Optimal Location of an Active Segment of Magnetorheological Fluid Layer in a Sandwich Plate

Jacek Snamina [1]

Abstract: In the present study a sandwich plate, of rectangular form, with magnetorheological (MR) fluid core is investigated. The plate consists of two outer layers made of aluminium and an MR fluid layer in between. The plate edges are clamped. It was assumed that the aluminium layers are pure elastic. The energy is dissipated in MR fluid layer. Additional assumptions concerning displacements, deformations and stresses are introduced in calculations. The active segment of MR fluid layer is placed in various parts of plate. The optimal positions of active segment for selected modes of vibration are determined.

Keywords: *Magnetorheological fluid, sandwich plate, optimal damping*

Introduction

Magnetorheological (MR) fluids provide significant and rapid changes of damping and stiffness under magnetic field influence [1, 2, 3]. Thanks to these properties MR fluids are increasingly used in semi-active vibration control systems. Applications of controllable MR damping devices are described in a number of studies. The most popular of such devices are MR dampers that can be mounted at selected locations of structures.

The MR fluids have also been implemented to achieve controllable layer between two elastic layers. This method of vibration reduction in continuous structures is effective over a broad range of frequencies through variations in distributed stiffness and damping properties in response to the applied magnetic field. In papers[4, 5] authors investigated the dynamic response of MR sandwich beam. The above studies considered fully treated beam structures with MR fluid layer over the entire beam length. Alternatively, MR fluid may be applied only over more critical section of beam to achieve more efficient damping of vibration. While a number of studies analyze sandwich beam structures, the applications of controllable fluids in sandwich plates are explored in a very few works [6].

In this study the sandwich plate, of rectangular form, with MR fluid layer is investigated. The aim of the work is do find the optimal placement of active segment of MR layer in order to eliminate vibrations with selected modes. Calculations of energy dissipated by MR layer are the base to solve optimization problems.

Properties of MR-fluid in Pre-yield Regime

MR fluid can operate in two main regimes, known as pre-yield regime and post-yield regime [2]. In the pre-yield regime the MR fluid behaves like a solid gel with visco-elastic properties. The stress in MR fluid is below the yield stress. The rheological behaviour of MR fluid in the post-yield regime is usually described by the visco-plastic model. The yield stress depends on the magnetic field and the properties of MR fluid carrier.

In the pre-yield regime, the MR fluid demonstrates a visco-elastic behaviour. The stress and strain are related using the complex shear modulus. The complex modulus G represents simultaneously the stiffness and damping properties of MR fluid. This modulus can be used only for harmonic motion. In this case displacements, strains and stresses can be described by complex exponential expressions. The real part of complex shear modulus is called storage modulus G_s and the imaginary part is called loss modulus G_l. The ratio of loss modulus to storage modulus is called the loss factor.

The relationship between magnetic field and complex shear modulus depends on the type of MR fluid. It is determined on the base of experimental data. In further consideration we use the following relationships for the moduli G_s and G_l vs. the magnetic field strength value H taken from [3]:

$$\begin{aligned} G_s &= \left(5\cdot 10^{-11}\cdot H^2 + 4.5\cdot 10^{-6}\cdot H + 5.78\cdot 10^{-1}\right)\cdot 10^6 \\ G_l &= \left(5.5\cdot 10^{-13}\cdot H^2 + 4.8\cdot 10^{-8}\cdot H + 6.31\cdot 10^{-3}\right)\cdot 10^6 \end{aligned} \tag{1}$$

Analysis of strain

We consider small transverse vibrations of the plate presented in Fig. 1. The geometric parameters of the beam are l_1, l_2 - lengths of the sides, h and h_0 - thickness of outer layers and the MR fluid layer.

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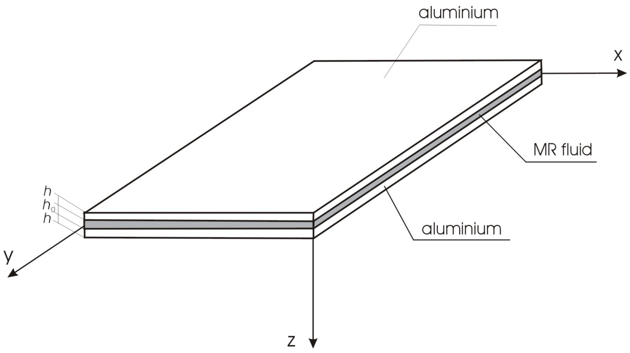

Fig. 1: Schematic diagram of plate.

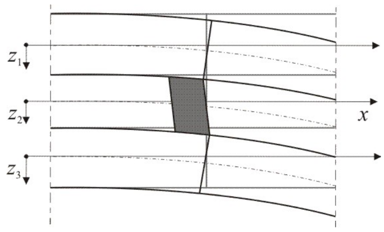

Fig. 2: Transversal section of plate.

The description of the motion of a three-layered plate is based on the following assumptions:

- all points placed on normal to the plate in its equilibrium state move with the same transverse displacement,
- aluminium layers are pure elastic, the energy is not dissipated in outer layers,
- there is no slip at the interface of MR fluid and outer layers,
- longitudinal direct stresses in the MR fluid layer are negligible even though the longitudinal direct strain is present,
- MR fluid layer undergoes shear, but no direct stress,

The transversal section of all layers in the non-deformed state and deformed state of plate is shown in Fig. 2. Additionally the coordinate systems used are depicted in this figure.

Due to the displacements of the points placed on the aluminium surfaces adjacent to MR layer, segments of MR fluid layer deform in shear. The deformed segment of the MR fluid layer is shown by the shaded area in Fig. 2. Analogical deformations are observed in yz plane.

The tangent forces between MR fluid layer and aluminium layers are small. Therefore we assumed that outer layers are not stretched (only bending deformations are present in aluminium layer).

The Lagrangian specification of displacement field is used in the next considerations. The motion of plate can be described in terms of vertical displacement $w(x,y,t)$.

Points given by the co-ordinate z_1 in the upper aluminium layer move according to the displacement vector $\vec{u}_1$. The components of $\vec{u}_1$ are:

$$\begin{cases} (u_1)_x = -z_1 \frac{\partial w}{\partial x} \\ (u_1)_y = -z_1 \frac{\partial w}{\partial y} \\ (u_1)_z = w(x,t) \end{cases} \tag{2}$$

Similarly the components of vector $\vec{u}_3$ describing the displacement of the points given by co-ordinate z_3 in the lower aluminium layer are:

$$\begin{cases} (u_3)_x = -z_3 \frac{\partial w}{\partial x} \\ (u_3)_y = -z_3 \frac{\partial w}{\partial y} \\ (u_3)_z = w(x,t) \end{cases} \tag{3}$$

Taking into account that there are no slips at the interfaces of the MR fluid layer and outer layers, we assume the following displacement $\vec{u}_2$ in the MR fluid layer:

$$\begin{cases} (u_2)_x &= z_2 \frac{h}{h_0} \frac{\partial w}{\partial x} \\ (u_2)_y &= z_2 \frac{h}{h_0} \frac{\partial w}{\partial y} \\ (u_2)_z &= w(x,t) \end{cases} \tag{4}$$

The components of strain in each layer can be calculated using the known strain-displacement relations. The strain components can be expressed in terms of the transverse displacement of the plate.

The non-zero components of strain are:

- in the upper aluminium layer:

$$\begin{cases} \varepsilon_x &= -z_1 \frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y &= -z_1 \frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} &= -2z_1 \frac{\partial^2 w}{\partial x \partial y} \end{cases} \tag{5}$$

- in the lower aluminium layer:

$$\begin{cases} \varepsilon_x &= -z_3 \frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y &= -z_3 \frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} &= -2z_3 \frac{\partial^2 w}{\partial x \partial y} \end{cases} \tag{6}$$

- in the MR fluid layer:

$$\begin{cases} \varepsilon_x &= z_2 \frac{h}{h_0} \frac{\partial^2 w}{\partial x^2} \\ \varepsilon_y &= z_2 \frac{h}{h_0} \frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} &= 2z_2 \frac{h}{h_0} \frac{\partial^2 w}{\partial x \partial y} \\ \gamma_{xz} &= \left(\frac{h}{h_0} + 1\right) \frac{\partial w}{\partial x} \\ \gamma_{yz} &= \left(\frac{h}{h_0} + 1\right) \frac{\partial w}{\partial y} \end{cases} \tag{7}$$

The last three equations in (7) can be written in the following concise form

$$\gamma_{kl} = L_{kl}(w) \tag{8}$$

where indices kl take values form the set $\{(xy),(xz),(yz)\}$ and $L_{kl}(.)$ are appropriate linear differential operators.

The shear deformations γ_{xz} and γ_{yz} of aluminium layers are equal to zero. In the MR fluid layer all shear components of the strain tensor are non-zero.

Dissipation of Energy

Taking into account the properties of MR fluid we assumed that the non-zero longitudinal strains ε_x and ε_y appear in MR fluid layer do not develop tensile stresses. Thus the dissipation of energy in MR fluid layer is associated only with shear deformation.

Free vibrations of damped system are usually expressed using complex natural modes[7]. Complex modes and corresponding complex eigenvalues are the solution of modal analysis of plate. For small damping, the real parts of complex modes are significantly large than the imaginary parts.

Complex displacement associated with only one complex mode can be expressed in the following form:

$$w^{(clx)}(x,y,t) = A_0 \hat{w}(x,y) e^{\beta t} \tag{9}$$

where A_0 is the amplitude, $\hat{w}(x,y)$ is the complex mode and $\beta = -\alpha + i\omega$ is the corresponding eigenvalue. In expression describing the complex eigenvalue the symbol α stands for the damping coefficient and the imaginary part ω is the circular natural frequency. The complex displacement is an abstract quantity. Physical interpretation has usually the real part of complex displacement:

$$w(x,y,t) = \mathrm{Re}\left[w^{(clx)}(x,y,t)\right] \tag{10}$$

In calculations, it is very convenient to introduce the complex components of strain by the following equation:

$$\gamma_{kl}^{(clx)}(x,y,t) = L_{kl}\left(w^{(clx)}(x,y,t)\right) \tag{11}$$

The complex strain can be expressed using the complex strain mode $\hat{\gamma}_{kl}$ as:

$$\gamma_{kl}^{(clx)} = A_0\hat{\gamma}_{kl}e^{\beta t} = A_0 e^{-\alpha t}\hat{\gamma}_{kl}e^{i\omega t} \tag{12}$$

The exponential term $e^{-\alpha t}$ is a slowly varying function. Thus we can assume that the amplitude $A = A_0 e^{-\alpha t}$ is nearly constant in one period of motion.

$$\gamma_{kl}^{(clx)} = A\hat{\gamma}_{kl}e^{i\omega t} \tag{13}$$

The complex stress $\tau_{kl}^{(clx)}$ and complex strain $\gamma_{kl}^{(clx)}$ in MR fluid can be related using complex shear modulus:

$$\tau_{kl}^{(clx)} = G\cdot\gamma_{kl}^{(clx)} \tag{14}$$

The average power, dissipated from the plate, can be expressed as:

$$P = \frac{1}{T}\int_0^T \int_V \sum_{kl}\left(\tau_{kl}\frac{\partial\gamma_{kl}}{\partial t}\right) dV dt \tag{15}$$

Using concept of complex strain and stress, the above equation can be transformed to the following form:

$$P = \frac{1}{2}\omega A^2 \operatorname{Im}(G)\sum_{kl}\left(\int_V |\hat{\gamma}_{kl}|^2 dV\right) \tag{16}$$

For small damping, real parts of complex modes are dominant. Thus the square of modulus of strain components can be written in the approximate way:

$$|\hat{\gamma}_{kl}|^2 \cong (\operatorname{Re}\hat{\gamma}_{kl})^2 = (L_{kl}(\operatorname{Re}\hat{w}))^2 \tag{17}$$

The final form of the average power can be expressed as

$$P = \frac{1}{2}\omega A^2 \operatorname{Im}(G)\sum_{kl}\left(\int_V (L_{kl}(\operatorname{Re}\hat{w}))^2 dV\right) \tag{18}$$

Numerical Calculation

The calculations were performed for the following geometric parameters of the plate: $l_1 = 0.45$ m, $l_2 = 0.55$ m, $h = 0.001$ m, $h_0 = 0.002$ m

Natural mode shapes In the first stage of calculation the modal analysis was done. The complex modes of plate were determined. Due to small damping the imaginary parts of modes are significantly less than the real parts. In approximate calculations imaginary parts of modes can be neglected as it was assumed in previous subsection. The real parts of the first-, second-, third- and fourth- complex mode of the plate are shown in Fig. 3. They are very similar to mode shapes of plate without damping.

Optimal location of an active segment Taking into account equation (18) the average power dissipated in MR fluid layer can be calculated for selected mode of vibration and assumed placement of active segment of fluid layer. Calculations were done for various locations of active segment of MR fluid. The following dimensions of active segment: 4 cm × 5 cm were established. The longer side of segment is parallel to the longer edge of plate. The results of calculations are presented in Fig. 4. Values of power are normalized. In each graph the maximum value of power was assumed to be one. The placement of active segment is given by coordinates (x and y) of its centre. The placement of active segment is optimal when the value of average power takes its maximum.

Conclusions

Numerical results of calculations show that optimal placements exist for each mode of vibrations. The number of optimal placement increases with increasing natural frequency. When active segment is nearby the optimal position the dissipated power increases significantly.

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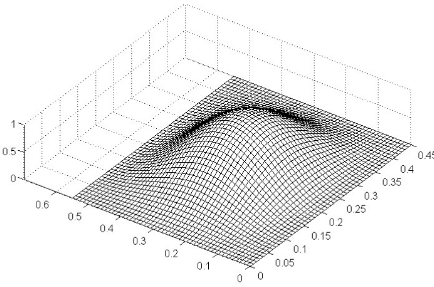
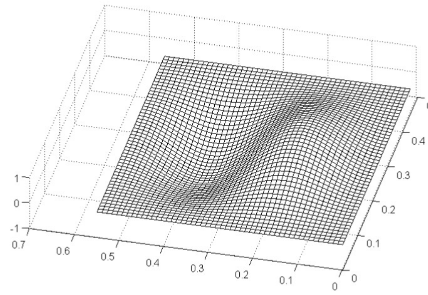
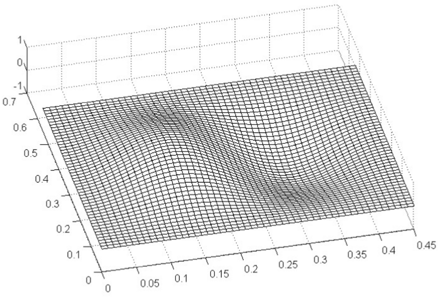


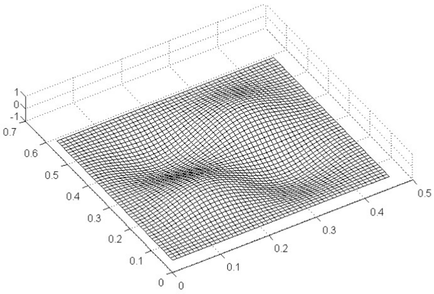
(a) first mode



(b) second mode

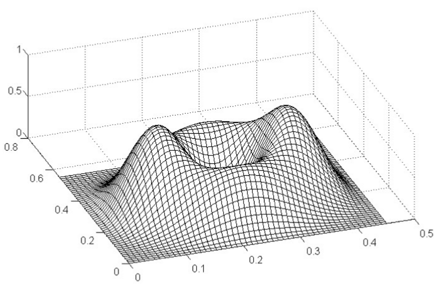


(c) third mode

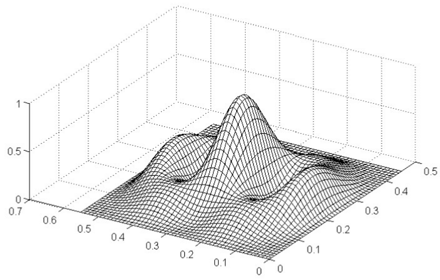


(d) fourth mode

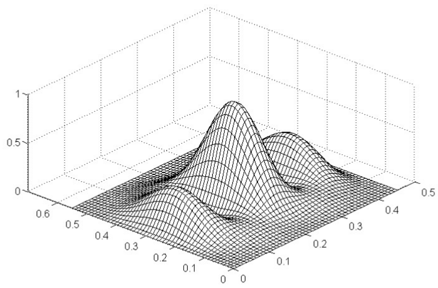
Fig. 3: The real parts of the first-, second-, third- and fourth- complex mode of plate.



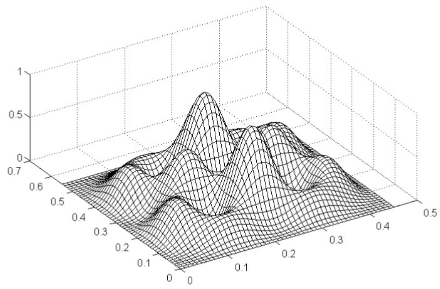
(a) first mode



(b) second mode



(c) third mode



(d) fourth mode

Fig. 4: Dissipated power for first-, second-, third- and fourth- mode for various placement of active segment.

References

[1] Q. Sun, J. X. Zhou, and L. Zhang, "An adaptive beam model and dynamic characteristics of magnetorheological materials," *Journal of Sound and Vibration*, 2003.

[2] K. D. Weiss, J. D. Carlson, and D. Nixon, "Viscoelestic properties of magneto- and electrorheological fluids," *Journal Intelligent Material Systems and Structures*, 1994.

[3] M. Yalcinitis and H. Dai, "Performance comparison of magnetorheological and electrorheological materials in adaptive structural applications," *Journal Intelligent Material Systems and Structures*, 1998.

[4] V. Rajamohan, S. Rakheja, and R. Sedaghati, "Vibration analysis of a partially treated multi-layer beam with magnetorheological fluid," *Journal of Sound and Vibration*, 2010.

[5] B. Sapiński and J.Snamina, "Modeling of an adaptive beam with mr fluid," *Solid State Phenomena*.

[6] J. Y. Yeh and C. L, W, "Dynamic stability of a sandwich plate with a constraining layer and electrorheological fluid core," *Journal of Sound and Vibration*, 2005.

[7] C. T. Sun and Y. P. Lu, *Vibration Damping of Structural Elements*. Prentice-Hall PTR, 1995.