Cartometric Analysis of Planimetric Accuracy of Topographic Maps 1:100 000 Created in S-46 Coordinate System

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Czechoslovak topographic maps in a scale of 1:100 000 in S-46 coordinate system on the Bessel ellipsoid represent relatively unknown state map work. This state map series originated in Czechoslovakia in the years 1949-1953 as a provisional work.

The paper deals with planimetric accuracy of maps determined by comparing positional deviations of control points. Control points are both measured in the maps themselves and in reference maps (cadastral maps, digital raster equivalents of topographic maps 1:25 000). The principle of transferring graphic coordinates to S-46 is based on sequence of Euclidean and affine transformations. Transformation removes the influence of inaccuracies of plotted S-46 grid points (which are used as identical) to control points and further suppress the influence of the map shrinkage. Calculated sets of variations are then statistically tested and analyzed.

***Key words**: cartometry, cartometric classification, topographic maps, S-46 coordinate system, Euclidean transformation, affine transformation, statistical tests*

Introduction

Old maps still have informational value. We can consider them as a mirror of olden ages. They are used for tracking landscape and forestation changes. They serve as a reference for reconstruction of old roads and waterways. We can use them also for reviewing topographic names. But these tasks need enough data about map parameters and features. We have to know the basic information about reference ellipsoid, map projection and map production. Non theoretical features of old maps are also important. Properties like shrinkage of map paper and map accuracy cannot be obtained from original documentation. Cartographical research methods are used in this case.

Topographic Maps 1:100 000 in S-46 Coordinate System

The Military Geographic Institute in Prague was restored after the World War II and proposal of the S-46 coordinate system was initiated. The new coordinate system was designed to replace the Křovák´s univers conform conic projection. The definition of the S-46 originates from results of transformations between Czech Datum of Uniform Trigonometric Cadastral Network (S-JTSK) and military grid of German army "Deutsche Heeresgitter". This coordinate system was created in times of German occupation of Czech lands. Maps utilize the Gauss projection of the Bessel ellipsoid in six-degree longitude zone (also know as Gauss-Krüger projection). Maps made in Křovák´s projection use Bessel ellipsoid too. The original division of map layout was one-fifth and Czechoslovak map key was used. Mapping in a scale of 1:25 000 started in 1949 but it was discontinued in 1952. Instead work began on so-called provisional topographic maps in the S-46 coordinate system in a scale of 1:50 000 and their derivates in a scale of 1:100 000 (TM100). Map layout division was one-sixth and Soviet map key was used. Works on the maps were completed in 1952-53. The paper deals with these topographic maps (Fig. 1). The S-46 grid is plotted in map face with two-kilometer spacing and the S-JTSK orientaion coordinates are shown in map frame [1], [2].

Method for Reviewing Planimetric Accuracy

The planimetric accuracy of maps is determined by comparing positional deviations of control points. Control points are both measured in the maps themselves and in reference maps (cadastral maps, digital raster equivalents of current topographic maps in a scale of 1:25 000 (RETM25)). Calculated sets of variations are then statistically tested and analyzed.

The principle of transferring graphic coordinates to the S-46 is based on sequence of Euclidean and affine transformations. Transformation removes the influence of inaccuracies of plotted S-46 grid points (which are used as identical) to control points and further suppress the influence of the map shrinkage.

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Transformations are performed for each control point separately using four identical points – points of the plotted S-46 grid.

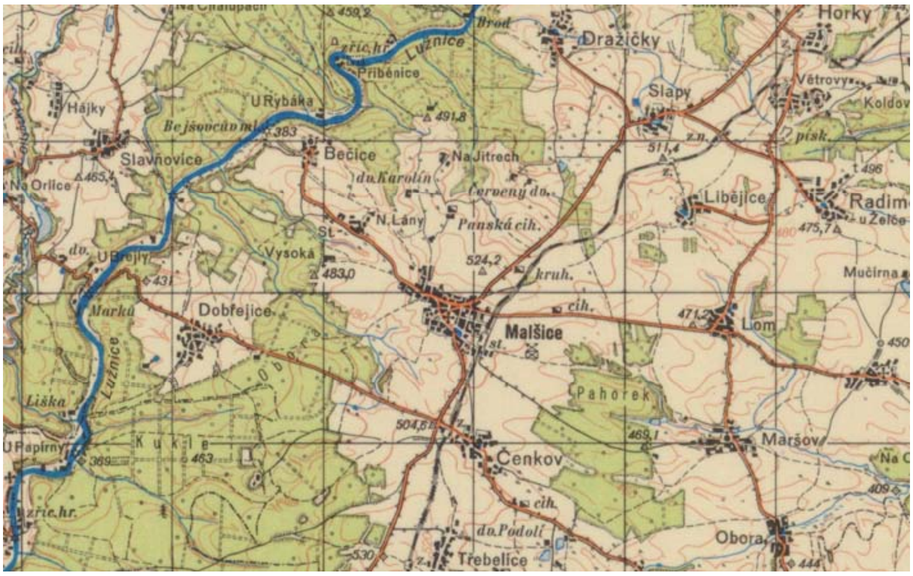

Fig. 1. The topographic map 1:100 000 in the S-46 coordinate system.

Cartometric Measurements

Only thirty-eight map sheets of western and southern Bohemia was available for research. Fourteen of them were selected for evaluation and they were scanned in 400 dpi resolution (Fig. 2).

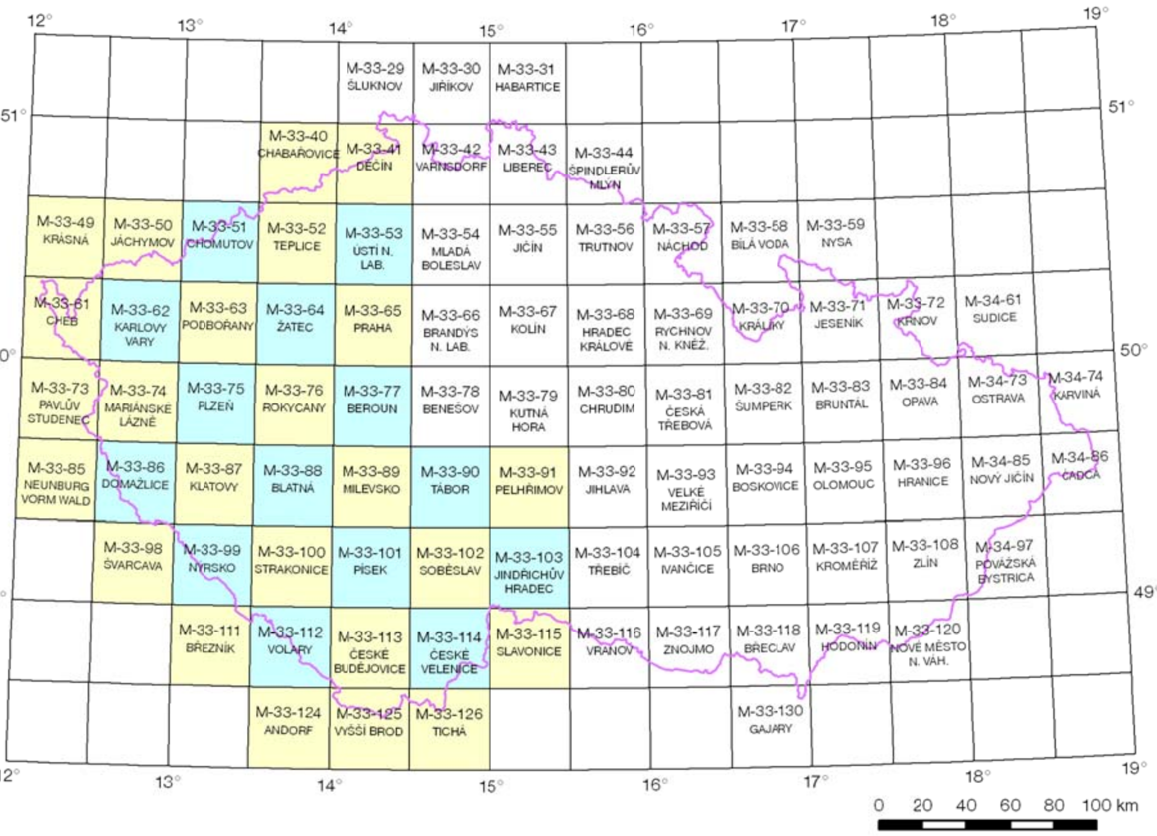

Fig. 2. Overview of the TM100 map layout in the Czech Republic. Blue – analysed map sheets. Yellow – available map sheets.

Eight control points of planimetry were chosen in the each map sheet. They were of two kinds – a road crossing and a trigonometric point (Fig. 3). Specific locations of control points were selected with regard to the greatest possible consistency in the clarity and they are spread over the entire area of map face. Churches and chapels as control points were not selected. They are useful for analysis of large scale maps but we have to consider generalization of map contents in the scale of 1:100 000 so they can be shifted.

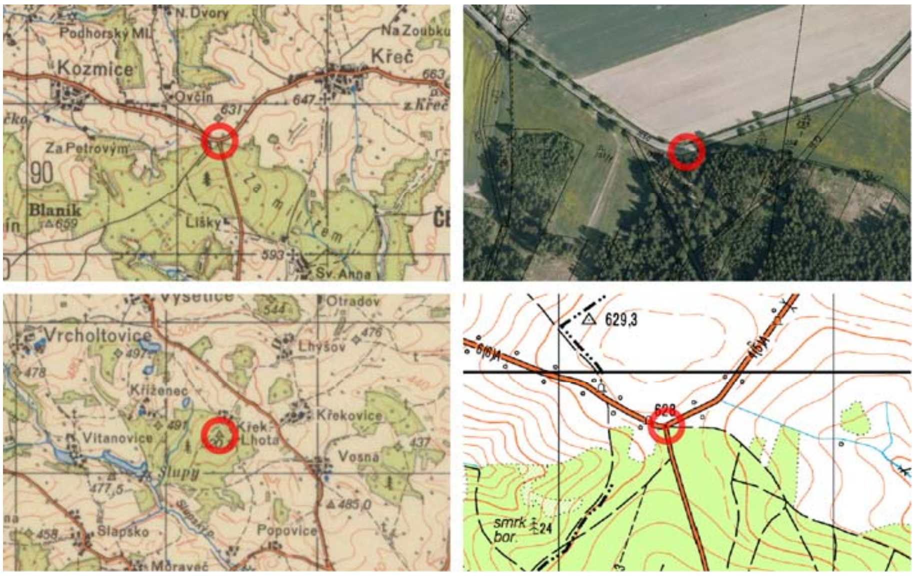

Fig. 3. Left – typical control points in TM100. Right – corresponding draw in cadastral map and in topographic maps 1: 25 000.

Map server IZGARD was used as a source of reference coordinates of road crossings. The IZGARD offers tools for searching and measuring point coordinates over RETM25 [3]. Cadastral map and orthophotomap in a form of web map service were utilized as a second source of reference. Czech Office for Surveying, Mapping and Cadastre provides these services [4]. Reference coordinates of trigonometric points were obtained from Land Survey Office database [5]. Standard error of position was set to fifteen meters. Control point which exceeded the limit was remeasured or replaced by more suitable point.

MicroStation and Groma software were used for reading and recording graphical coordinates of points. Measurements on each sheet (two times):

- Corners of map frame.
- Chosen points of the S-46 grid (corners of squares which bound control points).
- Control points of planimetry.
- All points of the S-46 grid with six-kilometer spacing on map frames and map face for mapsheets M-33-53, M-33-75, M-33-101 and M-33-103.

Graphical coordinates have negligible standard errors of position from the view of map scale. These errors have not exceeded one tenth of a millimeter at scale of a sheet.

Method for Reviewing Accuracy of the Plotted S-46 Grid in TM100

In the first phase it was necessary to determine theoretical coordinates of map frame corners and points of the S-46 grid. Geographic coordinates of frame corners were transferred to Gauss conformal projection of the Bessel ellipsoid in six-degree longitude zone with central meridian 15°. Equations utilized for this purpose can be found at scripts of mathematical cartography [6]. Theoretical coordinates of the plotted S-46 grid on the left and right frame were linearly interpolated from these values. The method could not be applied for points on the upper and bottom frame. Paralles bound map face and they are plotted as curves in general. So linear interpolation cannot properly capture their course. Curves may be proximized by section of circle or parabole. But if we know exact relation between geographic and Cartesian coordinates (i.e. cartographic equations nad their inverse forms) we can use them instead. Round values of X coordinate (divisible by 2000) and latitude (φ) for parallels with Y-axis or Y coordinate (divisible by 2000) and latitude for parallels with X-axis were known in this case. Unknown Cartesian coordinate was computed from these values via modified cartographic equations nad their inverse forms (Fig. 4).

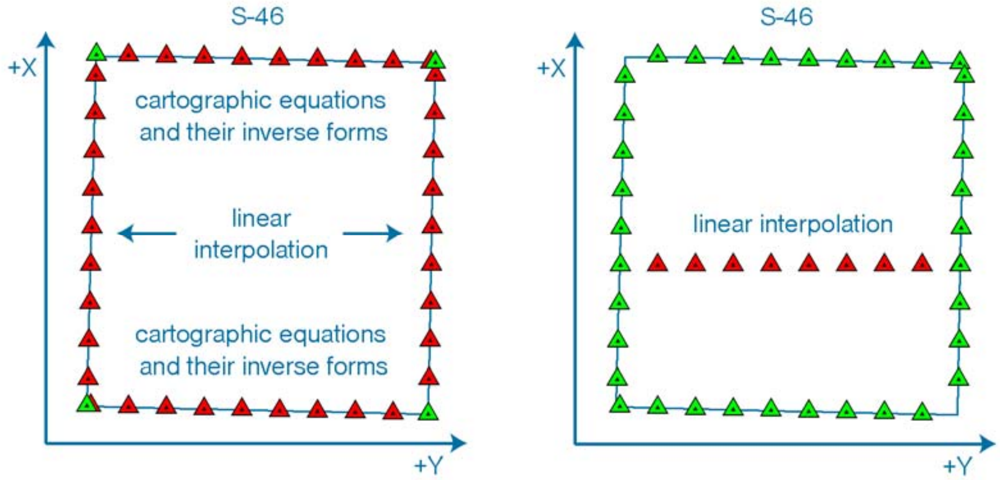

Fig. 4. Interpolation of theoretical positions of the S-46 grid points[2].

This method was not applied for points on the left or the right frame because difference between described method and linear interpolation is approximately one meter in real on the edge of six-degree longitude zone. So it is negligible.

Dimensions of the map face unaffected by map shrinkage were determined in the second phase – lengths of parallel and meridian arcs on Bessel ellipsoid were computed. Then the Map System was defined by these lengths divided by 100 (i.e. formula for map dimensions in millimetres). Origin of the Map System was set to the bottom left corner, positive y-axis points into the bottom right corner and positive x-axis is perpendicular to y-axis with geodetic sense.

The first transformation used in the method transferred theoretical coordinates of the S-46 grid points to the Map System. It was utilized by simple Euclidean transformation. Identical points were opposite corners of map sheet (Fig. 5).

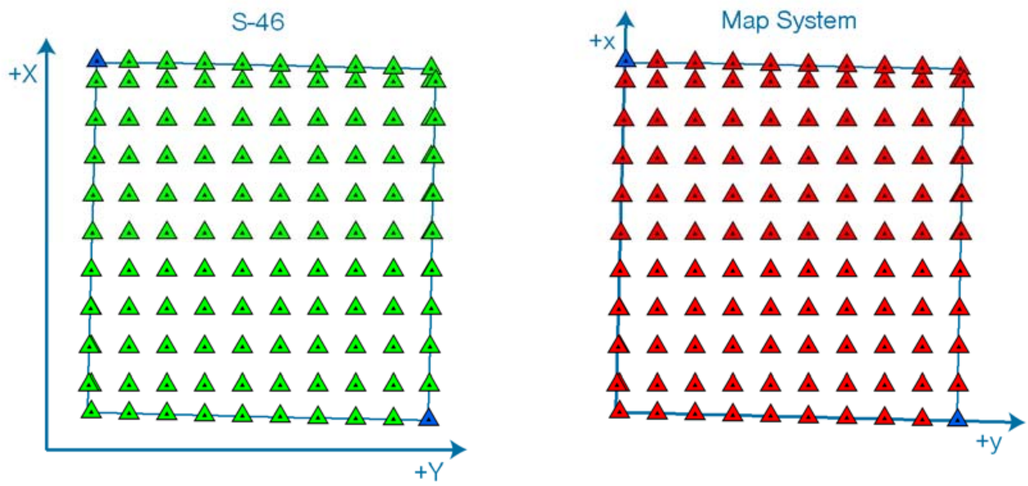

Fig. 5. Transformation of theoretical coordinates of grid points from the S-46 to the Map System.

First transformation was followed by four Euclidean transformations without adjustment. These transformations transferred theoretical coordinates of the S-46 grid points on map frames from Map System to Measurement System. Identical points were appropriate corners of map sheet. The purpose of this stage was preparation to elimination of map shrinkage (Fig. 6).

[2] The triangle indicates theoretical position of a point. The circle indicates measured position of a point. Known points are green. Computed point are red and blue is for identical points.

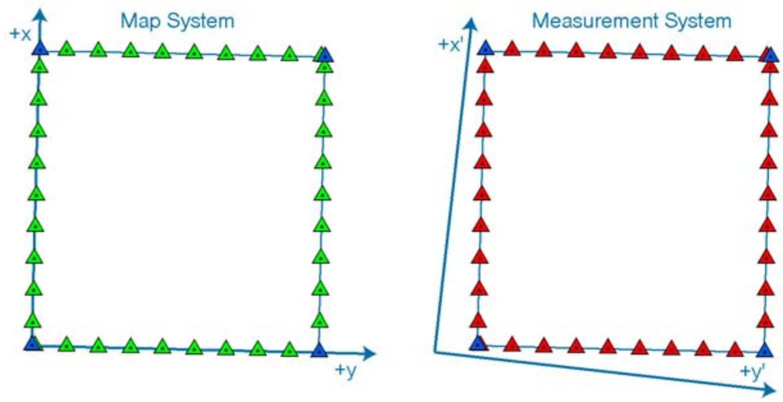

Fig. 6. Four transformations of theoretical coordinates of grid points from the Map System to the Measurement System.

The influence of map shrinkage to the S-46 grid points was suppressed by several Euclidean transformations without adjustment. Four transformations transferred measured coordinates of points on map frame to the Map System at first. The computation is very similar to the method which was described in previous paragraph. The difference lies in way of transformation. Transforming was carried out from the Measurement System to the Map System and transformation matrix was filled with measured non-theoretical data (Fig. 7).

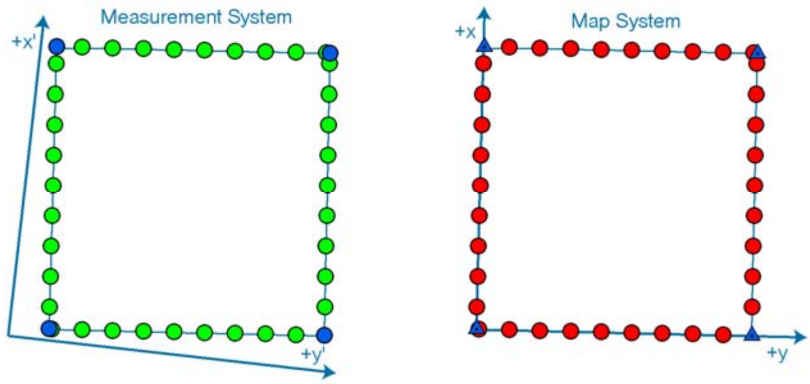

Fig. 7. Four transformations of measured coordinates of grid points from the Measurement System to the Map System.

Measured S-46 grid points within map face with same expected value of X coordinate are located approximately on a line. These lines are defined by points on map frames. Theoretical coordinates in the Map System and in the Measurement System of these edge points were known at this stage (previous two steps). Appropriate edge points were used as identical points for transformation of respective measured points to the Map System. This method determined measured coordinates of the S-46 grid in map face in the Map System without influence of map shrinkage (Fig. 8).

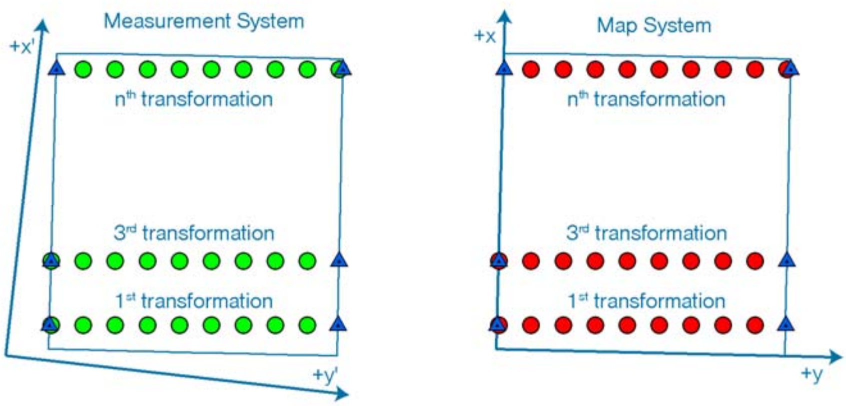

Fig. 8. Transformation of measured coordinates of map face grid points from the Measurement System to the Map System.

Last Euclidean transformation transferred measured values with shrinkage correction from the Map System to S-46 coordinate system. Identical points were opposite corners of map sheet (Fig. 9). Than these coordinates could be compared with theoretical coordinates and the shift of the plotted S-46 grid could be estimated.

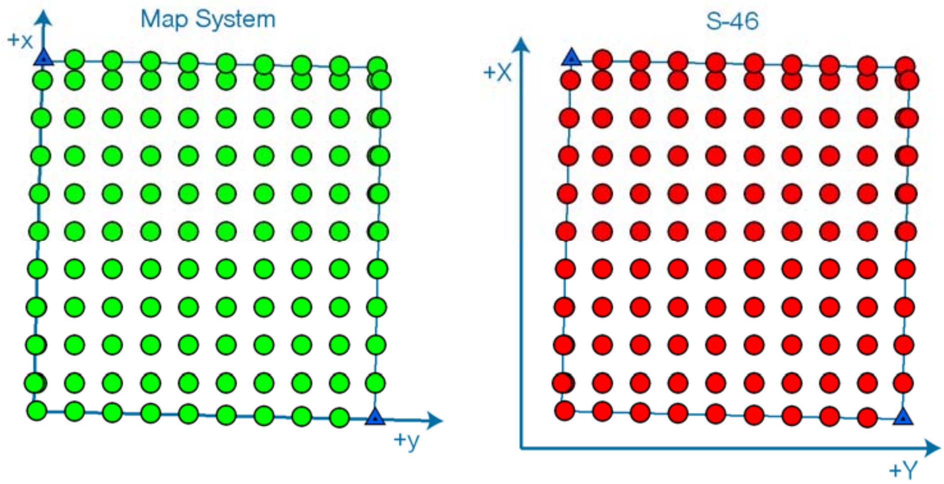

Fig. 9. Transformation of measured coordinates of grid points from the Map System to the S-46 system.

Whole sequence of Euclidean transformations without adjustment could be replaced by a single affine transformation with adjustment. Results are similar. But performed method is more suitable because it reduces uneven map shrinkage better. This state was verified at research of military topographic maps in a scale of 1:75000 which originate from the third military survey of Austria-Hungary [7, 8]. Whole described method is taken from these works with minor modifications.

Method for Reviewing Planimetric Accuracy of TM100

Every control point lies in a square of plotted S-46 grid. Every vertex of these squares is located within map face. It is possible to calculate real plotted coordinates of these quaternions in the S-46 coordinate system using process described above.

Each control point of planimetry was transformed to the S-46 coordinate system utilizing separate affine transformation with adjustment. The type of transformation was chosen with regard to expected map shrinkage in two major directions. Identical points were mentioned vertexes of the S-46 grid squares (Fig. 10)

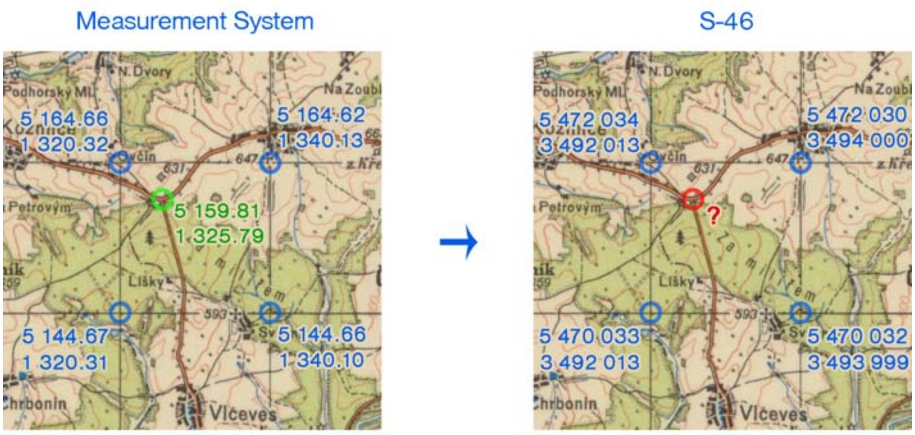

Fig. 10. The schema of affine transformation of a control point.

Coordinates of control points in the S-46 system obtained from measurement on topographic map 1:100 000 were known at this point. Map server IZGARD, Cadastral WMS neither database of geodetic control doesn't provide coordinates in S-46 system. So it was necessary to obtain reference coordinates in S-JTSK system. These coordinates were transferred to longitude and latitude on the Bessel ellipsoid and after that to the S-46 system. Formulas can be found at scripts of mathematical cartography [6]. The shift of planimetry was estimated by comparison of measured and reference values (i.e. by calculating position difference).

Average value was determined for these differences (shifts) which can be comprehend as a systematic error of control points coordinates. Experimental (sample) standard deviation and standard error of position were counted also.

Sets of shifts dY and shifts dX were statistically tested after that. The normality was checked utilizing central moments, Lilliefors test, Jarque-Bera test and Pearson's chi-squared test. In addition correlation coefficient between shifts dY and dX was calculated and statistically tested.

Three sets of dY and dX shifts were verified by test described above:

- Shifts of road crossings.
- Shifts of trigonometric points.
- Shift of both types of points.

Outliers measuring were excluded as well as trigonometric points with suspected change in position since the time of mapping. Reviewed sets were tested again. A set of shifts of the plotted S-46 grid was examined also.

The Research Results

The S-46 grid within map face of topographic maps 1:100 000 is plotted accurately. The same is true for points on frames of map. Sets of differences between theoretical and measured coordinates are normally distributed. They are not correlated and their expected values are close to zero. Experimental standard deviation of coordinates is approximately equal to 14 meters (Tab. 1).

Tab. 1. Planimetric accuracy of S-46 grid points in TM100[3] [meters].

$\overline{dY}^*$	$\overline{dX}^*$	s^*_{dY}	s^*_{dX}	s^*_{dYdX}	**n**
-6	-9	12	16	14	700

Some statistical tests indicate that sets of planimetric shifts of all crossroads control points and all trigonometric control points are not normally distributed. Sample correlation coefficient of both sets corresponds to base set which has zero correlation coefficient (Tab. 2 and 3).

Tab. 2. Planimetric accuracy of control points – whole set of road crossings [meters].

$\overline{dY}$	$\overline{dX}$	s_{dY}	s_{dX}	s_{dYdX}	**n**
-30	-67	44	41	42	65

Tab. 3. Planimetric accuracy of control points – whole set of trigonometric points [meters].

$\overline{dY}$	$\overline{dX}$	s_{dY}	s_{dX}	s_{dYdX}	**n**
-35	-57	70	142	112	44

However, normality of these sets is met when dubious values are excluded. Sample correlation coefficient of both sets corresponds to base set which has zero correlation coefficient again (Tab. 5 and 6). Plotted accuracy is roughly two and half times worse than accuracy of the S-46 grid as demonstrates tab. 1 and tab. 6.

Tab. 4. Planimetric accuracy of control points – set of selected road crossings [meters].

$\overline{dY}$	$\overline{dX}$	s_{dY}	s_{dX}	s_{dYdX}	**n**
-30	-72	37	32	35	60

[3] $\overline{dY}$ and $\overline{dX}$ are arithmetic means, s_{dY} and s_{dX} are experimental (sample) standard deviation, s_{dYdX} is experimental standard deviations of coordinates and *n* is size of set i.e. number of analysed points

Tab. 5. Planimetric accuracy of control points – set of selected trigonometric points [meters].

$\overline{dY}$	$\overline{dX}$	s_{dY}	s_{dX}	s_{dYdX}	n
-22	-69	35	33	34	36

Tab. 6. Planimetric accuracy of control points – set of selected road crossings and trigonometric points [meters].

$\overline{dY}$	$\overline{dX}$	s_{dY}	s_{dX}	s_{dYdX}	n
-27	-71	36	32	34	96

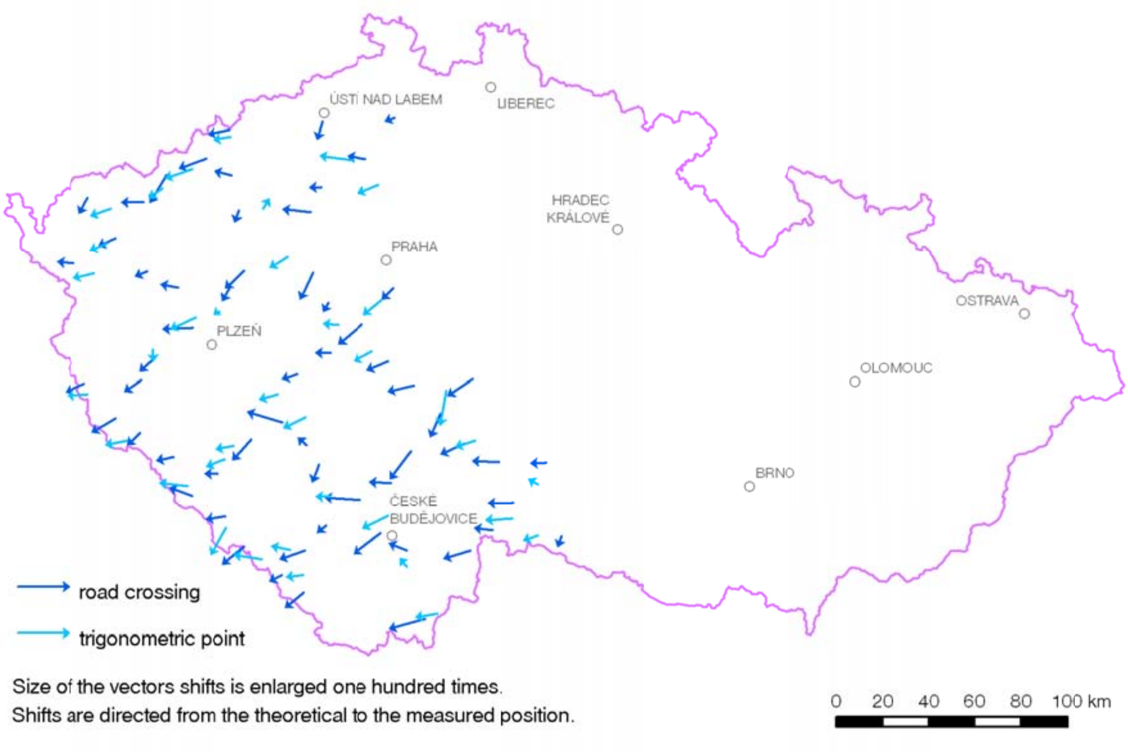

Fig. 11. The vector field of planimetric accuracy in TM100 with S-46 coordinate system in the Czech Republic.

Planimetric component of topographic maps is shifted about 27 meters on X-axis and 71 metres on Y-axis away from the origin. Systematic errors affect both directions and they are approximately constant. Values for road crossing set and trigonometric points set corresponds to each other. The accuracy of these important planimetric components is not excellent but is sufficient. Results don't exceed general accuracy of maps i.e. 0.5 millimetres at a scale of map.

Interesting results were obtained from calculation in which control points were transformed using theoretical coordinates of the plotted S-46 grid. Therefore coordinates divisible by 2000. Estimation of shifts is closer to zero. This brings the idea that the whole calculation of coordinates of the plotted S-46 grid was useless when it led to worse results. However, this is caused by small shift of the plotted S-46 grid which has the same direction as the shift of control points. If the grid would have been shifted in the opposite direction to planimetry or with greater value the procedure would be incorrect and results would not pose planimetric accuracy. It can be stated as a final opinion that the planimetry of topographic maps in the scale of 1: 100 000 is shifted towards the theoretical S-46 coordinates with roughly constant value, which however, doesn't greatly reduce the quality of maps.

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