Outlier Detection in 3D Coordinate Transformation with Fuzzy Logic

Yasemin Sisman[1], Aslan Dilaver[2]** and **Sebahattin Bektas[1]

Coordinate measurements inevitably contain outliers that affect the results of transformation. Conventional methods based on the leasts squared method determine the outliers by minimizing the selected objective function. Fuzzy Logic can be used to analyze the outliers. In this study, several outlier detection methods are described and applied to a real case consisting of a triangulation network. Results show that for outlier detection methods are not as efficient as Fuzzy provides a non-iterative solution in contrast to conventional methods.

***Key words:** Coordinate transformation, outliers detection, conventional methods, fuzzy logic*

Introduction

The use of satellite technologies in position navigating, the establishment of geodetic networks and determination of point coordinates has brought a new approach in which both the horizontal and the vertical coordinates are investigated together. It is known that the 3D coordinates of the points in various global datum can be determined faster and more accurately using the Global Positioning System (GPS) method. For example in the International Terrestrial Reference Frame (ITRF96) datum the position of points is determined by the GPS method, (URL1). The current application of new technologies in reference frame definition have made it necessary to form 3D point coordinates from defined different datum (coordinate systems) and to use the 3D datum (coordinate) transformation for efficiency. The known common coordinate points in both coordinate systems are needed to calculate the transformation parameters in coordinate transformation. First the transformation parameters are calculated by taking the coordinates of the common points as a measurement value and then the coordinates of other points are transformed using these transformation parameters. So, the selection of common point selection is very important and directly affects the results, (Kutoglu and Ayan 2006).

It is virtually impossible to avoid gross, systematic and random errors within a data set. While gross and systematic errors can be detected from the data set without calculation being required, outliers that are very close to random errors in terms of size can only be determined through the application of outlier tests. The literature reports different approaches have been used to determine the outlier measurements. The main approaches to determine outliers can be grouped as fallows; Conventional methods, Robust Estimation and Artificial Intelligence techniques. Conventional outlier detection procedures, introduced by Baarda (1968), use iterative approaches to find outliers in the data set, (Koch 1999). Lately, the Robust Estimation method was proposed by Huber (1981), and Hampel at al. (1986) as an alternative to conventional methods. A new approach is using Artificial Intelligence techniques for outlier detection for example Neural Networks and Fuzzy Logic technique. The latter was suggested by Zadeh (1965), can also be used to geodetic networks for outlier detection, (Berberan 1995. Sun 1994, Aliosmanoglu and Akyilmaz 2002).

In the coordinate transformation, the use of point coordinates as a measurement and in some transformation models, the acceptance of these measures as unerring are result in a negative situation. In order to obtain the significant results, both analyses of the models that measurements are erroneous and outlier detection has to be carried out on the results. The measurements are taken as erroneous in the conditional adjustment with unknown models and the total least squares models.

In this study, firstly the 3D coordinate transformation models, one of the main fields of mathematics and geodesy, are explained. The solution is calculated by conditional adjustment with unknown models. After the outlier detection by Conventional Methods and Fuzzy Logic are theoretically explained, the solutions can be obtained for these methods using a real case consisting of a triangulation network. The advantages and disadvantages of the outlier detection methods are assessed.

2. Methods

2.1. Coordinate (Datum) Transformation Models

When a geodetic network is set up, the datum has to be determined in order to orient the network space. Geodetic datum defines both the reference ellipsoid and the coordinate system. Therefore,

[1] *Yasemin Sisman, Sebahattin Bektas*, Ondokuz Mayis University Samsun,Turkey, ysisman@omu.edu.tr, sbektas@omu.edu.tr
[2] *Aslan Dilaver*, Karadeniz Tecnical University,Trabzon, Turkey, dilaver@ktu.edu.tr

the transformations between two datum and between two coordinate systems are in fact the same operation. Coordinate transformation is a common practice in geodetic studies.

For 3D coordinate transformation, many models have been developed including Bursa-Wolf, Molodensky-Badekas and Veis, (Thompson, 1976). These methods define 3D datum with 7 parameters (3 translations (X_0, Y_0, Z_0), 3 rotates ($\varepsilon_x, \varepsilon_y, \varepsilon_z$) and 1 scale parameter (κ)). Bursa-Wolf and Molodensky-Badekas models are the most popular. These models define and solve the relationship between two orthogonal coordinate systems with transformation parameters similarity transformation as given below.

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + (1+\kappa) \begin{bmatrix} 1 & \varepsilon_z & -\varepsilon_y \\ -\varepsilon_z & 1 & \varepsilon_x \\ \varepsilon_y & -\varepsilon_x & 1 \end{bmatrix} \begin{bmatrix} U_i \\ V_i \\ W_i \end{bmatrix} \tag{1}$$

Here, (U_i, V_i, W_i) and (X_i, Y_i, Z_i) show the positions in the first and secondary coordinate systems, respectively. In the Molodensky-Badekas model, the shifted and reduced values of the point coordinates according to a local starting point is used. Equation (1) is linearized according to the datum parameters used as unknown and shifted common point coordinates parameterized as below:

$$Ax + Bv + w = 0 \tag{2}$$

Equation (2) is the main equation of conditional adjustment with unknown models. The measurements are erroneous in this adjustment method. In Equation (2); A and B are the design matrices, x is an estimate of the transformation parameters, v is residual of measurements and w is the misclosure vector. In the solution of Equation (2) by Least Squared method (LSM), $Q_{\ell\ell}$, the inverse weights matrix of the coordinates used as measurements, Q_{xx}, the cofactor matrix of datum transformation parameters, and Q_{vv}, the cofactor matrix of the residuals, can be calculated (Vanicek, 1972).

$$\begin{gathered} N = BQ_{\ell\ell}B^T; \quad x = -\left(A^T N^{-1} A\right)^{-1} A^T N^{-1} w; \quad Q_{xx} = \left(A^T N^{-1} A\right)^{-1} \\ v = Q_{\ell\ell}B^T k; \quad Q_{vv} = Q_{\ell\ell}B^T \left(N^{-1} - N^{-1} A \left(A^T N^{-1} A\right)^{-1} A^T N^{-1}\right) BQ_{\ell\ell} \end{gathered} \tag{3}$$

The validity of the results of the solution produced by LSM depends on the completeness and accuracy of the mathematical model that has been built. In the test of the model hypothesis, the equality of a priori variance σ_0^2 and a posteriori variance s_0^2 should be statistically explicated. In the application, both null H_0 and alternative H_S hypotheses are formed and test size T is computed. The validity of the hypothesis which has been statistically explicated. In the case where H_0 is valid, it is assumed that the differences are derived from random errors, and the model may be accepted according to a certain probability level. Otherwise, H_S will be valid. In that case, the functional model is extended and tested. Afterwards, for the test of the stochastic model, the outlier detection process is started, (Thompson, 1976).

2.2. Outlier Detection Methods

It is unavoidable that there are gross or outlier measurements in the data set. These gross errors can be determined and eliminated while the linearization equations of the adjustment model are formed. According to error theory, other errors such as outliers that are very close to random measurement errors can be determined only by explicating the solutions found by the LSM.

Not all the outliers are due to bad data having gross errors. In some cases, these measurements may contain important information for the data set. If the model is built well for example, considering the distribution of the data, the outlier measurements may be removed from the group without an additional evaluation. Furthermore, there may be a failure in the shape of the mathematical model because of the excessive amount of outlier measurements, (Hampel, 1986).

2.2.1. *Conventional Outlier Detection Methods*

Conventional Outlier Detection Methods are the Data-Snooping Method (W-Test), Tau Test and t- Test. In these methods the hypothesis test is performed to determine the $\nabla\ell_i$ error in any ℓ_i measurement in a $\underline{\ell}$ measurement vector. Then, the test value is calculated for each measurement and the test value is compared with the critical value of the distribution table. If the test value is larger than critical value, then the largest value is considered to be the outlier and it should be removed from the data set. The iteration continues until there is no outlier left in the data set. The test value, critical value and distribution information are given in Table 1; α_0 is the significance level, f is the freedom degree, s_{01}^2 is the posteriori variance eliminated from the model errors, and N, τ, t represent respectively normal, tau and student distribution, (Koch 1999).

Tab. 1. Conventional outlier detection methods.

Method	Data-Snooping	Tau-Test	t-Test
Test Value	$W_i = \frac{\lvert v_i \rvert}{\sigma_0 \sqrt{Q_{v_i v_i}}}$	$T_i = \frac{\lvert v_i \rvert}{s_0 \sqrt{Q_{v_i v_i}}}$	$t_i = \frac{\lvert v_i \rvert}{s_{01} \sqrt{Q_{v_i v_i}}}$
Test Distribution	$\sim N(0, 1)$	$\sim \tau_{f,(1\text{-}\alpha_0/2)}$	$\sim t_{f\text{-}1,(1\text{-}\alpha_0/2)}$
Critical Value	$N_{(1-\alpha_0/2)}$	$\tau_{f,(1\text{-}\alpha_0/2)}$	$t_{f\text{-}1,(1\text{-}\alpha_0/2)}$

2.2.3. *Fuzzy Logic and Fuzzy Set Theory*

Recently, many approaches have been tested on decision making theories. Some of the Artificial Intelligence techniques that are used in outlier analysis are, Neural Networks, Support Vector Machine, Fuzzy Logic, (Cateni at al. 2008).

Fuzzy logic is a logical model providing a general idea about the decision process in the analysis of the data set. The fuzzy logic suggested by Zadeh (1965) is, essentially, an approach that allows transition values to make a definition between the conventional values, such as right/wrong, yes/no, high/low. The main purpose of the method is to bring a certainty to assigning a membership degree to the concepts, which are hard to express or have difficult meaning, (Shi at al. 1999). Fuzzy set theory was suggested as an alternative to classical set theory. In a fuzzy set, for any component to have a membership degree between 0 and 1, the limit of the classical set is extended and the degrees of a fuzzy event to occur or to exist are measured. In classical set theory, an object is either a component of a set or not, whereas in fuzzy logic an object can be given memberships in many sets. A fuzzy system consists of three main parts, which are fuzzification, rule base and defuzzification. Firstly, fuzzification can be defined as a transfer between a definite system and a fuzzy system and it describes a property of an object in a certain fuzzy set. The objects can belong to ‘low, middle, high’ property classes with membership functions and each object is assigned a membership degree between 0 and 1. Secondly, the rule base combines the membership functions from the fuzzificator with the rule handling data such as ‘if, and, although, if not’ which is based on the database and stored there. Thirdly, in the defuzzification unit, the rule results that are obtained from the rule handling unit are evaluated in the defuzzificator and turned into definite results, (Aliosmanoglu and Akyilmaz, 2002).

The membership functions define the degree to which an input belongs to a fuzzy set. These membership functions are chosen empirically and optimized using a sample input/output data. The If-then rules define a connecting the antecedent to the consequent (i.e. input to output). These rules are given weights based on their criticality, (Syed and Cannon, 2004). With this approach, measurements can be classified according to their membership degrees by adequate membership functions. In general, the membership degrees between 0 and 1 are real numbers, 0 shows that there is no membership and 1 shows full membership. The different forms of membership functions can be chosen while fuzzy sets are formed. The most common membership functions include the triangle, trapezoid, Gauss curve and sigmoid. As the membership functions represent the fuzzy set, the selection of their shape and form directly affects the decision process.

In classical set theory, the values of measurement are grouped into two parts. In fuzzy set theory, more groups are formed up. The classification of group members is done effectively by using the membership function. Many geodetic problems, deformation analysis, parameter estimation, geoid determination, data analysis in a geographical information system, outlier detection of leveling networks and GPS networks, can be solved with the help of fuzzy model, (Hampel at al., 1986; Konak at al., 2005; Kutterer, 2002).

The most commonly used methods in fuzzy set theory are Mamdani and Takagi–Sugeno methods, (Xue-Gong, 2001). These methods define a membership function, then measurements are grouped according to the membership degree. Consequently, the decision can be made as to whether a measurement is an outlier or not, thus an outlier data set can be formed. Also, an outlier data set can be formed using the membership values obtained from the fuzzy set theory in the LSM solution, (Konak at al., 2005).

2.2.3.1. *Outlier Detection By Fuzzy Logic*

According to the LSM, when the given functional model includes the $\nabla\ell$ measurement error vector, the mathematical relationship between the residuals and the measurement errors is established with *R* redundancy matrix. *R* can be written following equation.

$$\begin{bmatrix} v_1 \\ v_2 \\ . \\ v_n \end{bmatrix} = -\begin{bmatrix} r_{11} & r_{12} & . & r_{1n} \\ r_{21} & r_{22} & . & r_{2n} \\ . & . & . & . \\ r_{n1} & r_{2n} & . & r_{nn} \end{bmatrix} \begin{bmatrix} \nabla\ell_1 \\ \nabla\ell_2 \\ . \\ \nabla\ell_n \end{bmatrix} \tag{4}$$

From Equation (4), it can be seen that each measurement is affected by all measurement errors depending on the ratio of the size of the component corresponding to its redundancy matrix. Both in correlation and uncorrelation adjustment, the rank of the R is equal to the excess measurement number; hence, this matrix is not positively defined. In other words, errors cannot be calculated with a single meaning. In this case, instead of measurement errors ∇i, the size of residuals is dealt with in both conventional outlier tests based on statistic decision-making process.

However, in fuzzy logic, membership relationships determining the effects of the errors on residuals may be formed using the redundancy matrix. According to Equation (4), row elements of redundancy matrix show the total effect on one residual of errors, which can possibly occur in all measurement errors, the column elements of redundancy matrix show the total of effect on the other residuals of each measurement error. This provides a possibility of examining the errors, derived from the distribution of errors and also to absorb them.

The steps of the fuzzy set theory are given below:

Fuzzification: After the initial adjustment, taking the relationship given in Equation (4) into consideration, the test values for each relation are calculated. After comparing the statistical limits with the test value of each residual, the residuals in the fuzzy set are divided into two measurement groups: one, the normal residuals (the test values under the statistical limits) $N\{V_i\}$, and the other, the abnormal residuals (the test values above the statistical limits) $M\{V_i\}$. After the hypothesis tests, the membership function shows the residuals of the members. The components of the $N\{V_i\}$ subgroup consisting of the test values below the statistical limit are given zero membership value. The components of the $M\{V_i\}$ subgroup consisting of the test values above the statistical limits are given a membership value between 0 and 1, depending on the statistical limit deviations, degree of freedom and the errors probability of the test.

$$\mu_{\widetilde{M}(V_i)} = \begin{cases} 0 & T_i \le q \\ \dfrac{1}{1+\left(\dfrac{c_{ij}}{d}\right)^2} & T_i > q \end{cases} \tag{5}$$

If each residual has T_i test value of measurements, *q* critical value, c_{ij} relation between the test value and the critical value, and *d* the scale factor, membership function is calculated by the above equation (Kutoglu 2006).

Using the Equation (5), residuals, which are not affected from the outliers, can easily be calculated with fuzzy set theory.

$$\mu_{\widetilde{N}(V_i)} = 1 - \mu_{\widetilde{M}(V_i)} \tag{6}$$

Rule Base: To determine the relations between measurement errors and fuzzy relationships, $\tilde{r}_{ij}$, the components of the redundancy matrix normalized between 0 and 1, are used. Measurement errors can be separated into two groups, similar to the residuals. For example, the *A* subgroup may consist of the measurement errors, which have maximum affect on the abnormal residuals and the *B* subgroup, may consist of the measurement errors which have minimum effect on the normal residuals. If $\widetilde{R}$ normalized redundancy matrix is used to define the membership values of *A* and *B* sets and these membership values are taken as $\mu_{\widetilde{A}(\nabla\ell_i)}$ and $\mu_{\widetilde{B}(\nabla\ell_i)}$, in the calculation of the membership values. The membership function $\mu_{\widetilde{A}(\nabla\ell_i)}$

which is the maximum relative effect of i^{th} measurement errors of residuals having a $\mu_{\widetilde{M}(v_i)} \geq 0.5$ membership value in the $M_{(v_i)}$ fuzzy function set, and then; the membership function $\mu_{\widetilde{B}(\nabla \ell_i)}$ which is the maximum relative effect of i^{th} measurement errors of residuals having a $\mu_{\widetilde{N}(v_i)} \geq 0.5$ membership value, is determined.

$$\mu_{\widetilde{A}(\nabla \ell_i)} = \widetilde{r}_{ji}\, \mu_{\widetilde{M}(V_i)}\ ;\ \widetilde{r}_{ji} = \max(\widetilde{r}_{ki}) \tag{7}$$

$$\mu_{\widetilde{B}(\nabla \ell_i)} = (1 - r_{mi})\, \mu_{\widetilde{N}(v_i)}\ ;\ \widetilde{r}_{mi} = \max(\widetilde{r}_{ki}) \tag{8}$$

The measurements that are immensely affected by the gross errors have maximum affect on the abnormal residuals whereas these have minimum affect on normal residuals. The minimum value of the membership function given in Equation (7) and Equation (8) shows the degree of ℓ_i observations, where the minimum value is determined to be out of the limits. According to fuzzy set theory, the intersection of fuzzy sets *A* and *B* are composed of an *H* set.

$$\mu_{\widetilde{H}(\nabla \ell_i)} = \min\left[\mu_{\widetilde{A}(\nabla \ell_i)}, \mu_{\widetilde{B}(\nabla \ell_i)}\right] \tag{9}$$

Defuzzification: To determine a significant limit critical value, the equations given below calculated by weighted average fuzzification method can be used.

$$C_H = \frac{\sum P_i \mu_{\widetilde{H}(\Delta \ell_i)}}{\sum P_i}\ ;\ P \Rightarrow \begin{cases} \mu_{\widetilde{H}(\nabla \ell_i)} \in \mu_{\widetilde{A}(\nabla \ell_i)} & P_i = \widetilde{r}_{ji} \\ \mu_{\widetilde{H}(\nabla \ell_i)} \in \mu_{\widetilde{B}(\nabla \ell_i)} & P_i = 1 - \widetilde{r}_{mi} \end{cases} \tag{10}$$

Here *P* is weight matrix of the measurements. The obtained membership function $\mu_{\widetilde{H}(\nabla \ell_i)}$ is compared with the C_H value and if $\mu_{\widetilde{H}(\nabla \ell_i)} > C_H$, then it is decided whether the values are out of limit, which means that they are in a different set, (Aliosmanoglu and Akyilmaz 2002).

3. Result and discussion

For this study, a geodetic network (Fig. 1) formed by The Directorate of Land Registry and Cadastre, was chosen. Both the European Datum-50 and ITRF96 (URL1) coordinate values of the 45 points of the network are known.

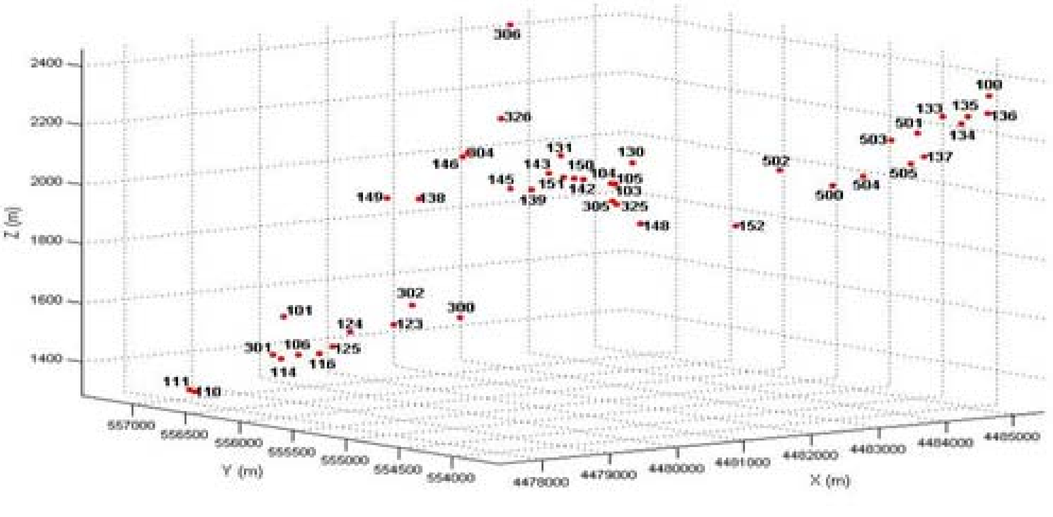

Fig. 1. The Network Selected for the Study.

The network points were produced according to coordinates and vectors at a 1998.00 reference epoch of Turkish National GPS Network points. All the measurements were performed in June 2005 using GPS receivers. Data reduction and post-processing were carried out using Leica LGO 2.0 software. Adjustment computation was performed accepting the coordinates of Turkish National GPS Network points as stable in the measurement epoch. Then, coordinates of points *(X, Y, Z)* were calculated using coordinated of points *(φ, λ, h)*.

3.1. Coordinate Transformation Methods

Between the European Datum-50 and ITRF96 datum, the coordinate transformation parameters were estimated using Molodensky-Badekas models, and the significance of the calculated parameters was examined (Table 2). In the result of these tests, it was found (Z_0) parameter was not statistically significant. After the examination, it was concluded that there might be an outlier in the data set and an outlier detection methods needs to be applied to the data set.

Tab. 2. Coordinate transformation parameters.

	X_0	Y_0	Z_0	ε_x	ε_y	ε_z	κ
Parameters	-0.142 m.	0.228 m.	-0.568 m.	-20.936’’	11.382’’	2.929’’	3.940 ppm
Significance Test	Valid	Valid	Invalid	Valid	Valid	Valid	Valid

3.2. Outlier Detection

Firstly, Conventional methods (Data-Snooping, Tau and t-test) were applied to detect outliers from the results obtained after the coordinate transformation was carried on the application network. The outliers were found and the results are shown in Table 3. Data-Snooping and t-test methods found the following outliers 501, 105, 100, 143, 148, 131, 302 101. In tau test, 503 point found as outlier additional these outliers. The iterative solution was estimated until a consistent measurement group was obtained.

Then the Fuzzy Logic method was also applied to outlier detection using the results obtained after the coordinate transformation. The outliers found by the fuzzy logic method are shown in Table 4.

Firstly, Fuzzy Logic method was determined the points of membership value owned in outlier group and the test value was calculated. Then, a comparison was made between the points of membership value and the test value. Here, eight points were determined to have membership values in outlier groups, three points were taken out of the outliers group due to having smaller membership values than the test value. The eight points yielded equivalent result to conventional methods.

As a result, only 5 points were considered to be outlier in Fuzzy Logic methods.

Tab. 3. Outliers determined by conventional methods.

Iteration	Data-Snooping		Tau Test		t-Test	
	PN	Test Value	PN	Test Value	PN	Test Value
1	501	3.010	501	3.735	501	3.721
2	105	4.589	105	3.505	105	3.492
3	100	4.359	100	3.459	100	3.446
4	143	4.062	143	3.346	143	3.333
5	148	4.107	148	3.529	148	3.515
6	131	4.047	131	3.655	131	3.640
7	302	3.707	302	3.513	302	3.498
8	101	3.908	101	3.871	101	3.854
9	Consistent		503	2.651	Consistent	
10			Consistent			

Tab. 4. Outlier measurement determined by Fuzzy Logic.

PN	Membership Value	Test Value	Result
501	0.44310	0.10554	Outlier
105	0.30923	0.10554	Outlier
148	0.28541	0.10554	Outlier
143	0.27012	0.10554	Outlier
131	0.23053	0.10554	Outlier
100	0.06271	0.10554	Consistent
302	0.05076	0.10554	Consistent
101	0.03808	0.10554	Consistent

4. Conclusion

The coordinate transformation models Molodensky-Badekas models have been discussed theoretically. The method was statistically tested using a real case consisting of a triangulation network data set. 3D coordination transformation results were then used for outlier detection using two approaches, which were analyzed both theoretically and practically. The main observations for the three methods are given below;

In conventional methods;

- an iterative approach is used to reach a solution,
- for each iteration only one outlier appeared and removed from data set,
- the results of method are directly affected by the outliers,
- outlier detection is carried out using residuals, a function of measurement errors and is not completely clear.

In the Fuzzy Logic method;

- results can be interpreted without any iteration operations,
- outlier detection is carried out using a redundancy matrix which matrix is created in the solution estimated with LSM and the relationship between the residuals and the measurement errors. Thus the solution is clearer than the other methods,
- a classification of outliers can be realized by using the membership value of measurements.

As a result, if measurements group include errors as a coordinate transformation methods, these errors are taken into consideration. In the solution the method can be carried out by using as conditional adjustment with unknowns or total least square methods to determined measurements group errors. The outlier detection using residuals instead of measurements errors is uncertain. Therefore, in the solution the measurement errors must be used for the outlier detection.

In this study, it can be seen that the solution of the conditional adjustment with unknowns is significant for 3D coordinate transformation and the Fuzzy Logic method is more applicable than the conventional methods. The further investigations should be focused on total least squares models for coordinate transformation and Artificial Intelligence techniques for outlier detection.

5. References

Aliosmanoglu S. and Akyilmaz, O.: A comparison between statistical & fuzzy techniques in outlier detection, *International Association of Geodesy Symposia*, *Eds.Adam J., Schwarz, K. P., Springer, 382-387, 2002.*

Baarda, W. A.: Testing procedure for use in geodetic networks, *Publications on Geodesy Netherlands Geodetic Commission, Delft, 1968.*

Berberan A.: Multiple outlier detection: a real case study, *Survey Review*, *33, 255, 41-49, 1995*.

Cateni, S., Colla, V., Vannucci, M.: Advances in robotics, *Automation and Control*, www.i-technoline.com, *Eds. Arámburo J. and Treviño, A. R., 272-276, 2008.*

Hampel, F., Ronchetti, E. M., Rousseeuw P. J., Stahel, W. A.: Robust statistics: the approach based on influence functions, *A Wiley-Interscience Publication John Wiley & Sons, New York, USA; 18-34, 100-116, 1986.*

Huber, P.J.: Robust statistics, *John Wiley and Sons Inc., New York,, USA; 1-17, 1981*.

Koch, K.R.: Parameter estimation and hypothesis testing in linear models, *Springer, Berlin-Heidelberg, NewYork, USA, 271-296, 302-307, 1999*.

Konak, H., Dilaver A. and Ozturk, E.: The effects of observation plan and precision on the duration of outlier detection and fuzzy logic: a real network application, *Survey Review, 38, 298, 331-341, 2005*.

Kutoglu, H. S. and T. Ayan: The role of common point distribution in obtaining reliable parameters for coordinate transformation, *Applied Mathematics and Computation, 176, 751-758, 2006*.

Kutoglu, H. S., Ayan T. and Mekik, C.: Integrating GPS with national networks by collocation method, *Applied Mathematics and Computation*, *117, 508-514, 2006*.

Kutterer, H.: Joint treatment of random variability and imprecision in GPS data analysis, *Journal of Global Positioning Systems, 1(2), 96-105, 2002*.

Shi, Y., Ebenhart R. and Chen, Y.: Implementaion of evolutionary fuzzy systems, *IEEE Transactions on Fuzzy Systems, 7(2), 109-119, 1999*.

Sun, W.: A new method for localisation of gross errors, *Survey Review, 32, 252, 344-358, 1994*.

Syed, S., Cannon M.E.: Fuzzy logic based-map matching algorithm for vehicle navigation system in urban canyons, *ION National Technical Meeting, San Diego, CA, 26-28, 2004*.

Thomson, D. B.: Combination of geodetic networks*, Department of Surveying Engineering, University of New Brunswick, Canada, 1976.*

http://itrf.ensg.ign.fr/general.php (accessed 19 September 2011).

Vanicek, P., Wells D.E.: The least squares approximation, *Department of Geodesy and Geomatics Engineering University of New Brunswick, Canada, 1972.*

Xue-Gong, Z.: Takagi-Sugeno Fuzzy logic versus mamdani fuzzy logic, *MSc Thesis, Advanced Technologies in Electronics, University of The West of England, 2001.*

Zadeh, L. A.: Fuzzy sets*, Journal of Information and Control*, *8(3), 338-353, 1965.*