Adjustment of positional geodetic networks by unconventional estimations

***Silvia Gašincová** and **Juraj Gašinec**[1]*

The content of this paper is the adjustment of positional geodetic networks by robust estimations. The techniques (based on the unconventional estimations) of repeated least-square method which have turned out to be suitable and applicable in the practise have been demonstrated on the example of the local geodetic network, which was founded to compose this thesis. In the thesis the following techniques have been chosen to compare the Method of least-squares with those many published in foreign literature: M-estimation of Biweight,M-estimation of Welsch and Danish method. All presented methods are based on the repeated least-square method principle with gradual changing of weight of individual measurements. In the first stage a standard least-square method was carried out in the following steps – iterations we gradually change individual weights according to the relevant instructions/ regulation (so-called weight function). Iteration process will be stopped when no deviated measurements are found in the file of measured data. MatLab programme version 5.2 T was used to implement mathematical adjustment.

***Key words**: Method of least squares, robust estimations, outlier detection*

Introduction

In geodetic practice the method of least squares (hereafter in the text referred to as MNS) is generally used to analyze measured results. This method has several advantages – in case of normality of the sets of geodetic data measured it leads to the most reliable estimates of unknowns and provides statistically unbiased and consistent estimates of parameters. Due to the fact that not always all the conditions for its application are fulfilled, in the second half of the last century unconventional estimation techniques were developed in the theory of linear programming alongside standard estimation procedures. Of all these methods the so-called robust estimation techniques are mainly used in geodetic applications. These techniques allow obtaining estimates with various necessary properties and avoiding hidden, serious and systematic errors in measurements which could essentially effect and devalue the parameters being fixed. The known methods include e.g. the Method of the least sum of absolute values of corrections (hereafter in the text referred to as MNAS) whose characteristic feature is robustness of the estimated parameters to the influence of outlying (deviating) measurements and the MINIMAX method (Cebecauer, D., 1994) that was applied mainly in the transformation of local networks. In foreign literature, especially in German and English (Jager et all, 2005) MNAS is defined as L1 standard.

The submitted paper compares the method of least squares with some types of robust methods based on the principle of its repetition (Weiss, et all, 2004),. Individual methods of adjustment are demonstrated on the local geodetic network which was established for the purpose of experimental measurement whilst preparing the thesis dissertation dealing with the issue of robust methods (Gašincová, S., 2007). The network situated in the locality of the Liptovská Mara water dam was measured by the combination of terrestrial and satellite measurements.

Adjustment of measurements

In geodesy the following processing procedures are used to evaluate the measured results (Böhm J. et. al., 1990):

a. adjustment of direct measurements where the only unknown was independently measured several times (angle, length),
b. adjustment of intermediate measurements where more unknowns were „determined“ by means of direct measurement of other quantities which were in functional relationship with them,
c. adjustment of conditional measurements where individual quantities are measured directly, but at the same time
d. they must fulfil a predetermined mathematical or geometrical condition (e.g. in the triangle $\alpha+\beta+\gamma-200^g = 0$);

[1] *Silvia Gašincová, MSc, PhD., assoc. prof. Juraj Gašinec, MSc., PhD.*, Faculty of mining, ecology, procces control, and geotechnology, Technical univerzity of Košice, Institute of Geodesy, Cartography and Geographic Information Systems, Park Komenského 19, 040 01 Košice, Slovak Republic, silvia.gasincova@tuke.sk, juraj.gasinec@tuke.sk
(Review and revised version 31. 5. 2010)

e. combined methods of adjustment: adjustment of intermediate measurements with conditions, adjustment of conditional measurements with the unknowns, collocation, the Kalman filter, the Chebyshev´s method, etc.

The above-mentioned methods of adjustment are based on the condition of the minimum of the correction vector norm. The norm is a number assigned to each n-dimensional vector vektoru $v_{(1,n)} = (v_1, v_2 ... v_n)$ (Bitterer, 2006). The following types of the objective functions are most often used in geodesy:

$$\rho(v_i) = \left(\sum_{i=1}^{n} |v_i|^p \right)^{\frac{1}{p}} = min. \qquad i \in \langle 1,n \rangle \tag{1}$$

Parameter *p* defines a special type of the objective function:

a) for *p=2* (*L2-norma*) the objective function has the following form

$$\rho(v_i) = \left(\sum_{i=1}^{n} v_i^2 \right)^{\frac{1}{2}} = min., \tag{2}$$

that leads to the method of least squares (MNS),

b) for *p=1* (*L1-norna*) the objective function is expressed in the following form

$$\rho(v_i) = \sum_{i=1}^{n} |v_i| = min., \tag{3}$$

this represents the method of the least sum of absolute values of corrections (MNAS).

c) for *p=∞* (*L∞-norma*) the objective function in the form

$$\rho(v_i) = \left(\sum_{i=1}^{n} |v_i|^{\infty} \right)^{\frac{1}{\infty}}, \tag{4}$$

is changed into the MINIMAX method (known as the Chebyshev method) which minimizes the correction within a given tolerance interval (Bitterer, 2006) and for which applies that the biggest correction (its absolute value) is the minimum correction.

$$max|v_i| = min. \tag{5}$$

The methods of linear programming such as simplex method, its modified form Friedrich´s method or solution in which the MNS is repeatedly used with gradual change of weights are applied when searching for the minimum by MINIMAX and MNAS methods.

Robust estimation techniques

There are, as we know, two types of robust estimation: *robust estimation applied to MNS* where the sum of squares of corrections is replaced by more appropriate functions of corrections, e.g. the most reliable estimations (Huber, 1981; Hampel, 1986; etc.) and *truly robust methods* which include both the *simplex* and *Friedrich´s* method.

The robust method of adjustment based on the MNS principle takes place when in the estimation process the appropriately chosen correction function $\rho(v_i)$, the so-called *loss (estimation) function* is minimized (Sabová, Jakub, 2005).

$$\rho(v_i) = min \ , \tag{6}$$

that for the estimation process generates the so-called *influence function* $\Psi(v_i)$ characterizing the influence of errors on the adjusted values for which applies

$$\sum \Psi(v_i) = 0 , \tag{7}$$

where

$$\Psi(v_i) = \frac{\partial \rho(v_i)}{\partial v_i} \ . \tag{8}$$

For the adjustment to be robust it is advisable to use iteration method with variable weights, so that the weights of individual observations in each iteration step are determined as functions of corrections:

$$p(v_i) = \frac{\Psi(v_i)}{v} , \tag{9}$$

where $p(v_i)$ is a weight function in the solution of the adjustment method. Solution algorithm of the iteration robust estimation consists of the following steps :

1. in the first iteration step a standard MNS adjustment with weights $p_1^{(1)} = 1$ ((1)- iteration step for the ith observation) is performed, in case of heterogeneous measurements it is necessary to carry out their homogenization using $\sqrt{\boldsymbol{P}}$ matrix (this function is available with MATLAB - rootm(P)) programme. An effective tool of solution without using $\sqrt{\boldsymbol{P}}$ matrix in processing of heterogeneous measurements is given (Hemikogle, 2002),
2. from the corrections obtained in the first step of iteration using weight function $p(v_i)$ new weights are determined which will be used in the next step and analogically in the subsequent steps.

Solution based on this principle represents an iteration solution with gradually changing weights p_i in accordance with the respective specification of observations so that with the sufficient number of iteration steps convergence of corrections takes place in the last steps.

Experimental measurement of the local geodetic network

Taking into consideration the fact that at present combining satellite and terrestrial measurements creates a welcomed option of building, control and assessment of the quality of geodetic networks, the present paper presents a model of joint processing of satellite and terrestrial measurements (Leick, A, 1995).

The method of least squares and estimation techniques based on the principle of its repetition are demonstrated on the local geodetic network which was established for the purpose of developing the thesis on the site of the Liptovská Mara water dam. By choosing the site in the vicinity of the water dam we assumed formation of sufficient horizontal thermal gradient over the water surface and adjacent lands which would have a disturbing effect on terrestrial measurements (Bajtala, Sokol, 2005). For that reason, the measurement was carried out in the warm sunny environment. Unfortunately, the changed weather conditions during the measurement mixed up the heat blocks which is on the contrary a welcomed phenomenon in routine geodetic measurements

The geodetic network consists of four points (1,2,3,2643LM-1005) (fig. 1) which were temporarily stabilized by a steel rod of 12 mm diameter and 40 cm length – point 2643LM-1005 is a point of the State spatial network.

In terrestrial measurement, reflection prisms were placed at these points. Considering the fact that by combining GPS and terrestrial measurements it may lead to the situation when it is not possible to use the positions of GMS measurements for the terrestrial measurement due to the lack of direct visibility between the points, eccentric positions were stabilized in the vicinity of each of these standpoints the same way as points 1, 2, 3 from which the terrestrial measurement was carried out. Eccentric positions were marked according to the respective standpoints at 5001, 5002, 5003, 5005.

A single frequency GPS Stratus Sokkia system was used to carry out the satellite measurement which was performed by the static method simultaneously above all the network points (1, 2, 3, 1005). Four GPS devices were used in the measurement of the demonstration network. The approximate coordinates of the individual network points and their heights above the ellipsoid GRS80 in the coordinate system ETRS89 were determined from the point 2643LM-1005 (State spatial network) in the software package Spectrum Survey and Gues. From these coordinates, positional coordinates were determined in Gauss – Krüger projection.

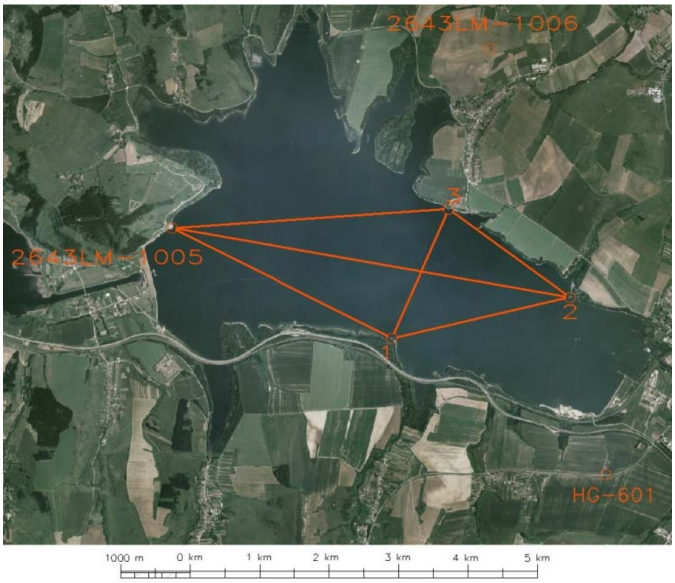


Fig. 1. Structure of the positional geodetic networks in the locality of the Liptovská Mara water dam.

Terrestrial measurement was performed from the eccentric positions 5001, 5002, 5003 and 5005. From each of these positions the sets of horizontal directions were measured in 12 groups to the individual points of the geodetic network. With respect to the equipment used, the slanted lengths were measured only from the positions 5001, 5003 and 5005. In performing terrestrial measurement 8 distances were measured. The sets of directions were measured from individual eccentric positions. In positions 5001, 5003 and 5005 *s=12 groups* were measured in *n=4* directions. In position 5002 s=11 groups were measured in *n=4* directions using the following instrumental equipment: *NIKON DTM 332 (position 5001*, *KERN DKM 2 – A (position 5002)*, *LEICA TCR 305 (position 5003)* a *LEICA TC 1800 (position 5004)*.

Fig. 2. Global Positional System STRATUS Sokkia.

	om	to	ne observation
Standpoint	:s]	**[h:m:s]**	**[h:m:s]**
1	14:12:08	18:14:56	4:02:48
2	15:08:37	18:07:22	2:58:45
3	15:43:07	17:49:37	2:06:30
1005	16:1537	17:24:21	1:08:44

Fig. 3. The time of observation on individual positions of the geodetic networks.

Processing of the positional geodetic network

Before the geodetic network measurement itself was performed, there was established a basis with 7 points on the campus of the Technical University for the purpose of verifying the functionality of electro-optical rangefinder and mainly determining an addition constant. The addition constant is calculated based on adjusting conditional adjustments with unknowns (Dobeš, J.,1990). The addition constant determined to the Zeiss reflection prisms for the devices of the Leica company was *c=-0,0142 m*, for Nikon DTM 352 the value of addition constant was set to *c=-0,0318 m.* The geodetic network was measured by combination of the static GPS method at the centric points with the terrestrial measurements where the devices were placed all at once in the eccentric positions 5001,5003,5003,5005.

Taking into consideration the mutual adjustment of the satellite and terrestrial measurements it was necessary to choose an appropriate processing – computing space before processing itself. Geographical coordinates of the points of the State spatial network as well as the coordinates of the reference stations SKPOS (Slovak spatial observation system) relate to ETRS-89 coordinate system and GRS80 reference ellipsoid whose parameters are very close to WGS84 ellipsoid (**W**orld **G**eodetic **S**ystem). Gauss conform projection of the ellipsoid in the meridian planes (also called Gauss-Kruger projection or Universal Transverse Mercator system – UTM) or Lambert conform projection (Buchar, Hojovec, 1996; Daniš, 1976) are most often used for the direct projection of these coordinates to 2D computing space.

Because of this reason, during the period of network pre-processing the measured elements were adjusted and subsequently reduced to the Gauss-Kruger projection chosen to be the computing space into which both satellite and terrestrial geodetic measurements were reduced and subsequently adjusted. In this projection as well as in its UTM modification in general both positive and negative plane coordinates x and y can occur which are called normal coordinates used for conversions between the neighbouring belts. Mainly negative y coordinates in each meridional belt appear to be a substantial disadvantage for our republic which is why the so-called *conventional* coordinates shifted 500 km to the west were introduced. In order to minimize the influence of longitudinal distortion and localization of all the points of the positional geodetic network in one meridional belt and in one coordinate system the Gauss Kruger projection is modified in such a way that the meridian belonging to the centre of the network is considered as a principal undistorted meridian. For that purpose the „ETRS 892Gauss.m" programme was written in Matlab programming language which rounded up the central meridian to full minutes. The conventional coordinates (x – and y-coordinates shifted by a certain constant) are reduced in such a way that the value of x coordinate of the point 1 is 1000.000 m (tab. 2). For the direct display of geographical coordinates of the points on the geodetic network in the 2D computing space the following relations were used (Hojovec, 1987; Buchar, 1996; Pick, 1998).

$$
\begin{aligned}
X &= S + (1/2) N \sin\varphi \cos\varphi \left(\Delta\lambda/\rho^2\right) + (N/24) \sin\varphi \cos^3\varphi (5 - t^2 + \\
&+ 9e'^2 \cos^2\varphi + 4e'^4 \cos^4\varphi)(\Delta\lambda/\rho)^4 + (N/720) \sin\varphi \cos\varphi^5 (61 - \\
&- 58t^2 + t^4 + 270e'^2 \cos^2\varphi / 330e'^2 \sin^2\varphi)\left(\Delta\lambda/\rho\right) \\
Y &= N \cos\varphi \left(\Delta\lambda/\rho\right) + \left(N/6\right) \cos^3\varphi \left(1 - t^2 + e'^2 \cos^2\varphi\right)\left(\Delta\lambda/\rho\right)^3 + \\
&+ \left(N/120\right) \cos^5\varphi \left(5 - 18t^2 + t^4 + 14e'^2 \cos^2\varphi - 58e'^2 \sin^2\varphi\right)\left(\Delta\lambda/\rho\right)
\end{aligned}
\qquad (10)
$$

<u>Notes .</u>

S- the length of the meridian from the equator to the given geographical latitude φ,
N- transverse radius of curvature ,
M- meridian radius of curvature ,
φ - geographical latitude of the converted point
λ -geographical length of the converted point,
e'-the second numerical eccentricity of the ellipsoid,
$t = \mathrm{tg}\varphi$,
ρ =57, 2957795°

Tab. 1. Geographics coordinates points of the local geodetic network.

point	φ [°]	[']	["]	λ [°]	[']	["]	H [m]
1	49	5	31.40194	19	31	59.66451	604.580
2	49	5	55.84541	19	34	03.85359	607.692
3	49	6	33.10905	19	32	33.47193	603.967
1005	49	6	15.38461	19	29	18.46236	608.596
5001	49	5	31.49713	19	31	59.45874	603.964
5002	49	5	56.02921	19	34	3.63487	606.891
5003	49	6	33.14883	19	32	33.27692	604.311
5005	49	6	15.28134	19	29	18.33564	607.969

Tab. 2. Coordinates in Gauss- Kruger projection.

point	x	Meridian y	Lenght h	influence convergency	lenght distortion	distortion
	[m]	[m]	[m]	[°]		[cm/km]
Normal coordinates						
1	5439865.248	-6.806	604.580	-0.000070431	1.000000000	-0.00
2	5440620.929	2512.401	607.692	0.026003759	1.000000078	-0.01
3	5441771.55	678.845	603.967	0.007028720	1.000000006	-0.00
1005	5441224.96	-3276.473	608.596	-0.033918521	1.000000132	-0.01
5001	5439868.189	-10.981	603.964	-0.000113629	1.000000000	-0.00
5002	5440626.605	2507.961	606.891	0.025957858	1.000000077	-0.01
5003	5441772.783	674.890	604.311	0.006987771	1.000000006	-0.00
5005	5441221.747	-3279.046	607.969	-0.033945114	1.000000132	-0.01
Conventional coordinates						
1	1000.000	4272.239	604.580	-0.000070431	1.000000000	-0.00
2	1755.680	6791.446	607.692	0.026003759	1.000000078	-0.01
3	2906.306	4957.891	603.967	0.007028720	1.000000006	-0.00
1005	2359.688	1002.572	608.596	-0.033918521	1.000000132	-0.01
5001	1002.941	4268.065	603.964	-0.000113629	1.000000000	-0.00
5002	1761.356	6787.007	606.891	0.025957858	1.000000077	-0.01
5003	2907.534	4953.936	604.311	0.006987771	1.000000006	-0.00
5005	2356.499	1000.000	607.969	-0.033945114	1.000000132	-0.01

Principle meridian: 19°32’ 0.00000”
Distortion of the central meridian: 1.0000
Dx= -5438865.248
Dy= 4279.046

Note: All the coordinates of the network are located in one meridional belt according to the average geographical length.

For the transformation of the distances it is necessary to know the size of the *longitudinal distortion* in individual points of the network that are determined according to the relation:

$$m = 1 + \frac{\lambda^2}{2}\cos^2\varphi\left(1+\eta^2\right) + \frac{\lambda^4}{24}\cos^4\varphi\left(5-4tg^2\varphi\right) \tag{11}$$

In order to convert azimuth to bearings we need to know the size of the *meridian convergence*.

$$\gamma = \lambda\sin\varphi + \frac{\lambda^3}{3\rho^2}\sin\varphi\cos^2\varphi\left(1+3\eta^2+2\eta^4\right) + \\ + \frac{\lambda^5}{15\rho^4}\sin\varphi\cos^4\varphi\left(2-tg^2\varphi\right) \tag{12}$$

Taking into consideration the fact that the SSN (State spatial network) coordinates as well as the coordinates of the SKPOS referential stations relate to the ETRS-89 coordinate system and GRS80 referential ellipsoid it was necessary to reduce individual parameters measured to the area of the mentioned ellipsoid before processing the geodetic network.

Distances measured terrestrially were also reduced into the plane of the cartographic projection. Before the reduction itself distances measured were modified by an additive constant which was calculated with the help of conditional adjustment with the unknowns. The reduction of individual distances into the cartographic projection itself was performed in the following sequence:

- calculation of the direct line connecting the endpoints of the distance measured,
- calculation of the chord length in the substitute ball of the *R* radius,
- the arc of a circle is calculated to the chord t,
- recalculation of the arc of a circle to the length of the geodetic line on the reference ellipsoid.

As the geodetic network was measured from the eccentric standpoint it was necessary to convert individual values obtained by terrestrial measurements to centric before the adjustment itself.

The values determined in this way were the input data which were entered into the adjustment model (the vector of measured observations **l**) in the sequence of the terrestrial measurement - centric distances, centric angles a and GPS measurements – cartographic distances, pointers

Methods of development the issue

The *method of least squares* was used for mutual adjustment of the satellite and terrestrial measurements which was compared with the following robust estimation procedures:

1. Danish method (Jäger, R. et al, 2005),
2. Robust M-estimate according to Biweight (Jäger, R.et al, 2005),
3. Robust M – estimate according to Welsch (Jäger, R.et al, 2005).

The above-mentioned estimation procedures are based on the principle of the repeated MLS. The geodetic network was adjusted as a free-network, i.e. all the coordinates of the geodetic network. 3 were adjusted. Before the network adjustment itself the homogeneity of precision of the measured quantities was tested. The testing was performed using an appropriate test to verify the homogeneity of variances of which Cochran statistics is used most frequently (Weiss, Sütti, 1997),(Weiss, et all, 2004), which was used in this case as well. Parameters of the 2nd order system were estimated by MINQUE (Minimum Norm Quadratic Unbiased Estimation) (Lucas, J. R. et all, 1998), separately for the satellite as well for terrestrial measurement for each device used.

Method of least squares

Tab. 3. Free adjustment of LGN (Local geodetic network): Method of least squares.

Measurement	l	l^	v	pi	Ti	s(v)	s(l^)	r *	f
	m	m	mm		*	mm	mm		
1- 3	2025.8600	2025.8662	6.1539	0.0103	0.6294	9.6426	2.0158	0.958	79.5
1- 2	2630.0910	2630.1065	15.4909	0.0103	1.6651	9.6708	1.8760	0.964	81.0
3- 2	2164.6750	2164.6867	11.7218	0.0103	1.2272	9.6612	1.9249	0.962	80.5
3- 1	2025.8560	2025.8662	10.1539	0.0103	1.0558	9.6426	2.0158	0.958	79.5
3-1005	3992.9020	3992.9124	10.4153	0.0103	1.0805	9.6763	1.8473	0.965	81.2
1005- 3	3992.9110	3992.9124	1.4153	0.0103	0.1430	9.6763	1.8473	0.965	81.2
1005- 2	5820.3110	5820.3001	-10.8843	0.0103	1.1306	9.6876	1.7870	0.967	81.9
1005- 1	3541.1120	3541.1118	-0.1781	0.0103	0.0180	9.6636	1.9126	0.962	80.6
	g	g	cc			cc	cc		
1-1005	0.0000	-0.0003	-2.7034	0.0658	0.8551	3.1420	2.3091	0.649	40.8
1- 3	96.8912	96.8914	1.8048	0.0658	0.5623	3.1593	2.2854	0.656	41.4
1- 2	156.3580	156.3581	0.8986	0.0658	0.2792	3.1496	2.2987	0.652	41.0
2- 1	0.0000	-0.0002	-2.3728	0.0658	0.7416	3.1665	2.2754	0.659	41.6
2-1005	25.1708	25.1711	2.8340	0.0658	0.8905	3.1675	2.2740	0.660	41.7
2- 3	54.2303	54.2303	-0.4612	0.0658	0.1426	3.1621	2.2815	0.658	41.5
3- 2	0.0000	-0.0001	-1.2940	0.0658	0.4046	3.1372	2.3157	0.647	40.6
3- 1	86.3025	86.3027	1.7007	0.0658	0.5298	3.1570	2.2886	0.656	41.3
3-1005	155.5796	155.5796	-0.4067	0.0658	0.1261	3.1510	2.2968	0.653	41.1
1005- 3	0.0000	0.0004	3.6644	0.0658	1.1624	3.1775	2.2600	0.664	42.0
1005- 2	15.3611	15.3615	4.0722	0.0658	1.3019	3.1769	2.2609	0.664	42.0
1005- 1	33.8326	33.8318	-7.7365	0.0658	2.7856 *	3.1755	2.2629	0.663	42.0
	m	m	mm			mm	mm		
1005- 2	5820.3030	5820.3001	-2.8843	0.1524	1.6300	1.8350	1.7870	0.653	30.2
1005- 3	3992.9110	3992.9124	1.4153	0.1524	0.7909	1.7742	1.8473	0.664	27.9
1-1005	3541.1100	3541.1118	1.8219	0.1524	1.0730	1.7037	1.9126	0.664	25.3
1- 2	2630.1060	2630.1065	0.4909	0.1524	0.2755	1.7439	1.8760	0.663	26.8
1- 3	2025.8680	2025.8662	-1.8461	0.1524	1.1785	1.5803	2.0158	0.513	21.3
2- 3	2164.6860	2164.6867	0.7218	0.1524	0.4190	1.6898	1.9249	0.480	24.8
	g	g	cc			cc	cc		
1005- 2	106.6186	106.6185	-0.9969	1.0913	1.1735	0.8567	0.4270	0.801	55.4
1005- 3	91.2574	91.2574	-0.4047	1.0913	0.5099	0.7802	0.5546	0.664	42.1
1-1005	325.0887	325.0888	1.1945	1.0913	1.6748	0.7419	0.6049	0.601	36.8
1- 2	81.4471	81.4472	0.7964	1.0913	1.0885	0.7348	0.6135	0.589	35.9
1- 3	21.9805	21.9805	-0.2973	1.0913	0.4092	0.7128	0.6390	0.554	33.2
2- 3	335.6777	335.6777	-0.2920	1.0913	0.4266	0.6717	0.6820	0.492	28.8

v* - Outlier detection Ti ~ t(alfa,f-1) = 2.080
r* - Measurement uncontrollable for the presence of a gross error (r*<0.30)

2nd order PARAMETERS

Standard deviation of the bases measured electronically: 9.85 mm
Standard deviation of measured directions: 3.90 cc
Standard deviation of measured GPS bases: 2.56 mm
Standard deviation of measured GPS pointers: 0.96 cc

Tab. 4. Coordinates.

	X° [m]	Y° [m]	dX^ [mm]	dY^ [mm]	dC^ [mm]	X^ [m]	Y^ [m]	sX^ [mm]	sY^ [mm]	sxy [mm]	sp [mm]
1	1000.000	4272.239	-0.8412	-0.4074	0.9346	999.9992	4272.2386	1.4435	1.1988	1.33	1.88
2	1755.680	6791.446	-0.3081	0.2231	0.3804	1755.6797	6791.4462	1.8365	1.1107	1.52	2.15
3	2906.306	4957.891	2.0926	0.5098	2.1538	2906.3081	4957.8915	1.4495	1.1566	1.31	1.85
1005	2359.688	1002.572	-0.9433	-0.3255	0.9979	2359.6871	1002.5717	2.4965	1.1133	1.93	2.73

Points exceeding the limit of linearization

Significance level alpha chosen*	=	0.050
Number of critical measurements*	=	1
Standard deviation a priori	=	1.000
Critical limit s0_a posterior	=	1.242
s0_aposter^2/s0_aprior^2	=	1.000
Crit. ratio s0_aposter^2/s0_aprior^2	=	1.542
Average side length [m]	=	3274.215
Average coordinate error in the network [mm]	=	1.543
Average positional error in the network [mm]	=	2.182
Effectiveness of adjustment	=	0.786
Redundancy	=	22.000
tr(R)	=	22.000
Critical values of the distribution functions		
t(Alfa,f)	=	2.074
chi^2(Alfa,f)	=	33.924
Decrease of the rank of the configuration matrix	=	2

Tab. 5. Error ellipses.

Standard error ellipses				Confidence errors of ellipses For the probability 95.0 percent		
point	a [mm]	b [mm]	convolution(a) [g]	a [mm]	b [mm]	convolution(a) [g]
1	1.448	1.193	9.0190	3.800	3.131	9.0190
2	1.839	1.106	4.5280	4.827	2.902	4.5280
3	1.456	1.148	389.9889	3.822	3.013	389.9889
1005	2.508	1.088	6.7189	6.581	2.854	6.7189

Tab. 6. Free adjustment of the LGN: MLS after introducing the change of scale.

	Measurement	l	l^	v	pi	Ti	*	s(v)	s(l^)	r	f
		m	m	mm				mm	mm		
Terresstrial measurements	**1- 3**	2025.8600	2025.8645	4.5478	0.0100	0.4637		9.6230	2.7653	0.924	72.4
	1- 2	2630.0910	2630.1043	13.2855	0.0100	1.4347		9.4907	3.1898	0.899	68.1
	3- 2	2164.6750	2164.6849	9.8516	0.0100	1.0296		9.5824	2.9027	0.916	71.0
	3- 1	2025.8560	2025.8645	8.5478	0.0100	0.8836		9.6230	2.7653	0.924	72.4
	3-1005	3992.9020	3992.9091	7.1017	0.0100	0.7750		9.0758	4.2282	0.822	57.8
	1005- 3	3992.9110	3992.9091	-1.8983	0.0100	0.2043		9.0758	4.2282	0.822	57.8
	1005- 2	5820.3110	5820.2952	-15.8395	0.0100	2.1328	*	8.0298	5.9808	0.643	40.3
	1005- 1	3541.1120	3541.1088	-3.2146	0.0100	0.3421		9.1981	3.9553	0.844	60.5
		g	g	cc				cc	cc		
	1-1005	0.0000	-0.0003	-2.7071	0.0657	0.8559		3.1428	2.3098	0.649	40.8
	1- 3	96.8912	96.8914	1.8187	0.0657	0.5661		3.1603	2.2858	0.657	41.4
	1- 2	156.3580	156.3581	0.8884	0.0657	0.2757		3.1505	2.2992	0.652	41.0
	2- 1	0.0000	-0.0002	-2.3838	0.0657	0.7446		3.1674	2.2759	0.660	41.6
	2-1005	25.1708	25.1711	2.8246	0.0657	0.8870		3.1683	2.2747	0.660	41.7
	2- 3	54.2303	54.2303	-0.4408	0.0657	0.1361		3.1630	2.2820	0.658	41.5
	3- 2	0.0000	-0.0001	-1.2785	0.0657	0.3992		3.1380	2.3163	0.647	40.6
	3- 1	86.3025	86.3027	1.7089	0.0657	0.5318		3.1580	2.2890	0.656	41.3
	3-1005	155.5796	155.5796	-0.4304	0.0657	0.1333		3.1517	2.2976	0.653	41.1
	1005- 3	0.0000	0.0004	3.6564	0.0657	1.1598		3.1784	2.2605	0.664	42.0
	1005- 2	15.3611	15.3615	4.0737	0.0657	1.3031		3.1777	2.2615	0.664	42.0
	1005- 1	33.8326	33.8318	-7.7301	0.0657	2.8029	*	3.1764	2.2634	0.663	42.0
GPS measurements		m	m	mm				mm	mm		
	1005- 2	5820.3030	5820.3005	-2.5089	0.1559	1.4629		1.7610	1.8202	0.653	28.1
	1005- 3	3992.9110	3992.9128	1.7586	0.1559	1.0377		1.6979	1.8791	0.664	25.8
	1-1005	3541.1100	3541.1120	2.0286	0.1559	1.2429		1.6532	1.9186	0.664	24.2
	1- 2	2630.1060	2630.1067	0.6943	0.1559	0.3991		1.7048	1.8729	0.663	26.0
	1- 3	2025.8680	2025.8664	-1.5968	0.1559	1.0471		1.5285	2.0193	0.483	20.3
	2- 3	2164.6860	2164.6868	0.8341	0.1559	0.4944		1.6565	1.9157	0.449	24.4
		g	g	cc				cc	cc		
	1005- 2	106.6186	106.6185	-1.0045	1.0755	1.1738		0.8634	0.4296	0.802	55.5
	1005- 3	91.2574	91.2574	-0.4218	1.0755	0.5265		0.7872	0.5570	0.666	42.2
	1-1005	325.0887	325.0888	1.1917	1.0755	1.6551		0.7492	0.6072	0.604	37.0
	1- 2	81.4471	81.4472	0.7871	1.0755	1.0631		0.7427	0.6152	0.593	36.2
	1- 3	21.9805	21.9805	-0.2826	1.0755	0.3835		0.7216	0.6397	0.560	33.7
	2- 3	335.6777	335.6777	-0.2699	1.0755	0.3891		0.6796	0.6842	0.497	29.1

Change of scale for electro-optical rangefinders = 1.00000092
v* - Outlier detection; Ti ~ t(alfa,f-1) = 2.086
r* - Measurement uncontrollable for the presence of a gross error (r*<0.30)

The method of least squares found two outlier detections in measuring the direction from the standpoint ***1005-1*** and the distance from the standpoint ***1005-2***. After the adjustment of the network by this method we came to the conclusion that corrections in the terrestrially measured bases indicate a systematic trend the cause of which can be attributed to the insufficient introduction of the physical corrections taking into account the size of the network. Because of this reason the change of scale was also estimated in electro-optical rangefinders (Nevosád, Z. at al, 2002).

2nd order PARAMETERS

Standard deviation of the bases measured electronically: 10.01 mm
Standard deviation of measured directions : 3.90 cc
Standard deviation of measured GPS bases : 2.53 mm
Standard deviation of measured GPS pointers : 0.96 cc

Tab. 7 Coordinates.

	X° [m]	Y° [m]	dX^ [mm]	dY^ [mm]	dC^ [mm]	X^ [m]	Y^ [m]	sX^ [mm]	sY^ [mm]	sxy [mm]	sp [mm]
1	1000.000	4272.239	-0.9359	-0.4363	1.0326	999.9991	4272.2386	1.4446	1.1958	1.33	1.88
2	1755.680	6791.446	-0.3076	0.3780	0.4874	1755.6797	6791.4464	1.8455	1.1132	1.52	2.16
3	2906.306	4957.891	2.2165	0.6095	2.2988	2906.3082	4957.8916	1.4537	1.1598	1.31	1.86
1005	2359.688	1002.572	-0.9730	-0.5512	1.1183	2359.6870	1002.5714	2.5097	1.1361	1.95	2.75

Tab. 8. Error ellipses.

	Standard error ellipses			Confidence errors of ellipses For the probability 95.0 percent		
point	a [mm]	b [mm]	convolution(a) [g]	a [mm]	b [mm]	convolution(a) [g]
1	1.449	1.191	8.6914	3.815	3.135	8.6914
2	1.849	1.108	4.6732	4.868	2.917	4.6732
3	1.459	1.153	390.8976	3.842	3.035	390.8976
1005	2.522	1.109	6.9268	6.640	2.921	6.9268

As it is obvious from the output of the solution there was a positive adjustment of corrections in the distances measured electro-optically. Introducing the change of scale into the adjustment resulted in a better conditionality of the criteria matrix S where the number of conditionality cond(S) decreased from $4{,}828.10^3$ to $2{,}853.10^3$ and subsequently faster convergence of the variance components (average errors) of the devices used. The MLS revealed two outlier detections. In the case where the method reveals deviating measurements the standard procedure includes exclusion of such measurements from the file of the measured data.

ABSOLUTE CONFIDENCE ERROR ELLIPSES α=0,05 %

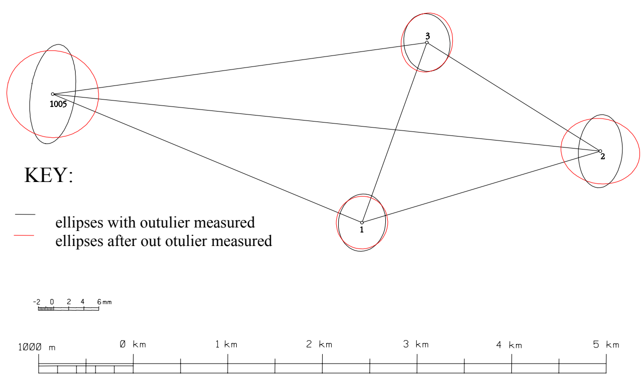


Fig.4. Absolute confidence error ellipses - Method of least squares.

Danish method

The Danish method is one of the most widely used robust methods because of its simplicity. The key feature of this method lies in the fact that it reduces (decreases) the influence of outlier (deviating) detections on the estimations of parameters. The method is purely *heuristic,* i.e. it is not based on stochastic and statistic theories. The principle of this method is based on indicating outlier detections by large corrections appertaining to them. After the standard adjustment of estimations of the first order parameters by the method of least quarters by means of Gauss – Markov model, the a priori weights of measurements are replaced by functions of corrections that results in the increase of absolute values of corrections of outlier detections and at the same time decrease of their deformation influence on the network geometry. The iteration cycle is repeated until the desired results are achieved. In conventional positional networks the solution requires not more than (10-15) iterations. At present various types of exponential functions are used for the weight function. The following weight function is used in practical solution:

$$p(v_i) = \begin{cases} 1 & pre \quad \dfrac{p_i|v_i|}{s_0} < c \\ exp\left(-\dfrac{|v_i p_i|}{cs_0}\right) & pre \quad \dfrac{p_i|v_i|}{s_0} > c \end{cases}, \tag{13}$$

p_i: of i- measurement , vi: correction of i- measurement, s0: a posterior variance factor.

with standard:
for the first step of iteration $a = 0$,
for the second and third step of iteration $a = 0.05$, and in the whole iteration process $b = 3, c = 3$.
The equation (13) is therefore defined by the interval

$$-c < |v_i| \sqrt{p_1} / s_0 < c \quad , \tag{14}$$

from which the individual weights of measurements start to be defined. The constant c is usually chosen between 2 and 3 and is dependent on the redundancy (abundance of measurements) determined by GMM and the quality of the measured data. In the case that the constant c < 2, the method used is robust, in the case that c>3 the processing method is changed to the MLS. Taking into account the vast amount of calculation involved only parameters of the 2nd order estimated by the respective method as well as coordinates of the individual points and absolute confidence ellipses are given in the following outputs

Free adjustment LGN: Danish method

2nd order PARAMETERS

Standard deviation of the bases measured electronically: 3.98 mm
Standard deviation of measured directions : 2.29 cc
Standard deviation of measured GPS bases : 2.40 mm
Standard deviation of measured GPS pointers : 0.63 cc

Tab. 9. Coordinates.

-----	X° [m] ----	Y° [m]---	dX^ [mm] ---	dY^ [mm] --	dC^ [mm] -*-	X^ -[m] ------	Y^ [m] ----	sX^ [mm] –	sY^ [mm]---	sxy sp [mm]–[mm]
1	1000.000	4272.239	-1.6864	-0.5605	1.7771	999.9983	4272.2384	1.2590	1.0004	1.141.61
2	1755.680	6791.446	-0.3368	0.2859	0.4418	1755.6797	6791.4463	1.4201	1.0134	1.231.74
3	2906.306	4957.891	2.0140	0.6125	2.1051	2906.3080	4957.8916	1.1410	0.9330	1.041.47
1005	2359.688	1002.572	0.0092	-0.3379	0.3380	2359.6880	1002.5717	2.1939	1.0847	1.732.45

Tab. 10. Error ellipses.

Standard error ellipses				Confidence errors of ellipses For the probability 95.0 percent		
point ----------	a [mm] ---	b [mm] -------	convolution(a) [g] ---	a ----[mm] -----	b [mm] ---	convolution(a) [g] –
1	1.306	0.938	25.0535	3.440	2.469	25.0535
2	1.420	1.013	1.9394	3.740	2.667	1.9394
3	1.146	0.927	389.9487	3.017	2.441	389.9487
1005	2.212	1.047	9.3324	5.825	2.756	9.3324

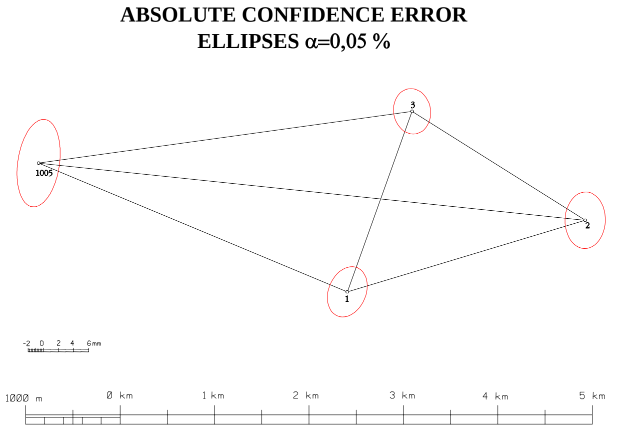
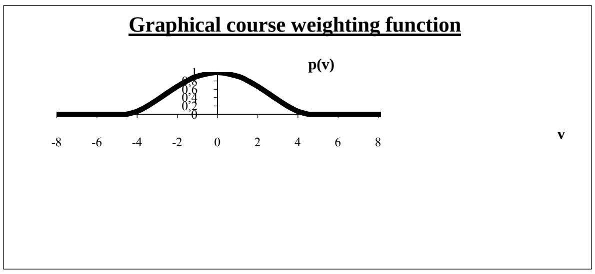


Fig. 5. Absolute confidence error ellipses - Danish method.

In adjustment the particular geodetic network, the relationship 13 defined by the interval 14 dependent on the so-called *damping constant c* which is chosen by the designer was used for calculation of individual weights. After the network adjustment in the first cycle two deviating measurements (length ***1005-2*** and direction ***1005-1***) were also detected. In further cycles the weights of measurements began to decrease. Iteration process stopped in the fifth cycle at the damping constant c=1.57.

Robust M- estimation according to Biweight

For this estimation the following constants are used:
weighting function



$$p(v)=\begin{cases}\left(1-\left(\frac{v}{a}\right)^2\right)^2 & |v|\le a\\ 0 & |v|>a\end{cases}$$

Fig. 6. Graphical course weighting function.

Free adjustment LGN: M – estimation according to Biweight

2nd order PARAMETERS

Standard deviation of the bases measured electronically: 3.19 mm
Standard deviation of measured directions : 1.71 cc
Standard deviation of measured GPS bases : 2.03 mm
Standard deviation of measured GPS pointers : 0.62 cc

Tab. 11. Coordinates.

	X° [m]	Y° [m]	dX^ [mm]	dY^ [mm]	dC^ [mm]	*	X^ [m]	Y^ [m]	sX^ [mm]	sY^ [mm]	sxy [mm]	sp [mm]
1	1000.000	4272.239	-1.3854	-0.4544	1.4580		999.9986	4272.2385	1.2097	0.9443	1.09	1.5
2	1755.680	6791.446	-0.6736	0.2044	0.7039		1755.6793	6791.4462	1.4749	0.9722	1.25	1.7
3	2906.306	4957.891	2.0423	0.4016	2.0814		2906.3080	4957.8914	1.0922	0.8860	0.99	1.4
1005	2359.688	1002.572	0.0167	-0.1516	0.1525		2359.6880	1002.5718	2.1675	1.0904	1.72	2.4

Tab. 12. Error ellipses.

Standard error ellipses				Confidence errors of ellipses For the probability 95.0 percent		
point	a	b	convolution(a)	a	b	convolution(a)
------	[mm]	---------[mm]	---- [g] ----	---------[mm]	------- [mm]	---- [g] ----
1	1.230	0.918	17.4338	3.238	2.417	17.4338
2	1.475	0.972	0.0681	3.884	2.560	0.0681
3	1.098	0.879	388.8753	2.892	2.314	388.8753
005	2.182	1.061	8.4690	5.746	2.793	8.4690

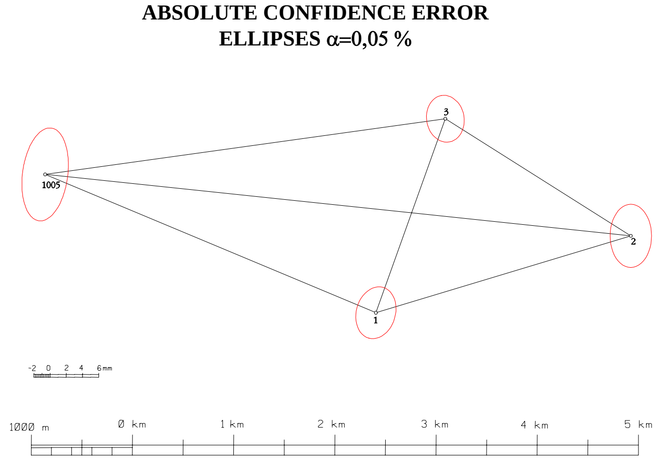


Fig. 7. Aabsolute confidence error ellipses- Biweight.

This method stopped in the fourth iteration cycle using the damping constant c=4.685.

Robust M-estimation according to Welsch

Weighting function:

$$p(v) = e^{-(v/a)^2}$$

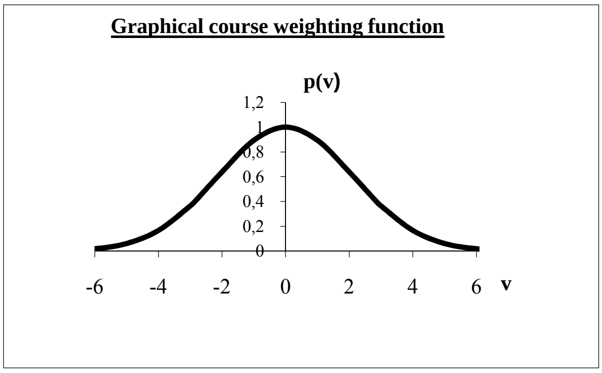


Fig. 8. Graphical course weighting function.

Free adjustment LGN: M – estimation according to Welsch

2nd order PARAMETERS

Standard deviation of the bases measured electronically: 3.12 mm
Standard deviation of measured directions : 1.70 cc
Standard deviation of measured GPS bases : 1.95 mm
Standard deviation of measured GPS pointers : 0.58 cc

Tab. 13. Coordinates

	X°	Y°	dX^	dY^	dC^	X^	Y^	sX^	sY^	sxy	sp
------	[m] -----	[m] ----	[mm] --	[mm] -	[mm] -*---	[m]-------	[m]-- --	[mm]---	[mm]--	[mm]--	[mm]-
1	1000.000	4272.239	-1.4353	-0.5024	1.5207	999.9986	4272.2385	1.1982	0.9173	1.07	1.51
2	1755.680	6791.446	-0.6779	0.1320	0.6906	1755.6793	6791.4461	1.4276	0.9520	1.21	1.72
3	2906.306	4957.891	2.0245	0.4853	2.0818	2906.3080	4957.8915	1.0614	0.8578	0.96	1.36
1005	2359.688	1002.572	0.0887	-0.1148	0.1451	2359.6881	1002.5719	2.1219	1.0867	1.69	2.38

* Points exceeding the points of linearization

Tab. 14. Error ellipses.

Standard error ellipses				Confidence errors of ellipses For the probability 95.0 percent		
point	a [mm]	b [mm]	convolution(a) [g]	a [mm]	b [mm]	convolution(a) [g]
1	1.217	0.892	16.6105	3.205	2.349	16.6105
2	1.428	0.952	399.2342	3.759	2.507	399.2342
3	1.066	0.852	390.3326	2.806	2.244	390.3326
1005	2.137	1.057	8.6604	5.627	2.784	8.6604

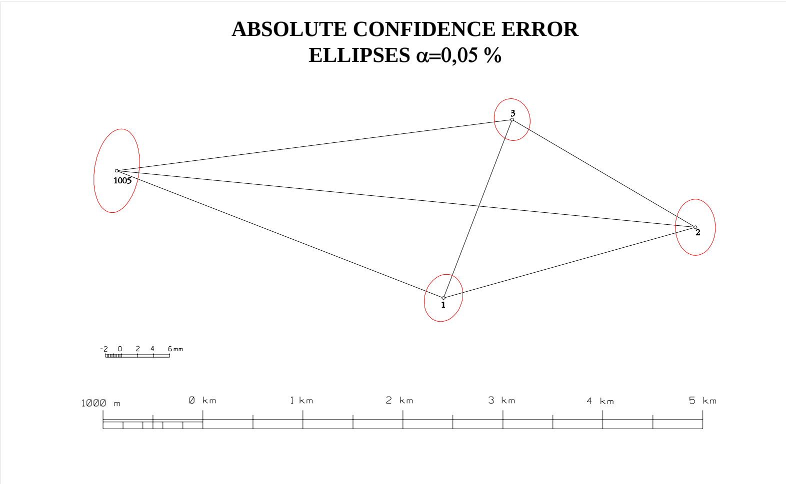


Fig. 9. The absolute confidence error ellipses – Welsch.

Tab. 15. Results of adjustment.

	Measurand	MLS v	MLS pi	DANISH METHOD v	DANISH METHOD pi	BIWEIGHT v	BIWEIGHT pi	WELSCH v	WELSCH pi
TERRESTRIAL		mm		mm		mm		mm	
	1- 3	4.5478	0.0100	5.8820	0.0023	5.3933	0.0147	5.5169	0.0130
	1- 2	13.2855	0.0100	14.5303	0.0000	14.0052	0.0000	14.0584	0.0000
	3- 2	9.8516	0.0100	10.5074	0.0001	10.6777	0.0000	10.5905	0.0000
	3- 1	8.5478	0.0100	9.8820	0.0001	9.3933	0.0000	9.5169	0.0001
	3-1005	7.1017	0.0100	8.2596	0.0022	7.6230	0.0115	7.7535	0.0108
	1005- 3	-1.8983	0.0100	-0.7404	0.0632	-1.3770	0.0890	-1.2465	0.0903
	1005- 2	-15.8395	0.0100	-13.8105	0.0003	-14.3997	0.0007	-14.3600	0.0007
	1005- 1	-3.2146	0.0100	-1.5056	0.0632	-1.9104	0.0694	-1.8565	0.0666
		cc		cc		cc		cc	
	1-1005	-2.7071	0.0657	-2.4379	0.1901	-2.6903	0.1378	-2.7118	0.1196
	1- 3	1.8187	0.0657	1.7563	0.1901	1.4824	0.2309	1.4703	0.2152
	1- 2	0.8884	0.0657	0.6816	0.1901	0.6072	0.3141	0.5474	0.3100
	2- 1	-2.3838	0.0657	-2.5222	0.1901	-2.3108	0.1680	-2.3069	0.1495
	2-1005	2.8246	0.0657	2.9647	0.1901	3.0818	0.1185	3.1075	0.1024
	2- 3	-0.4408	0.0657	-0.4425	0.1901	-0.2952	0.3357	-0.2580	0.3358
	3- 2	-1.2785	0.0657	-1.3513	0.1901	-1.2351	0.2808	-1.2285	0.2717
	3- 1	1.7089	0.0657	1.6437	0.1901	1.6245	0.2379	1.6455	0.2231
	3-1005	-0.4304	0.0657	-0.2924	0.1901	-0.2588	0.3370	-0.2581	0.3366
	1005- 3	3.6564	0.0657	1.4123	0.0075	-0.0385	0.0933	0.1176	0.0743
	1005- 2	4.0737	0.0657	1.7606	0.0052	0.3623	0.0651	0.5127	0.0502
	1005- 1	-7.7301	0.0657	-9.8458	0.0002	-11.3278	0.0000	-11.1609	0.0000
GPS		mm		mm		mm		mm	
	1 005- 2	-2.5089	0.1559	-2.7077	0.1733	-2.9384	0.0778	-3.0390	0.0696
	1005- 3	1.7586	0.1559	1.3882	0.1733	0.9976	0.1540	1.0318	0.1494
	1-1005	2.0286	0.1559	2.3822	0.1733	2.1955	0.1293	2.1640	0.1225
	1- 2	0.6943	0.1559	0.9324	0.1733	0.5694	0.2208	0.5591	0.2361
	1- 3	-1.5968	0.1559	-1.0380	0.1733	-1.4019	0.1693	-1.3272	0.1706
	2- 3	0.8341	0.1559	0.6614	0.1733	0.9650	0.2137	0.8256	0.2278
		cc		cc		cc		cc	
	1005- 2	-1.0045	1.0755	-0.8910	0.0237	-0.8505	0.0885	-0.8409	0.0744
	1005- 3	-0.4218	1.0755	-0.2393	2.4856	-0.2513	1.9269	-0.2361	2.0265
	1-1005	1.1917	1.0755	1.5026	0.0494	1.4594	0.2340	1.4855	0.1980
	1- 2	0.7871	1.0755	0.6221	2.4856	0.7569	1.0105	0.7447	0.9572
	1- 3	-0.2826	1.0755	-0.3032	2.4856	-0.3679	2.3301	-0.3324	2.5631
	2- 3	-0.2699	1.0755	-0.2982	2.4856	-0.2275	2.3742	-0.2065	2.6334

The above-mentioned method found two outlier detections in the first cycle and stopped in the fourth iteration cycle. Table 19 gives the results of the adjustment directions measured terrestrially, electro-optically measured lengths and GPS vectors reduced to bases and pointers in the positional computational space in the cartographic plane.

Tab. 16. 2nd order parameters estimated by individual methods.

	MLS	Danish	BIWEIGHT	WELSCH
Standard deviation of the bases measured electronically [mm]	10.01	3.98	3.19	3.12
Standard deviation of measured directions [cc]	3.90	2.29	1.71	1.70
Standard deviation of measured GPS bases [mm]	2.53	2.40	2.03	1.95
Standard deviation of measured GPS pointers [cc]	0.96	0.63	0.62	0.58

Tab. 17. Adjustment coordinates complemets.

	MLS	Danish	BIWEIGHT	WELSCH
	dC^[mm]	**dC**^[mm]	**dC**^[mm]	**dC^**[mm]
1	1.0326	1.7771	1.4580	1.5207
2	0.4874	0.4418	0.7039	0.6906
3	2.2988	2.1051	2.0814	2.0818
1005	1.1183	0.3380	0.1525	0.1451

Conclusion

The objective of the submitted paper is to compare the method of least squares with some types of robust estimation procedures. The following procedures: M-estimation according to Biweight, M-estimation according to Welsch and the Danish method were chosen from a number of estimation procedures published in the foreign literature and compared with the MLS MatLab programme version 5.2 T was used to implement the mathematical adjustment. **Table 15** shows the conformity of the individual tested robust estimation methods and their effect on the corrections with respect to the MLS. Based on the results obtained in the processing of this experimental geodetic network it seems reasonable to suggest that the adjustment methods used produce comparable results shown not only in the above-mentioned Table but also in the similar graphic course of the functions of individual estimation procedures. All the methods stopped after the fourth iteration cycle whereas the biggest corrections took place in the measurements of directions from the standpoint 1005-1 and measurement of the length from the standpoint **1005-2.**

In spite of the fact that the standard method used in geodetic surveying is the MLS it is necessary to pay sufficient attention also to alternative estimation methods. The current significance of the topic is supported by the fact that the use of unconventional estimation procedures in the recent years has been more and more often discussed also in foreign literature (Jager et all, 2005). These methods are applicable mainly in the cases where it is not possible to influence the intersection of outlying-deviating measurements to the file of measured data. Though in the paper these methods are presented only on the positional geodetic network, they can be applied in other fields of geodetic survey such as deformation survey.

*Tento článok bol vytvorený realizáciou projektu **Centrum excelentného výskumu získavania a spracovania zemských zdrojov,** na základe podpory operačného programu **Výskum a vývoj** financovaného z Európskeho fondu regionálneho rozvoja a grantového projektu číslo 1/0786/10 Vedeckej grantovej agentúry MŠ SR **Výskum dynamiky ľadovej výplne jaskynných priestorov bezkontaktnými metódami z hľadiska ich bezpečného a trvalo udržateľného využívania ako súčasti prírodného dedičstva Slovenskej republiky**.*

References

[1] Bajtala, M., Sokol, Š: Odhad variančných komponentov z meraní v geodetickej sieti. *Acta Montanistica Slovaca, roč.2, č. 10, s. 68-77, Košice, 2005.*

[2] Bitterer, L.: Vyrovnávací počet. *Žilinská univerzita, Žilina, 2006.*

[3] Böhm, J., Radouch, V., Hampacher, M.: Teorie chyb a vyrovnávací počet, Praha, *Geodetický a kartografický podnik, 1990.*

[4] Buchar, P, Hojovec, V.: Matematická kartografie 10, *ČVUT, Praha, 1996*.
[5] Cebecauer, D.: Kombinované použitie metód najmenších štvorcov, najmenšej sumy absolútnych hodnôt opráv a MINIMAX, *Geodetický a kartografický obzor, 40 (82), 1994 č.11, s. 221-225*.
[6] Daniš, M.: Matematická kartografia, *Stavebná fakulta, Bratislava, 1976*.
[7] Dobeš, J. a kol.: Presné lokálne geodetické siete, *Výskumný ústav geodézie a kartografie, Bratislava 1990*.
[8] Gašincová, S.: Spracovanie 2D sietí pomocou robustných metód. *Doktorandská dizertačná práca. Košice 2007, str.91*.
[9] Hampel, F. et al: Robust statistics, the approach based of influence functions. *J.Willey&Sons, New York, 1986*.
[10] Hojovec, V.: Kartografie, Geodetický a kartografický podnik, *Praha, 1987*.
[11] Huber, P. J.: Robust estimation of a location parameter. *Ann Math Stat 35: 73-101, 1964*.
[12] Hekimoglu, S., Berber, M.: Effectiveness of robust methods in heterogeneous linear models., *Journal of Geodesy, 76, 706-713, 2003*.
[13] Jäger, R., Műlle, T., Saler, H. Schväble, R.: Klassischle und robuste Ausgleichungsverfahren, *Herbert Wichmann Verlag, Heidelberg, 2005*.
[14] Leick, A.: GPS satellite surveying. *John Wiley & Sons, 560 s ,1995*.
[15] Lucas, J.R., Dillinger, W. H.: MINQUE for block diagona bordered systems such as those encountered in VLBI data analysis. *Journal of Geodesy, 72, 343-349, 1998*.
[16] Pick, M.: Geodézie. Súradnicové systémy a zobrazení, STU, *Bratislava, 1998*.
[17] Nevosád, Z.,Vitásek, J., Bureš, J.: Geodézie IV. Souřadnicové výpočty, *CERM, s.r.o Brno, 2002*.
[18] Sabová, J., Jakub, V. Geodetické deformačné šetrenie. ES/AMS, Fakulta BERG TU v Košiciach, *ISBN 978-80-8073-788-7, Košice, 2007*.
[19] Weiss, G., Sűtti, J.: Geodetické lokálne siete I, *Košice 1997*.
[20] Weiss,G., Labant, S., Weiss, E., Mixtaj, L., Schwarczová, H.: Establishment of Local Geodetic Nets. *Acta Montanistica Slovaca, Ročník 14 (2009), číslo 4, 306-313, ISSN 1335-1778*.
[21] Weiss G., Jakub, V., Weiss. E.: Kompatibilita geodetických bodov a jej overovanie. *TU Košice, 2004, ISBN: 80-8073-149-7*.