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A comparative study between MP and DLQ control strategies for a flexible link

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In this paper we developed a control scheme for a Quanser Flexible Link experiment. The purpose of the paper is to design a controller which reduces (eliminates) the undamped oscillations of the tip of the beam while commanding the tip to a desired position. First, we design a control scheme with a model predictive controller. Model predictive control (MPC) is a well-known technique used to (sub)optimally control of dynamical processes. Then, we design a Digital Linear Quadratic Regulator (DLQR) and we compare the results obtained by MPC scheme with those obtained by using the DLQR control scheme.

Key words: *Quanser Flexible Link, MPC, DLQR, Optimal Control.*

Introduction

The Flexible Link module consists of a Quanser DC servomotor and a Flexgage module.

For typical tracking plus regulation problem – move the link within a range of angles, while maintaining a small deflection at the tip - we developed two control schemes, one with a model predictive controller and other with DLQ controller. A predictive control algorithm solves an on-line and optimal control problem subject to system dynamics and variable constraints. The neglected nonlinearities of the real experiment in the controller design step lead to important differences between the simulation and experimental results, including vibrations of the real beam and/or stationary errors due to insensibility zone. In order to eliminate these tracking errors and reduce the vibrations of the beam we used an improved control law by using some blocks with variable structure.

A Digital Linear Quadratic Regulator yield a state feedback that can achieve the stabilization (regulation) as well as minimizing a performance index. In order to eliminate the tracking error, we use an integral action additionally with a feedback control.

Simulation and experimental results are included in order to illustrate the performances of these controllers.

Quanser Flexible Link Experiment

Description of the system

The SRV02 rotary plant module serves as the base component for the Quanser rotary family of experiments.

Measurement of the flexible arm deflection α is obtained using a strain gage at the motor end of the link.

For the measurement of the angular position of the shaft θ an optical encoder attached to the shaft of the DC motor is used. Consider the Flexible Link schematic shown in fig.1.

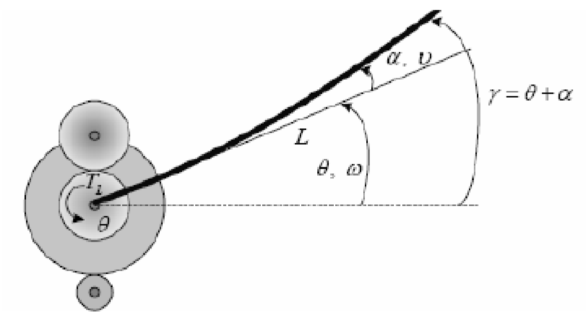

Fig. 1. Schematic representation of the Flexible Link.

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Mathematical model of the system

We consider for the combined servomotor and the flexible link module the state variables $x(t) = [\theta \quad \alpha \quad \omega \quad \upsilon]^T$, where $\omega = \dot{\theta}$, $\upsilon = \dot{\alpha}$ and obtain the following state-space model:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{\omega} \\ \dot{\upsilon} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{stiff}}{J_{eq}} & -\frac{\eta K_m^2 K_g^2 + B_{eq} R_a}{J_{eq} R_a} & 0 \\ 0 & -\frac{K_{stiff}(J_{eq} + J_{arm})}{J_{eq} J_{arm}} & \frac{\eta K_m^2 K_g^2 + B_{eq} R_a}{J_{eq} R_a} & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \omega \\ \upsilon \end{bmatrix} + \tag{1}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{\eta K_m K_g}{J_{eq} R_a} \\ -\frac{\eta K_m K_g}{J_{eq} R_a} \end{bmatrix} v_i$$

where R_a is the armature resistance, K_m is the motor voltage constant, J_{arm} is the link moment of inertia, K_g high gear ratio, K_{stiff} is the equivalent spring constant, J_{eq} is equivalent moment of inertia, B_{eq} equivalent viscous friction and v_i is the input voltage.

Model Predictive Control

The first control type used is MPC. The benefits compared to general controllers are it's ability to handle constraints on the control and output signals. The model predictive control system uses model based predictions of the process outputs to manipulate the process inputs in such a way that deviations from setpoints are minimized, subject to constraints on inputs and outputs. Consider the discret-time system model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{2}$$

where $x(k) \in R^{n_x}$ are the states, $u(k) \in R^{n_u}$ are manipulated inputs and $y(k) \in R^{n_y}$ are the measured outputs.

The cost function used in MPC is

$$J(k) = \sum_{i=1}^{N_p} \left\| \hat{y}(k+i/k) - r(k+i) \right\|_{Q(i)}^2 + $$

$$+ \sum_{i=1}^{N_u} \left\| \Delta u(k+i-1) \right\|_{R(i)}^2 \tag{3}$$

subject to constraints specified on the inputs, outputs and inputs increments:

$$\begin{aligned} u_{min} &\le u(k) \le u_{max} \\ y_{min} &\le y(k) \le y_{max} \\ \Delta u_{min} &\le \Delta u(k) \le \Delta u_{max} \end{aligned}$$

where: $Q(i)$ - positive definite error weighting matrix; $R(i)$ - positive semi-definite control weighting matrix; $\hat{y}(k+i/k)$ - vector of predicted output signals; $r(k+i)$ - vector of future set-point; $\Delta u(k+i)$ - vector of future control actions; N_p - prediction horizon; N_u - control horizon.

The control law has regular form:

$$\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u-1) \end{bmatrix} = K_{MPC} e(k) = \begin{bmatrix} K_0 \\ K_1 \\ \vdots \\ K_{N_u-1} \end{bmatrix} e(k) \tag{4}$$

where $e(k)$ is the tracking error.

DLQ Control

We consider the discrete-time system (2) with (*A, B*) a controllable pair. The DLQ control law which minimizes the following LQ performance index:

$$J(x_i, u_i) = \sum_{i=1}^{\infty}[x_i^T Q x_i + u_i^T R u_i] \tag{5}$$

is given by

$$u_k = -(R + B^T PB)^{-1} B^T PAx_k = -Kx_k \tag{6}$$

where P is the positive definite solution of the following algebric Riccati equation:

$$P = A^T PA + Q - A^T PB(R + B^T PB)^{-1} B^T PA \tag{7}$$

Experimental results

MP controller

In MP controller case, we consider the following input data:

- control horizon: $N_u = 4$
- prediction horizon: $N_P = 21$
- sample time: $Ts = 0.2$
- input constraints: $-10 \le u(k) \le 10$

K_{MPC} from relation (4) have particular form:

$$K_{MPC} = [K_r \ K_x]^T$$

where:

K_r - reference gain; K_x - state gain; K_v - measured disturbance gain;

K_{MPC} is computed using an internal model of the plant with discretization form of the continuous system (1):

$$K_{MPC} = [3.9757 \ -0.0001 \ -4.1878$$
$$0.2957 \ -0.2949 \ -0.1897 \ -3.9757 \ 0.0001]^T$$

DLQ controller

In DLQ controller case, we used a dynamic state feedback control, for tracking reference and disturbance rejection.

The dynamic state feedback control includes integrated control error as part of state.

$$w_{k+1} = w_k + e_k \tag{8}$$

Augmented state space representation of the system is:

$$\begin{bmatrix} x_{k+1} \\ w_{k+1} \end{bmatrix} = \hat{A}\begin{bmatrix} x_k \\ w_k \end{bmatrix} + \hat{B}u_k + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_k$$
$$y_k = \hat{C}x_k \tag{9}$$

where:

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 1 \end{bmatrix}; \ \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}; \ \hat{C} = C$$

Discrete matrices A, B, C are obtained by using a ZOH discretization of continuous system (1) with sample time $T_S = 0.2$.

The combined control law is:

$$u_k = -[K \ K_I]\begin{bmatrix} x_k \\ w_k \end{bmatrix} \tag{10}$$

The output performance weighting matrix Q and the control increment matrix R are chosen with the particular form:

$$Q = diag([0.01 \ 0.1 \ 0.01 \ 0.1 \ 1]) \quad R = 1$$

With this input data, K, K_I has following form: $K = [2.4101 \ 0.1752 \ 0.1115 \ 0.0708]$

$K_I = [-0.6339]$

Variable structure part

In both cases we used a linear model of the nonlinear system. In order to compensate the neglected nonlinearities of the real plant, we add to our standard control scheme a variable structure part. The tuning parameters that influence the controllers are the thresholds of the two switches (one for the stationary error and the other to compensate the dead zone of the plant).

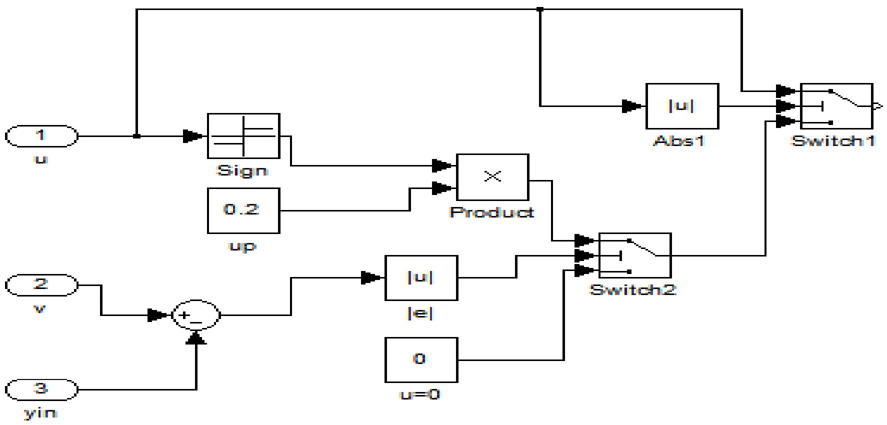

Fig. 2. Variable structure part of the controller.

The structure implements the following algorithm:

$$\textbf{\textit{if}}\ \left|e(k)\right| \geq 0.3,\quad y(k)_{Switch2} = 0.2 * sgn(u(k))$$

$$\textbf{\textit{else}}\ \ y(k)_{Switch2} = 0$$

$$\textbf{\textit{if}}\ \left|u(k)\right| \geq 0.3,\quad y(k)_{switch1} = u(k)$$

$$\textbf{\textit{else}}\ \ y(k)_{Switch1} = y(k)_{Switch2}$$

To illustrate necessity of previous algorithm, we use a control scheme which includes a DLQ controller. Fig. 3 illustrate angular position of the shaft θ, and input u obtained by use a real time scheme without variable structure. Output signal follows the reference (square waveform), but we observe a dead zone in term of command, which influence the overall performance.

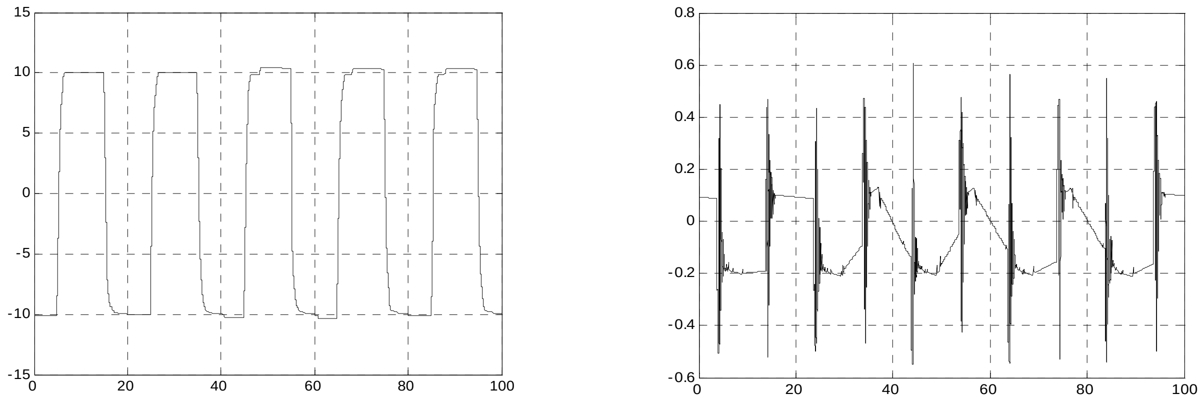

Fig. 3. DLQ experimental response without variable structure part of the controller.

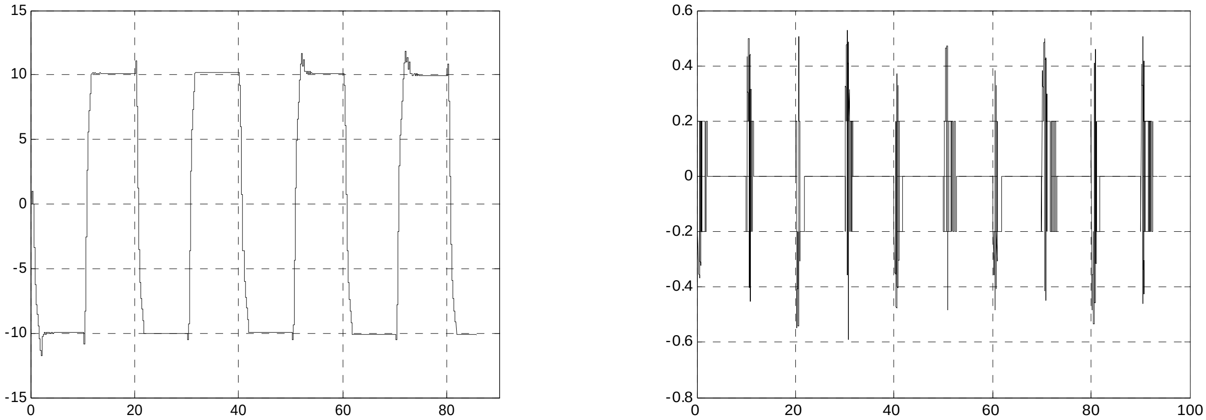

Fig. 4. DLQ experimental response with variable structure part of the controller.

With variable structure part, added to controller, experimental response is presented in fig. 4.

We remark a good tracking performance and compensation of the dead zone.

Finally, we presented in Fig. 5, the experimental response obtain by using a MPC controller with variable structure part.

It remarks a good tracking performance, a faster response time and no overshoot.

Of course, by using the variable structure, we accept a maximum position error ($e = 0.3$).

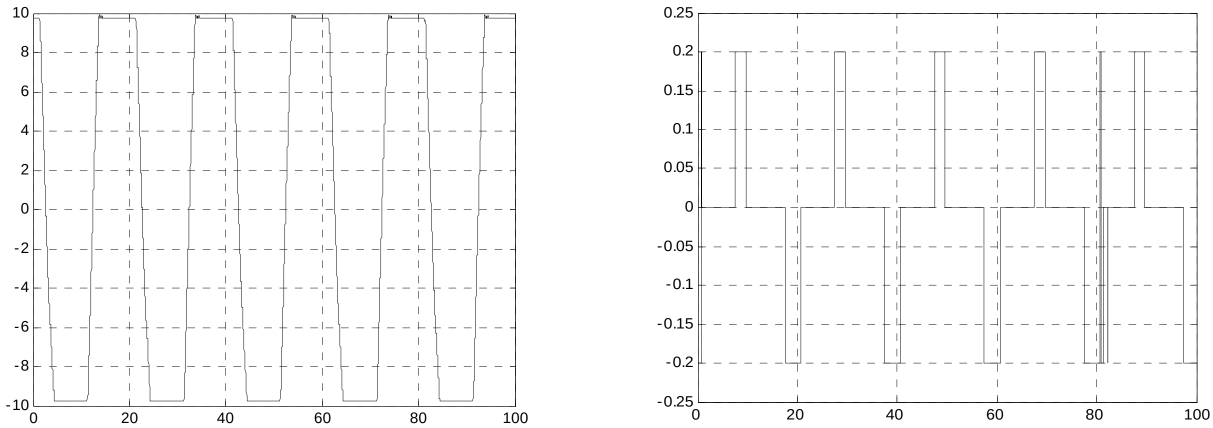

Fig. 5. MPC experimental response with variable structure part of the controller.

Conclusion

In this paper we presented a Quanser real-time experiment: flexible link.

For control scheme we proposed two controllers: MP and DLQ.

In order to compensate nonlinearities of the real experiment we completed our control scheme by adding a variable structure part.

From experimental results, we observed a good performance tracking in both cases, but presence of overshooting in DLQ controller case. With MPC scheme we obtained a robust control.

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