Photogrammetric observation of deformation of the vertical mining shaft

Marek Fraštia [1], ***Marián Plakinger*** [2] *and* ***Jozef Beck*** [3]

Mining works are often specific in their shape, dimension and localisation. Therefore also the alternative measuring processes are adapted for acquisition of the reliable information about object geometry, especially for the objects inaccessible and non-measurable using conventional even the high tech surveying equipment. Mentioned team of authors designed and realised the displacement measurement of selected signalised points within the vertical mining shaft with a diameter 5m and up to the depth approx. 40 m. The aim of the measurements is to estimate the phase displacement of these points which represent the possible deformations of shaft walls and with the other observations they can predict the possible collapse of this mining work.

Key words: *deformation measurements by image acquisition and processing.*

Introduction

With progressing mining activity there was a necessity to release the coal resources located in the protective pillar of G-shaft for their exploitation. Special problem was physical liquidation of shaft itself which could effectively serve for the supply of fresh wind and for pumping the mining waters during the chopping off the coal resources in protective pillar or for quarrying other parts of the mine. For this purpose the task was assigned which should solve the possibility to maintain the functionality of the shaft as long as possible for mentioned purposes.

Mining Works are often specific in their shape, dimension and localisation. Therefore also the alternative measuring processes are adapted for acquisition of the reliable information about the object geometry, especially for the objects inaccessible and non-measurable using conventional even the high tech surveying equipment. Mentioned team of authors designed and realised the displacement measurement of selected signalised points within the vertical mining shaft with a diameter 5m and up to the depth approx. 40 m. The aim of the measurements is to estimate the phase displacement of these points which represent the possible deformations of shaft walls and with the other observations they can predict the possible collapse of this mining work.

Selection of the measurement methodology was dependent mainly on specific conditions characteristic for single object, its environment and required outputs and costs as well:

- Shaft is vertical with diameter 5m.
- Observation is possible only from the overground grid which is on the terrain level.
- There is no access to the observed points after their building up, necessary contactless measurement.
- Measurement is necessary to be done 1-2 times per 2 weeks.
- Required accuracy of displacement determination is more than 1cm in horizontal direction.

Considering previous facts the methods like polar measurement, laser scanning, vertical measurements and physical inclination measurements were disproved and the project of photogrammetric observation of object was prepared. Reasons for the selection of the measurement methodology and the attained results are in detail presented in the submitted paper.

Characteristic of the observed shaft and surrounding, selection of the measurement methodology

The shaft “G” is situated in the area of liquidated factory Bana Novaky, Hornonitrianske bane Prievidza a.s. with geographic coordinates $48^0 44`35.78``$; $18^0 35`32.01``$ or JTSK (Datum of Uniform Trigonometric Cadastral Network) coordinates 458297 m; 1224489 m; and elevation 275,6 m in Bpv system (Baltic Vertical Datum - After Adjustment). Mine surrounding and mine itself is directly affected by mining activity of parallel wall coalface quarried in the depth 260 m (Fig. 1), whereby over these walls there is a vertical terrain depression up to 8,96 m and beyond the quarried walls there are horizontal displacements

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as well. Shaft "G" itself is therefore in the impact of the horizontal and vertical forces activated by mining activity of brown coal mining.

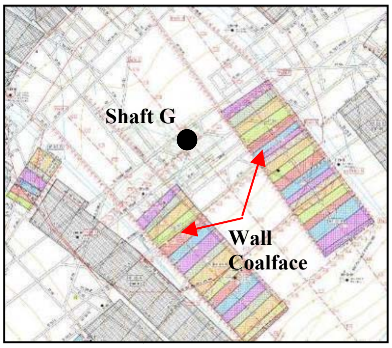

Fig. 1. Position of shaft related to the neighbour mining Works.

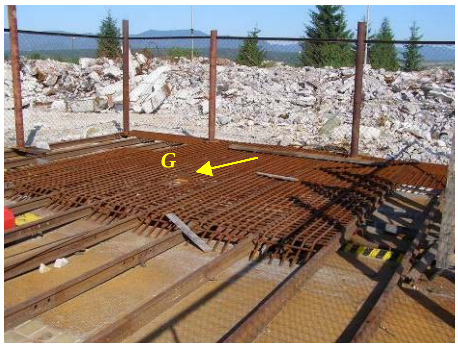

Fig. 2. Overground grid and axis of the shaft.

Shaft "G" is 332,1 m deep in circular shape with 5m diameter. Shaft walls are stiffened by panel concrete ring reinforcement 20cm thick with ring height 32 cm and 12 segments in the ring. There is a steel reinforcement (beams and crossbars) in the whole shaft profile which are anchored to the concrete reinforcement every 9 m (Fig. 3). Reinforcement is oriented in north eastern direction.

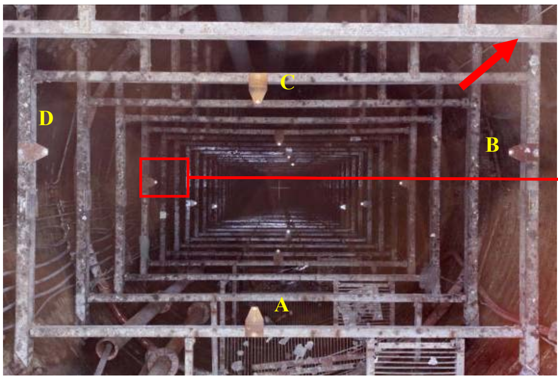

Fig. 3. Nadir sight to the shaft.

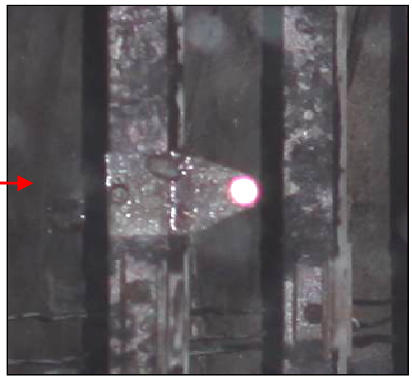

Fig. 4. Signalized point .

Selection of characteristic points for displacement observations depends on the possibility of stabilisation and signalisation and from measurement methodology itself as well. The non-selected observation of the shaft was left out by reason of many technological installations on the shaft walls and the only possible measurement method – laser scanning. Using this technology appeared to be problematic due to more reasons – scanner would have been necessary rotated by 90°, scanning is possible only over the grid what lead to large obscuration, huge steel reinforcement in the shaft is obstruction as well, long post-processing time, too high costs. That's why it was decided that the potential shaft deformation would be represented by displacements of the artificially signalised points on the steel reinforcement A-B-C-D (Fig. 4) in four horizontal levels -9 m, -18 m, -27 m and -36 m from the overground grid, all together 16 points.

Method of measuring the vertical using optical checkweigher was refused by reason of necessity to have at least 4 stations, overall complexity of measurement of more points underneath, need of installation and renovation of signal marks and missing optical checkweither as well with down sight in the inventory of the company.

Spatial polar method would be on place only if the measurements are done from bottom of the shaft to the top. This is not possible from the already mentioned reasons; therefore the polar method was refused as well.

Physical methods of inclination and displacements would be too expensive, would require the complete installation with electrical line what would be problematic in these conditions of permanent water discharge. The displacements values could be larger than range of the sensor and it would be difficult to refer the relative changes to the reference coordinate system. Moreover the risk of loss of the measuring equipment is too high considering their price.

Photogrammetric observations methodology meets the requirements of contactless measurements, low costs, fast acquisition and processing the data and the required precision as well. Considering the shape of the shaft, using the photogrammetric method it is not possible to determine the spatial displacements but it is possible to determine the displacements in the horizontal plane YX. Searched parameters are to be determined from 1 image but with using more images at the same time the processing can be completed using LSM.

For photogrammetric determination of the YX coordinates or displacements we need to know following parameters:

i. parameters of inner orientation of the camera,
ii. parameters of outer orientation of the camera,
iii. elevations of observed points,
iv. image coordinates of observed points.

1. Parameters of inner orientation of the camera were determined using calibration on the test point field (Fraštia, 2008). Self calibration with equal parameters for all images was used. Calibration results and camera settings can be found in the table 1. Also the coefficients of the radial symmetric and decentration distortion of the camera lens were determined (not shown in the table).

Tab. 1. Technical parameters of the camera/image.

Canon EOS 50D, digital SLR camera			
Number of pixels	15 000 000	**Format**	JPEG
Size of CCD sensor	(24.263x16.173) mm^2	**Resolution**	4752 x 3168
Focal length (f)	56.195 mm	**Colour depth**	16.7 x 10^6 colours
Pixel size	5.105 μm	**File size**	around 4 MB
Coordinates of main point	11.967mm , 8.012mm	**Image scale number**	200 – 1 600

2. Parameters of outer orientation define the position and orientation of the image at the time of exposure in reference coordinate system. This task is usually solved using perspective orientation on the control points and enables the calculation of the point coordinates directly in the reference system. Control points needs to be distributed over the whole area of the image to ensure the quality result and their number needs to be more than 3. In our case the position of the camera station (coordinates of the projection centre X_0 Y_0 Z_0 = X_G Y_G Z_G) is known from the GNSS measurement of the point G – shaft axis what allows to monitor its displacements in every phase and this fact was used for partial determination of the parameters of outer orientation. The next task was to determine the angle orientation ΩΦK of the camera at time of image exposition. For this purpose the orientation cross in the depth around 85m was used where the length of its arms are known (Fig. 5, 6) and we can determine the local coordinates of ends of the cross arms. Cross centre with coordinates 0, 0, Z is located on the shaft axis and its arms are oriented in the same direction as the shaft reinforcement. Rotation of the cross to the axis X_{JTSK} was determined as u = 38^0. Then the local coordinates of the ends of the cross arms and its centre were derived from the arms dimensions (Fig. 6) and thereafter transformed (1) about the rotation u into the system parallel to JTSK where the centre of the cross stayed at the value 0,0.

$$\begin{aligned} X &= x \cdot \cos u - y \cdot \sin u \\ Y &= x \cdot \sin u + y \cdot \cos u \end{aligned} \qquad (1)$$

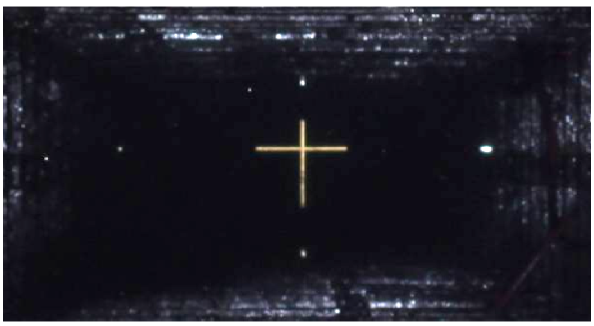

Fig. 5. Photo detail of orientation cross.

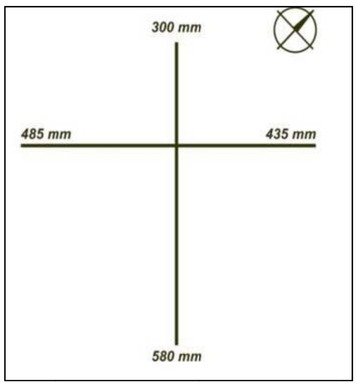

Fig. 6. Size of orientation cross.

3. Vertical displacements or elevations of observed points are determined using direct measurement of slope distances using laser EDM and based on the known approximate geometry of point distribution. Point elevations refer to the station of the distance measurements – to the point G on the top of shaft axis.

Fig. 7. Distance measurement to observed points.

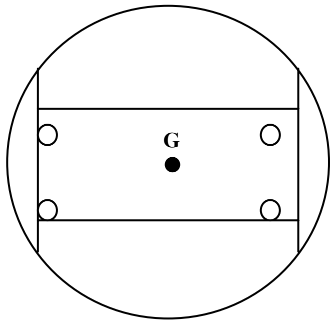

Fig. 8. Camera stations.

Signalisation of observed points

Method of signalisation was selected to enable the possibility of sub pixel measurement of image coordinates of observed points using specialised photogrammetric software. Points are circular in shape and they are manufactured from the reflex foil (Fig. 4.) which after illumination returns the light back to the source of the radiation. Similar foil is used in transport on traffic signs. It ensures the high luminance and contrast of the target marks, especially in the dark environment of the shaft.

The requirement is to have the source of the radiation. It can be secured by camera flash or the natural light of the sky. The size of the marks on the image should be in range 5-15 pixels (Luhman et. al, 2006). During monitoring there was a problem with the damage and destruction of the marks because of the permanent presence of the huge amount of water discharging from the sides of the shaft and because of the siltation of the targets by dirt and rust. Because of this reason the renovation of the targets is done every 6 months. Orientation cross is signalised by reflex foil as well.

Image capturing

Image capturing is realised always using the same camera Canon EOS 50D and the same lens EF 50 mm f/1.8 II. For the coordinates determination of the observed points it is sufficient to take pictures from the camera station „G“ which is stabilised on the shaft axis and special holder of the camera was created for the purpose of image capturing (Fig. 9) to ensure the identical position with GNSS measurements of the point G.

Depth of focus in the whole image field was ensured using the lens aperture higher than 11, focusing was always manually set to infinity. For checking purposes also the images from the other position were captured besides the shaft axis and these are suitable to process together by LMS in one project.

Fig. 9. Taking images.

Processing the measured data

Problems and a prior precision estimation

Solution of this task involves several problems:

i. The most precise and simplest photogrammetric method for measuring the displacements in plane is time baseline. It requires static camera station and invariable parameters of inner and outer orientation. In this case it is not possible to ensure this because there are changes in the shaft and on the surface as well. Solution is determination of relative displacements towards the shaft axis. Identical position XY of the camera in every phase is ensured by installed equipment in G point, orientation is ensured by orientation cross however the problem is the changing height of the camera station Z. Because the position of the G point is periodically monitored by GNSS measurements, we decided to use this fact and to make the calculation through the perspective transformation in 2 variants – related to the axes and related to the zenith.

ii. Main expectation is positional stability of orientation cross and the information about the length of its arms. Orientation cross is located in the 85m depth. According to the static calculations the surrounding mining works have no impact on the horizontal stability of the shaft so the mentioned assumption is reasonable.

iii. Problem of the physical focal length estimation and physical height of the projection centre. Using calibration the mathematical focal length and mathematical projection centre are determined but they are not identical with the physical optic values. That's why we don't know where this point is physically located. Naturally, when we get its coordinates in reference system using perspective transformation, thereafter it is possible to set it out in this system. Anyway, the inaccuracy in determination of the focal length and in the height of projection centre are expressed as a change of the image scale number M_S and therefore also in the different scale of reference system XY. In calculations always the same value of the focal length f = 56,195 mm is used and the value Zo = Z_{GNSS} which constantly differs from the physical position Zo about the value ΔZo. Then for the difference ΔZo = 10 cm and displacement 10 mm, the inaccuracy of the determined displacement will be in range 0,03 mm – 0,1 mm depending on the point elevation. That's why it is possible to ignore these errors.

iv. Inhomogeneity of the precision of the rotation parameters ΩΦK. While there is an expectation of very precise determination of the image rotation ΩΦ (view axis) towards the orientation cross, the problem is the precision of the image rotation K around the view axis because the cross arms represent only the short distance on the image (117 pixels ~ 0.6 mm, size of the image is 22,3 mm). Considering the fact that the only stable part of the object displayed on the image is the cross and its close surrounding, the only possibility to raise the precision of rotation K is to signalised other points on the extension of the cross arms up to the side of the steel reinforcement (Fig. 5). By doing so the reference distance is prolonged to 417 pixels ~ 2 mm, 3 times more. The precision of the rotation K determination should accordingly increase.

v. Estimation of a priory precision. As long as we use the exact time baseline without stereo photogrammetric evaluation, the calculation of the precision of displacement determination is simple:

$$m_X = \frac{M_S \cdot pix \cdot m_x'}{\sqrt{k}}, \quad m_Y = \frac{M_S \cdot pix \cdot m_y'}{\sqrt{k}}, \tag{2}$$

$$m_{XY} = \sqrt{m_X^2 + m_Y^2} = \sqrt{2} \cdot m_X = \sqrt{2} \cdot \frac{M_S \cdot pix \cdot m'}{\sqrt{k}} \tag{3}$$

where m_{XY} is position (plane) precision of the point in reference plane, M_s is image scale number, pix is size of the image element CCD/CMOS of sensor, m` is precision of the pixel coordinates measurement and k is number of images. Than the displacement precision is calculated from the precision of the point position in comparing phases i,j:

$$m_{pij} = \sqrt{m_{XYi}^2 + m_{XYj}^2} \tag{4}$$

Mentioned calculation considers correct parameters ΩΦK and XoYoZo what is the fundamental of time baseline. Then we get the following values of final displacement precision for the above object distances and precision of pixel coordinates measurement 0,5 pix and 1 image:

Tab. 2. A priory precision of displacement, simplified version.

horizontal level of point	1 (-9 m)	2 (-18 m)	3 (-27 m)	4 (-36 m)
m_{pij} [mm]	0,9	1,8	2,7	3,6

Displacements precision changes in dependence on the change of object distance, i.e. distance of the observed point from the image plane and thus it is constant for all points in one horizontal level.

In our case it is not possible to ensure the exact time baseline because the image capturing station in not static and also it is not possible to ensure the same camera orientation in every phase. A priory precision estimation is therefore more complicated because we have to consider also the errors of the camera rotation and final reference precision will be affected by location of the x`y`point on the image. Precision estimation is realised by Law of errors cumulation, where functional formulas of inverse perspective transformation (5) are partially derive by rotary angles ΩΦK and image coordinates x`y`.

$$X = X_{0i} + (Z - Z_{0i})\frac{\cos\Phi\cos K(x') + (-\cos\Phi\sin K)(y') + \sin\Phi\cdot f}{(\sin\Omega\sin K - \cos\Omega\sin\Phi\sin K)(x') + (\sin\Omega\cos K + \cos\Omega\sin\Phi\sin K)(y') + \cos\Omega\cos\Phi\cdot f}$$

$$Y = Y_{0i} + (Z - Z_{0i})\frac{(\cos\Phi\sin K + \sin\Omega\sin\Phi\cos K)(x') + (\cos\Omega\cos K - \sin\Omega\sin\Phi\sin K)(y') + (-\sin\Omega\cos\Phi)\cdot f}{(\sin\Omega\sin K - \cos\Omega\sin\Phi\sin K)(x') + (\sin\Omega\cos K + \cos\Omega\sin\Phi\sin K)(y') + \cos\Omega\cos\Phi\cdot f} \tag{5}$$

We assume that the precision of measured image coordinates is 0,5-1pixel, image coordinates $x`y`$suffice the approximate values on 0,1 mm acquired from the reconnaissance image and approximate values of the reference points coordinates can be for example from the drawing documentation of the shaft and proposal of the observed points stabilisation. Precision of rotation angles are derived from the precision of image coordinates determination of the orientation cross centre m`, image scale M_s and distance of the cross D from the station of the image capturing:

$$m_{\Omega\Phi}{}^{g} = \frac{M_s\cdot m^{`}}{D}\cdot\rho^{g} = \frac{\frac{D}{f}\cdot\frac{0.5\cdot pix}{\sqrt{n}}}{D}\cdot\rho^{g} = \frac{0.5\cdot pix}{f\cdot\sqrt{n}}\cdot\rho^{g} \tag{6}$$

where *f* is focal length, pix is size of pixel and n is a number of points used for determination of the rotation (in our case 5 = 4 outer points of cross arms + 1 central point).

For angle *K* we have used the following formula:

$$m_{K}{}^{g} = \frac{m^{`}}{r}\cdot\rho^{g} = \frac{0.5\cdot pix}{r\cdot\sqrt{2}}\cdot\rho^{g} \tag{7}$$

where *r* is distance between 2 outer orientation points. From the formula it results that the longer the distance is, the more accurate the angle *K* is determined. After the substitution into the formulas (6) and (7) we have the following values of the a prior precision of determination of the displacement on individual points:

Tab. 3. A prior precision of displacement, values in mm.

Homogenous precision in both compared phases; f = 50 mm ; pix =0,005 mm ; r = 2 mm $m_\Omega = m_\Phi = 0{,}001^g$; $m_K = 0{,}057^g$																
Point	**A1**	**A2**	**A3**	**A4**	**B1**	**B2**	**B3**	**B4**	**C1**	**C2**	**C3**	**C4**	**D1**	**D2**	**D3**	**D4**
m`= 0.5 pix	1,5	2,2	3,0	3,8	2,5	3,0	3,6	4,4	1,3	2,1	2,9	3,8	2,6	3,0	3,6	4,4
m`= 1 pix	2,2	3,6	5,2	6,8	2,9	4,1	5,6	7,1	2,0	3,5	5,2	6,8	3,0	4,2	5,6	7,1

From the table 2 we can see that the precision of the point is dependant not only on the distance from the image capturing station but also on the position of the point on the image (points A, C have shorter radial distance than points B, D). Influence of the angle precision ΩΦ is practically negligible; on the other hand the precision of the angle K in combination with radial distance of the image point plays an important role with the precision of the measurement of the image coordinates and image scale. Damage of the target leads into the precision decrease up to 1 pixel, what is 2-10 times lower precision of determination of point position. That’s why it is important to ensure the targets in the way to allow automatic sub pixel measurement of the image coordinates.

Reference coordinate systems

Determination of displacements or deformations requires definition of the reference system where all the displacements are determined. In this case we chose 2 variants of positional reference system:

1. shaft axes (Fig. 10a), where rotation around this axes is defined by orientation cross with its arms. Shaft axis itself is realised by overground point – camera station and orientation cross. Both points are geodetically set out. Overground point of axis is monitored using GNSS equipment and therefore also the change in position of the axis is determined. Thus the displacements in horizontal direction are determined towards this shaft axis. Non-perpendicularity of system axis can be ignored according to relatively little displacement of Z axis and long distance "cross-G". Displacements in this system are relative to the shaft axes and inform about the shaft deformation.
2. zenith (Fig. 10b) going through orientation axis, whereby the rotation around this axis is defined by the orientation cross itself with its arms. In this system it is possible to express the displacements practically identical with the system SJTSK. Displacements in this system tell about changing the point position in the system JTSK.

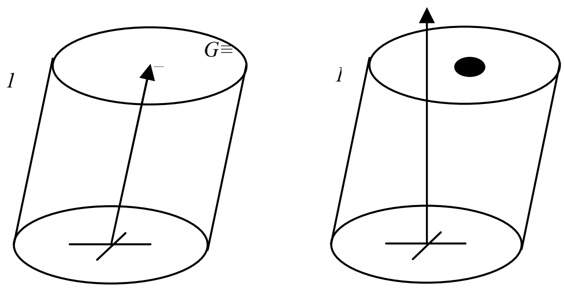

Fig. 10. Reference systems - axis of mine shaft 10a) and zenith 10b).

Both systems are realised by orientation cross coordinates (0, 0, Z in both systems), ends of cross arms (identical in both systems) and coordinates of point G – 0, 0, Z_G in system "shaft axis" and X_G,Y_G,Z_G measured by GNSS device in the system „zenith". Of course the main assumption is positional stability of orientation cross; nevertheless the deformational effects should have no impact in such a depth according to the geotechnical calculations.

Image coordinates measurement

Images were redrawn into the idealised form using module "Idealise" in software Photomodeler6. Idealised form of the image is digital image without optical radial and decentration distortion and main point is located exactly in the centre of the image. Image coordinates measurement was realised as well using specialised photogrammetric software Photomodeller6. This software allows the automatic measurement of image coordinates with precision 0.1-0.3 pixel. Precision of the manual measurement (in case of damage of the circular shape of the target) is 0.5 pixels.

Reference coordinates calculation

Measured pixel coordinates were further transformed into image coordinates using conformal transformation and these further enter into the calculation of YX coordinates of observed points together with determination of the parameters of outer orientation. Calculations are realised using software MathCAD. Unknown coordinates can be calculated in different ways:

1. In case of one image the calculation consists of 2 steps:
   i. Determination of camera rotations based on the points of the orientation cross and points on the extension of the cross arms – calculation from the 7 cross points, which coordinates were derived from the cross arms lengths. Formulas of perspective transformation were used for the calculation with LSM adjustment (Kraus, 2007):

$$x' = -f\,\frac{m_{11}(X\text{-}X_0)+m_{21}(Y\text{-}Y_0)+m_{31}(Z\text{-}Z_0)}{m_{13}(X\text{-}X_0)+m_{23}(Y\text{-}Y_0)+m_{33}(Z\text{-}Z_0)}$$

$$y' = -f\,\frac{m_{12}(X\text{-}X_0)+m_{22}(Y\text{-}Y_0)+m_{32}(Z\text{-}Z_0)}{m_{13}(X\text{-}X_0)+m_{23}(Y\text{-}Y_0)+m_{33}(Z\text{-}Z_0)} \qquad (8)$$

$$
\begin{aligned}
m_{11} &= \cos\Phi * \cos \mathrm{K} \\
m_{12} &= -\cos\Phi * \sin \mathrm{K} \\
m_{13} &= \sin\Phi \\
m_{21} &= \cos\Phi * \sin \mathrm{K} + \sin\Omega * \sin\Phi * \cos \mathrm{K} \\
m_{22} &= \cos\Omega * \cos \mathrm{K} - \sin\Omega * \sin\Phi * \sin \mathrm{K} \\
m_{23} &= -\sin\Omega * \cos\Phi \\
m_{31} &= \sin\Omega * \sin \mathrm{K} - \cos\Omega * \sin\Phi * \sin \mathrm{K} \\
m_{32} &= \sin\Omega * \cos \mathrm{K} + \cos\Omega * \sin\Phi * \sin \mathrm{K} \\
m_{33} &= \cos\Omega * \cos\Phi
\end{aligned}
$$

where x‘, y‘ are image coordinates, X, Y, Z are reference coordinates, x'_0, y'_0 are coordinates of main point, m_{11} – m_{33} are parameters of the orthogonal rotation matrix and X_0, Y_0, Z_0 a are coordinates of projection centre. Unknowns are in this case 3 angles ΩΦK hidden in the orthogonal rotation matrix. Number of correction formulas is 14.

ii. Calculation of the reference coordinates of 16 observed points from the inverse colinearity formulas:

$$
\begin{aligned}
X &= X_{0i} + \left(Z\text{-}Z_{0i}\right)\frac{m_{11}(x') + m_{12}(y') + m_{13}f}{m_{31}(x') + m_{32}(y') + m_{33}f} \\
Y &= Y_{0i} + \left(Z\text{-}Z_{0i}\right)\frac{m_{21}(x') + m_{22}(y') + m_{23}f}{m_{31}(x') + m_{32}(y') + m_{33}f}
\end{aligned}
\qquad (9)
$$

After coordinates calculation the characteristics of accuracy were calculated using the Law of errors cumulation, because in this case it is not possible to realise the LSM adjustment and there is no covariance matrix. Standard deviations m_{YX} were calculated from the standard deviations of camera rotations angles ΩΦK and from the precision of measured image coordinates (0,5 pix ~ 2,5 μm).

2. If we use 2 or more images captured from the shaft axis G, it is possible to do the LSM adjustment together with determination of the unknown reference coordinates XY and also the parameters of outer orientation of images Ω,Φ,K and Zo (projection centre elevation) using the formulas (8). Coordinates of projection centres Xo, Yo are known. Plan matrix $A_{(92,39)}$ has in this case of 2 images, 7 orientation and 16 observed points the following form:

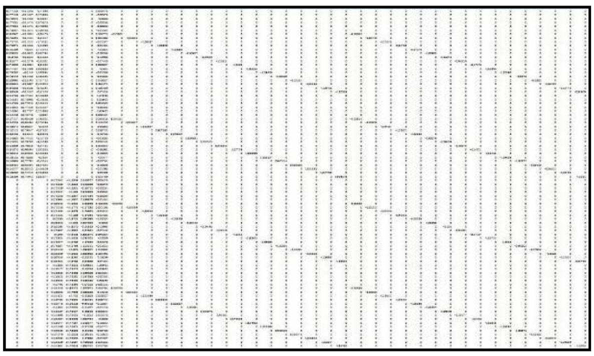

Fig. 11. Structure of plan matrix, 2 images from G point.

Final precision of point coordinates is then determined by covariance matrix. The results of the 15[th] phase control measurement are shown below:

№ of unknowns	n = 39
№ of measurements	k = 88
№ of iterations	3
№ of images	2
Standard error of unit weight	m_0 = 0,0024 mm
Maximal correction	v_{max} = 0,006 mm

$\Omega 1 = 0{,}0913^g \pm 0{,}0008$	$\Omega 2 = -0{,}0514^g \pm 0{,}0008$	$X_G = Xo = 0{,}050\ m \pm 0{,}003$ (GNSS)
$\Phi 1 = 0{,}0988^g \pm 0{,}0008$	$\Phi 2 = -0{,}0485^g \pm 0{,}0008$	$Y_G = Yo = -0{,}002\ m \pm 0{,}003$ (GNSS)
$K1 = 218{,}3705^g \pm 0{,}0696$	$K2 = 217{,}6784^g \pm 0{,}0696$	$Z_G = Zo = 274{,}468\ m \pm 0{,}00015$

Tab. 4. A posterior accuracy of XY position of points, values in mm.

15th control measurement																
Point	**A1**	**A2**	**A3**	**A4**	**B1**	**B2**	**B3**	**B4**	**C1**	**C2**	**C3**	**C4**	**D1**	**D2**	**D3**	**D4**
m`= 0.44 pix	1,2	1,4	1,7	2,0	2,1	2,2	2,4	2,7	0,9	1,1	1,5	1,9	2,2	2,4	2,6	2,9

Table 3 doesn't show the displacement precision but the positional precision, because there is not equal precision of point positions in individual phases.

3. When we process 2 or more images of the same phase, one from the shaft axis and other from the positions beyond the shaft axis (Fig. 8), we use the joint adjustment LSM with the simultaneous determination of the unknown reference coordinates XY and the parameters of outer orientation of images as well using the formulas (8). Plan matrix $A_{(92,41)}$ has in the case of 2 images, 7 orientation and 16 determined points the following form:

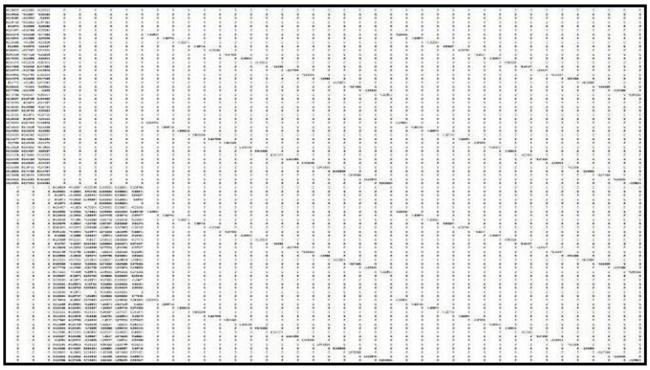

Fig. 12. Structure of plan matrix.

where the first 3 columns are derivations of formulas (8) by ΩΦK of image from the point G, next 6 columns by ΩΦK and XoYoZo of images beyond the point G and other columns express the derivations by unknown reference coordinates. Index G and 2 represents the image number, other images would analogue raise the number of matrix raw and number of columns for parameters of outer orientation.

4. Indeed it is possible to arbitrary combine the shown alternatives. But the efectivity of the solution should be always on mind.

Plan matrix is very sparse and in case of problems with the matrix singularity of normal formulas $\mathbf{N} = \mathbf{A}^T\mathbf{A}$ it is necessary to adopt the g-inverse matrix **N** into the LSM adjustment. Because we don't know the unknown parameters (parameters of outer orientation and reference points coordinates) with sufficient precision, the calculation needs to be done iteratively.

Determination of positional (XY) displacements

Provided the stability of orientation cross and considering the position change of overground point of shaft axis it is possible to derive the positional change of observed points for certain phase from previous and base measurement.

The largest displacements were assumed to be on the top of the shaft or in the upper parts of the shaft up to the depth 15-20 m from the shafthead. Positional and vertical measurements showed considerable displacements of points in 3rd, resp. 4th horizontal level, while the shaft axis changed the position in the period January – June in range <-7 cm;+7 cm> in direction of Y axis and <-2cm;4cm> in direction of X axis (Fig. 13).

The movement direction correlates with the position and process of coal getting next to the observed shaft. Observed points in the shaft itself shows the movements similar to the overground point of the shaft, however this movements is with larger depth smaller (Fig. 14). Displacement analysis of observed points shows the variant change of points position on individual horizontal levels, what was confirmed also by visual checks of shaft walls, material fallen from the walls and frames of steel reinforcement torn off. Also the orientation cross 85 m deep was affected by vertical change up to 50 cm.

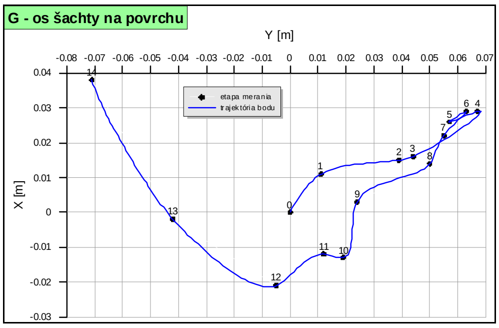

Fig. 13. Trajectory of shaft axis during 14 measurement phases (January -June).

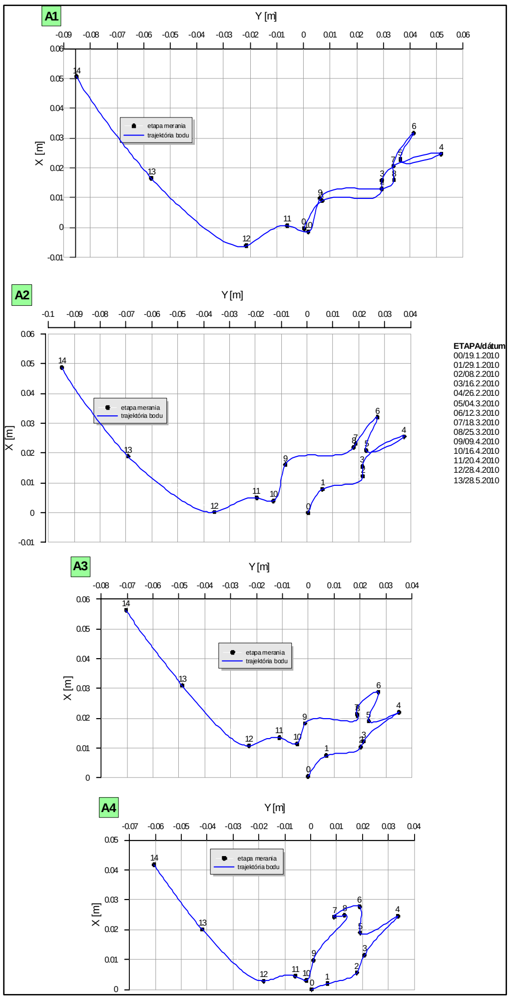

Fig. 14. Trajectory of observed points in vertical "A" profile.

Conclusion

Image processing and photogrammetry found their applications in wide range of human activity. Without doubt the measurement of displacements and deformations is one of them.

Introduced method of photogrammetric observation of the shaft is low cost, fast and simple from the point of view of installation of observed points and primary (image capturing) and secondary data acquisition. Secondary data acquisition (pixel coordinates measurement) requires the software for subpixel measurement which is provided by all available photogrammetric softwares for convergent photogrammetry. The custom calculation program needed to be developed for measured data processing, because this process is modification of standard software solutions. Semi-automatic measurement of signal marks enables up to 5 times higher reference precision of the point positions. Therefore it is important to renovate the damaged and dirty targets. It happens during the visual checks of the shaft walls by mining emergency worker specialised for working in heights.

Part of the multi-images convergent photogrammetry is also the geodetic monitoring of affects of undermining with wall coalfaces. This monitoring is elaborated in separate paper "The "G-Shaft" and its Stability inside the Mining Licensed Area Novaky I. The Mining Field Novaky" from authors Jozef Beck and Marián Plakinger.

Shown monitoring and application of results of measured values was the base for admitting the important resolutions about the retaining or the physical liquidation of the G-shaft.

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