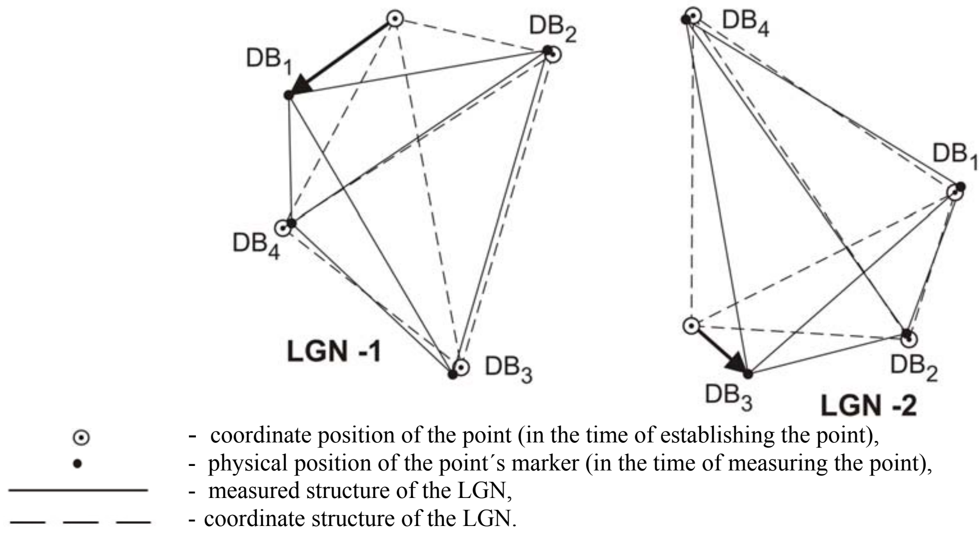

Fig. 3. Coordinate and physical structures of the LGN-1 and LGN-2, each with a single incompatible point DB_1 (for LGN-1) and DB_3 (for LGN-2)

The indicated procedure of identifying incompatible DB points from their set of 5 DB points, for example for their network structures in LGN-1and LGN-2 (Fig. 3.), is performed in two stages:

- determining the difference $dif(d_{ij})$ between the coordinate and measured distances for all the distances between the DB points (to be used as DB points for the LGN), testing them by (8), making up a survey by (9) or (10) for the LGN-1 and LGN-2 and finding out, in their structure, for which distances are their dif(dij) values of significance (let them be for example the distances lengths as follows: DB_1-DB_2, DB_3-DB_1, DB_4-DB_1 in LGN-1, or the lengths of DB_3-DB_1, DB_2-DB_3, DB_3-DB_4 in LGN -2), whereby one can also find out for which distances in LGN-1 and in LGS-2 the $dif(d_{ij})$ values are of no significance.
- based on the results, one is then ready to produce a survey of distances in compliance with (10), by which, let there arise a situation in the LGN-1 and LGN-2 (fig. 3.) by

$$\text{LGN-1: } dif(d_{ij}) = \begin{bmatrix} (12) & & \\ (13) & 23 & \\ (14) & 24 & 34 \end{bmatrix}, \qquad \text{LGN-2: } dif(d_{ij}) = \begin{bmatrix} 12 & & \\ (13) & (23) & \\ 14 & 24 & (34) \end{bmatrix}.$$

Based on the structure of $dif(d_{ij})$, it follows for the LGN-1 that the point of incompatibility is presented by point DB_1 (it is component in all the 3 incompatible distrances d_{12}, d_{13}, d_{14} . As for the LGS-2 structure, it follows that the incompatible point is only the DB_3, because it is present in all the 3 incompatible distances d_{13}, d_{23}, d_{34}).

Conclusion

When establishing an LGN including both the existing points (from the ŠTS) functioning as the datum for DB points to the LGN and also the newly determined UB points in the given area, it is appropriate for the quality of the new LGN, that the compatibility of the DB points used for the LGN be verified. This principle of using suitable DB points for processing of LGNs is valid also for bounded and free adjustment. Applying unsuitable DB points would inevitable be reflected in the values of the resulting quantities as well as in their accuracy characteristics. The implied procedure of veryfing DB points to be used for the development of LGN is a simple, not requiring special ways of measurements nor analyses and is applicable to all LGN adjustments performed by LSM (Least Squares Method).

References

Böhm, J., Radouch, V., Hampacher, M.: Teórie chyb a vyrovnávací počet. *Praha: Geodetický a kartografický podnik, s. p., 1990, ISBN 80-7011-056-2.*

Cimbálník, M.: K přesnosti souřadnicového systému. *Geodetický a kartografický obzor 22/64, 11, p. 311-315, 1976.*

Cimbálník, M.: Návrh spřesnení souřadnicového systému. *Geodetický a kartografický obzor 39/81, 2, p. 22-26, 1993*.

Gašinec, J., Gašincová, S., Rajniak, M.: K problematike vyrovnania 2D geodetických sietí. *In: Aktuálne otázky meračstva a inžinierskej geodézie: zborník z medzinárodného vedeckého seminára: Herľany, 9.-10. október 2000. FBERG, TU Košice, p. 38-42, 2000, ISBN 80-7099-595-5.*

Jakub, V.: Posudzovanie stability geodetických bodov. *Dizertačná práca, FBERG, TU Košice, 2001.*

Reissmann, G.: Die Ausgleichugsrechnung. *Vlg.f. Bauwesen, Berlin, 1980.*

Stichler, S.: Ubntersuchung von Methoden der Identifizierung stabiler Punkte. *Vermessungstechnik, 33, 11, p. 203-206, 1985.*

Sütti, J.: O kompatibilite geodetických sití. In: Zbor. „Aktuálne problémy inžinierskej geodézie", *STU Bratislava, 1996.*

Sütti, J., Weiss, G., Kuráň, Š.: Analýza identity bodov v polohových sieťach. *Acta Montanistica Slovaca, roč. 3 (1998), 2, p. 159-166.*

Weiss, G., Jakub, V., Weiss. E.: Kompatibilita geodetických bodov a jej overovanie. *TU Košice, 2004, ISBN: 80-8073-149-7.*

Weiss, G., Sütti, J.: Geodetické lokálne siete I. *FBERG, TU Košice, 1997, ISBN: 80-967 636-2-8.*