Processing of a geodetic network determined in ETRS-89 with application of different cofactors

Slavomír Labant[1], ***Gabriel Weiss***, ***Pavel Kukučka***, ***Silvia Gašincová*** *and* ***Katka Pukanská***

At present, manufacturers characterize the accuracy of vectors measured by the static method of GNSS technology using relationship $5\ mm + 1 \cdot D\ ppm$. *The advantage of the GNSS system over other terrestrial technologies is that it is not affected by uncertainties in the ground layers of the atmosphere. The paper presents experimental measurement of the 3D geodetic network using the technology of global navigation satellite systems, processing and analysis of measurements taken at the Čierny Váh pumping hydro-power station. Observations were carried out in July 2008. The aim of the paper is to assess parameters used in the model to estimate parameters of the first and second order of the network structures.*

Key words*: 3D local geodetic net, GNSS surveying, LS method, MINQUE*

Introduction

Nowadays, observations of terrain stability are carried out using various methods, mainly of terrestrial nature. These are terrestrial measurements performed by an electronic total station, very precise leveling (VPL), digital photogrammetry, laser scanning and global navigation satellite systems (GNSS). Recent years have seen the expansion of GNSS in different industries and fields, as well as in various areas of land surveying.

The advantage of using GNSS is mainly that it doesn't require a direct line of sight between points under survey. Its use is also advantageous in the areas of the increased nature protection as it eliminates the need to cut paths into the forest for this purpose. When taking measurements using GNSS, there is only one condition that must be fulfilled: clear skies or direct visibility between the antenna of the receiver and the satellites.

Selection of processing space

3D GNSS network structures require only 1 point to be chosen for accurate positioning in a certain 3D coordinate system for constrained adjustment from the set of the reference points available where the selected point is stationary in relation to the surrounding environment. The advantage of this connection is that there is no network deformation caused by the influence of other reference points. If this point itself is not compatible, repeated measurements will show similar distortion of the coordinates of individual points, but not in the differences of the network coordinates. After surveying, the data is transmitted to the respective programming environment where the GPS (GNSS) vectors measured between the surveyed points are processed. The processed output values are provided either in *.txt or *.html format which include information about the vector, its length, accuracy of the measurement, duration of surveying, frequency of the received signal, temperature and pressure input values for atmospheric corrections, Cartesian and geographic coordinates of the initial and terminal points of the vectors as well as other details.

The GNSS systems work in the reference coordinate system called World geodetic system 1984 (WGS-84). The GNSS receiver provides standard results in Cartesian geocentric coordinates related to WGS-84. Three dimensional coordinates obtained by measurement using the GNSS method generally refer to the European Terrestrial Reference System 89 (ETRS-89) with a reference ellipsoid GRS-80 for measurements performed in our continent.

ETRS-89 system was adopted due to the movement of the Eurasian continental plate causing a change in the coordinates of the fixed points. There is a shift between WGS84 and ETRS89 of approximately 2.5 cm per year. Heights are the same in both systems (Hefty et al., 2003; Ferianc et al., 2007).

[1] *MSc. Slavomír Labant, PhD., Prof. MSc. Gabriel Weiss, PhD., MSc. Pavel Kukučka, PhD., MSc. Silvia Gašincová, PhD., MSc. Katka Pukanská , PhD.,* Technical University of Košice, Faculty of Mining, Ecology, Process Control and Geotechnologies, Institute of Geodesy, Cartography and Geographic Information Systems, Letná 9, 042 00 Košice, slavomir.labant@tuke.sk, gabriel.weiss@tuke.sk, pavel.kukucka@tuke.sk, silvia.gasincova@tuke.sk, katka.pukanska@tuke.sk

The values used for computing are geodetic coordinates (ϕ, λ, h) determined on the basis of GNSS measurements which can be converted into 3D Cartesian coordinates within the selected 3D geodetic system (Weiss, 1997). 3D Cartesian coordinates obtained from the approximate geographical coordinates and GNSS measurements are used for further processing.

Processing of the geodetic network by Gauss-Markov estimation model

Gauss–Markov model (GMM) is most commonly used for adjustment of the general geodetic network (GN) which reads:

$$\begin{aligned} & v = Ad\hat{C} - dL = A(\hat{C} - C^\circ) - (L - L^\circ), && \text{- functional segment,} \\ & \Sigma_L = s_0^2 Q_L, && \text{- stochastic segment,} \end{aligned} \qquad (1)$$

where v is a vector of corrections of the observed values, $dL = L - L^\circ$ is a vector of reduced observations, $d\hat{C} = \hat{C} - C^\circ$ is a vector of complements of the adjusted values of the determined coordinates, A is a design matrix.

The initially measured and software-processed data of the GNSS vectors can be processed based on Gauss-Markov estimation model (adjustment of observation equations) as a constrained adjustment. Three dimensional Cartesian coordinates of the reference point obtained using software solution are considered as fixed. The adjusted coordinates $\hat{C}$ of the network points are determined from the pre-processed values measured in the network. In this case, GNSS observation vectors ΔXYZ_{ij} are arranged in L vector in the following order (Weiss et al., 2004; Gašinec et al., 2005; Labant, 2008):

$$\underset{(m,1)}{L} = \begin{pmatrix} \Delta XYZ_{12} \\ \vdots \\ \Delta XYZ_{i,j} \\ \vdots \\ \Delta XYZ_{m-1,m} \end{pmatrix}, \; where: \underset{(3,1)}{\Delta XYZ_{i,j}} = \begin{pmatrix} \Delta X_{i,j} \\ \Delta Y_{i,j} \\ \Delta Z_{i,j} \end{pmatrix} \text{ - observation components.} \qquad (2)$$

The observation vector of the performed measurements L is given by $m-$ of the observation vectors, i.e. $n = 3m$ of the observation components.

$\underset{(k,1)}{C^\circ}$ - is a vector of approximate coordinates of the calculated points with column structure:

$$\underset{(k,1)}{C^\circ} = \begin{pmatrix} X_i^\circ \\ Y_i^\circ \\ Z_i^\circ \end{pmatrix}, \; where \; i = 1, \ldots, b \; of\ the\ points. \qquad (3)$$

$\underset{(n,1)}{L^\circ}$ - is a vector of approximate values of observations obtained from the vector of approximate coordinates $\underset{(k,1)}{C^0}$, it can be expressed as a vector of functions $L^\circ = f(C^\circ)$.

$\underset{(n,1)}{dL}$ - is a vector of components of the values measured:

$$\underset{(n,1)}{dL} = \underset{(n,1)}{L} - \underset{(n,1)}{L^\circ}. \qquad (4)$$

$\underset{(n,1)}{v}$ - a vector of corrections is obtained:

$$\underset{(n,1)}{v} = \underset{(n,k)}{A} \cdot \underset{(k,1)}{d\hat{C}} - \underset{(n,1)}{dL}, \qquad (5)$$

where A is a design matrix – a matrix of partial derivations of the function $\underset{(n,1)}{L^\circ} = f(C^\circ)$ according to the vector of the parameters $\underset{(k,1)}{C^\circ}$:

$$\underset{(n,k)}{A} = \left(\frac{\partial L^\circ}{\partial C^\circ} = \frac{\partial f(C^\circ)}{\partial C^\circ} = \{-1,\ 0,\ 1\} \right)_{\hat{C} \doteq C^\circ}. \qquad (6)$$

$\underset{(n,n)}{Q_L}$ - a diagonal cofactor matrix of the vector $\underset{(n,1)}{L}$ that will have the structure:

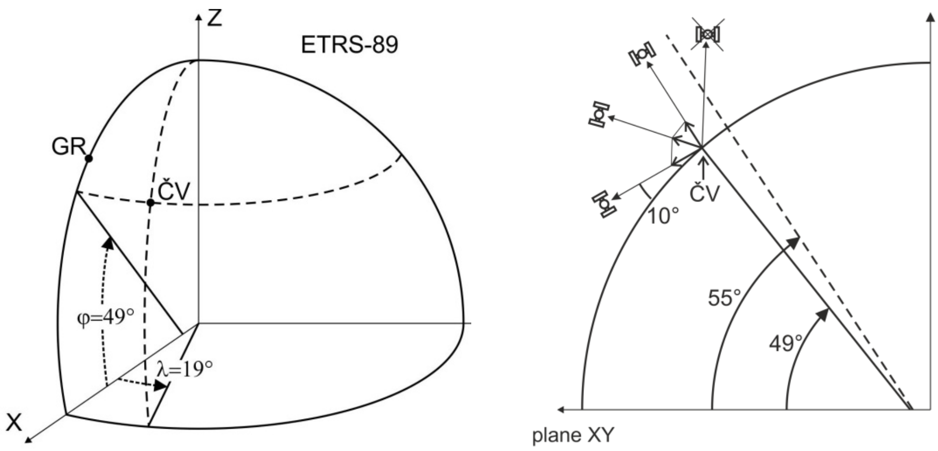
$$\underset{(n,n)}{Q_L} = \begin{pmatrix} \underset{(3,3)}{Q_{l_{12}}} & & & & \\ & \ddots & & & \\ & & \underset{(3,3)}{Q_{l_{i,j}}} & & \\ & & & \ddots & \\ & & & & \underset{(3,3)}{Q_{l_{m-1,m}}} \end{pmatrix} \quad \text{with cofactor submatrices,} \quad \underset{(3,3)}{Q_{l_{i,j}}} = \begin{pmatrix} q_{\Delta X \Delta X} & q_{\Delta X \Delta Y} & q_{\Delta X \Delta Z} \\ q_{\Delta Y \Delta X} & q_{\Delta Y \Delta Y} & q_{\Delta Y \Delta Z} \\ q_{\Delta Z \Delta X} & q_{\Delta Z \Delta Y} & q_{\Delta Z \Delta Z} \end{pmatrix}_{i,j}, \qquad (7)$$

elements $Q_{l_{i,j}}$ correspond to individual GNSS observation vectors $\Delta XYZ_{i,j}$, elements in the submatrix $Q_{l_{i,j}}$ on the main diagonals are cofactors of the values $(\Delta X_{i,j}, \Delta Y_{i,j}, \Delta Z_{i,j})$, elements off the main diagonal are mixed cofactors.

Apart from the measured observations $L_{i,j}$ and approximate values of coordinates $C_i°$, cofactors $q_{i,j}$ are also used as input data for thus built up model. The following 4 cofactors were used for processing (Caspary, 1987; Pukanská et al., 2007; Sabova et al., 2007) (Tab. 2):

1. ${}^{(1)}q_{l\,i,j} = 5\ mm\ +\ 1 \cdot D\ ppm$,where D is length of vector, and ppm is parts per million, which is recommended by the manufacturer of GNSS receivers for the static method,
2. ${}^{(2)}q_{l\,i,j}$ from covariance matrices $\Sigma_{l\,i,j}$ obtained from processing of observations in the programming environment for individual GNSS vectors with empirical a posteriori unit variance $s_0^2 = 1$,
3. ${}^{(3)}q_{l\,i,j}$ from covariance matrices $\Sigma_{l\,i,j}$, with empirical a posteriori unit variance $s_0^2 = RMS$ (root mean square) also obtained from the same pre-processing of values,
4. ${}^{(4)}q_{l\,i,j}$ using the MINQUE method for estimation of cofactors.

The Slovak republic (SR) lies 49° north latitude and 19° east longitude (Fig. 1a). The area of the SR is not symmetrically distributed with respect to the individual axes of the ETRS-89 coordinate system. Taking into account the fact that the orbital inclination of GNSS satellites, GPS in particular, is 55°to the equator (Fig. 1b), there occurs an unfavorable geometric configuration of satellites and the accuracy of determining the Z – coordinate is worse. The evidence of that is the fact that numerical values of covariance matrices from the initial processing of observations in the respective programming environment usually refer the highest weights to the components in the direction of the axis Y, lower weights in the direction of the axis X and the lowest in the direction of the axis Z.



a) b)

Fig. 1. a) The area of SR in the coordinate system ERTS-89 b) configuration of the satellites for the SR area.

One of the methods used for estimation of the elements of matrix Q_L is the MINQUE method – Minimum Norm Quadratic Unbiased Estimation (Skořepa et al., 1998; Gašinec et al., 2005). The solution of the task is based on the functional part of the Gauss-Markov estimation model $v = A \cdot d\hat{C} - dL$, for the vector of residuals. Combination of various geodetic devices and measuring methods used to obtain a vector of the measured values in the terrain are described by the stochastic part of the model as follows:

$$C_l = \vartheta_X^0 V_X + \vartheta_Y^0 V_Y + \vartheta_Z^0 V_Z\,, \tag{8}$$

where coefficients of the linear combination ϑ_i^0 are a priori variance components, V_i are corresponding positive semi definite matrices of order n. Impartial invariant quadratic estimate (MINQUE) with a minimum norm of variance components $(\vartheta_X^0, \vartheta_Y^0, \vartheta_Z^0)^T$ for the estimation of parameters of order 2 is given by the relation:

$$\begin{pmatrix} tr(MV_X MV_X) & \cdots & tr(MV_X MV_Z) \\ \vdots & \ddots & \vdots \\ tr(MV_Z MV_X) & \cdots & tr(MV_Z MV_Z) \end{pmatrix} \begin{pmatrix} \hat{\vartheta}_X^0 \\ \hat{\vartheta}_Y^0 \\ \hat{\vartheta}_Z^0 \end{pmatrix} = \begin{pmatrix} dL^T MV_X MdL^T \\ \vdots \\ dL^T MV_Z MdL^T \end{pmatrix}. \tag{9}$$

Matrix M is calculated from the relation:

$$M = C_l^{-1} - C_l^{-1} A (A^T C_l^{-1} A)^{-1} A^T C_l^{-1}. \tag{10}$$

The estimated coefficients of the linear combination $(\hat{\vartheta}_X^0,\ \hat{\vartheta}_Y^0,\ \hat{\vartheta}_Z^0)^T$ are used again in the relation (8). The relation (9) then shows the improved values. The iteration cycle is completed upon fulfillment of the condition:

$$\left|\hat{\vartheta}_i - \vartheta_i\right| \le \delta, \qquad i = 1,\ 2,\ \ldots,\ n, \tag{11}$$

where δ is a preselected constant from the set of real numbers by which the required degree of accuracy of the estimated parameters is defined.

Experimental measurement of GN and evaluation of GNSS signals

Sokkia Stratus GPS receivers were used for receiving GNSS signals as they are well suitable for local measurements with vectors up to the distance 10-12 km, where the accuracy claimed by the manufacturer for the static method on the ground surface is $5\ mm + 1 \cdot D\ ppm$ in the horizontal direction and $10\ mm + 1 \cdot D\ ppm$ in the vertical direction. The GN measurement consisting of 7 points was carried out in July 2008 within one day (Fig. 2). The measurement was performed using the static method based on the principle of relative determination of the position at the Čierny Váh pumping hydro-power station (PHPS). Primary evaluation of the results was carried out in the post processing regime in the software environment of Spectrum Survey.

Observation was performed using 4 single frequency GPS receivers. Measurement by the static method was taken in turn over all the points of the network: 5001 to 5007. Point 5001 was chosen as a GN reference point as it was establishing in the largest and flattest area. The GPS receiver was left in the operating in this point during the whole measurement process. After achieving the number of observations necessary for the processing of GPS vector 25 to 30 km long, GPS receivers were subsequently moved to other points of the network. Measurement of the GPS network was carried out using 11 vectors (Fig. 2).

Tab. 1 shows the sequence and duration of observation in individual stations. Reception of signals in individual network points was 1 do 7 hours (for reference point 5001). During the measurement, the number of satellites tracked varied from 4 to10. The data obtained was processed in the software environment of Spectrum Survey. The results of the measurement were geographical coordinates φ, λ and height h in the individual points of the network.

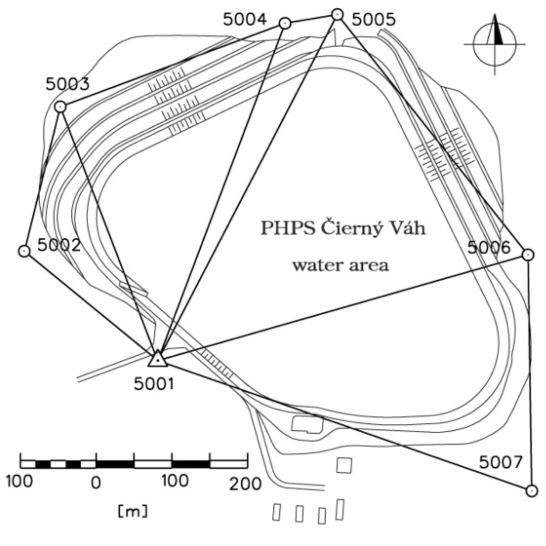


Fig. 2. 3D network structure at a scale 1:10 000.

WGS-84 system was transformed into ETRS-89 system using Trimble Geomatics Office for the reference point 5001 where also files in RINEX 2.11 format were imported from SKLM (Cadastral Registry Liptovský Mikuláš) and GANP (Gánovce) reference stations into software environment for post processing of the network.

Tab. 1. The sequence and duration of observation in individual stations in July 2008.

Measuring Day	Point number	Antenna number	Start time	Stop time	Time of observation
			[h:m:s]	[h:m:s]	[h:m:s]
25.07.2008	5001	MSD01420577	11:13:00	18:16:20	7:03:20
	5002	MSD01420584	11:56:40	14:12:30	2:15:50
	5003	MSD01420629	12:24:30	15:08:20	2:43:50
	5004	MSD01420557	14:09:30	15:53:50	1:44:20
	5005	MSD01420584	14:36:20	17:16:20	2:40:00
	5006	MSD01420629	15:39:20	17:43:20	2:04:00
	5007	MSD01420557	16:24:00	17:33:40	1:09:40

Comparison of standard deviations from estimates of the measured values with the choice of different cofactors

Processing of GN was carried out with the number of observation vectors $m = 11$, observation components $n = 3m = 33$, then with the number of the calculated points $b = 6$ and coordinates $k = 3b = 18$. These quantities determined the size of vectors and matrices both entering into and coming out of GMM. In the diagonal matrix Q_L, the values on its main diagonal line showed variable results for different cofactors being used (chapter 3). Input and output values in the processing and adjustment with different cofactors are given in Tab. 2. Output values in comparison with cofactors were not so varied. Average values of accuracy of the adjusted observations measured from covariance matrices $\Sigma_{\hat{L}}$ were calculated based on the input parameters.

Higher weight value should be assigned to a measurement with higher accuracy and should receive a minor part of the total correction. Software processing assigns observations to weights according to the number of satellites (from which data is received), configuration of satellites (or their constellation), frequency of signal reception as well as other factors.

Tab. 2. Processing and adjustment of GN with selection of cofactors: 1.) 5mm+1 D ppm, 2.) s_0^2=1, 3.) s_0^2=RSM, 4.) MINQUE.

	vector		L	$L°$	dL	q_L				$\hat{L}$ [m]				v [mm]			
			[m]	[m]	[mm	1	2	3	4	1	2	3	4	1	2	3	4
1	5001	ΔX	-38.645	-38.645	0	25.39	44.72	4.24	56.59	-38.647	-38.648	-38.648	-38.647	-1.73	-3.27	-3.27	-1.73
2	5002	ΔY	-210.804	-210.804	0	27.15	27.73	2.63	27.29	-210.808	-210.809	-210.809	-210.808	-4.44	-5.14	-5.14	-4.31
3		ΔZ	78.134	78.134	0	25.79	66.80	6.33	32.94	78.137	78.138	78.138	78.137	2.89	3.93	3.93	2.95
4	5001	ΔX	-205.632	-205.632	0	27.10	75.89	7.20	56.59	-205.633	-205.634	-205.634	-205.632	-0.54	-2.00	-2.00	-0.46
5	5003	ΔY	-234.013	-234.013	0	27.40	44.87	4.26	27.29	-234.010	-234.009	-234.009	-234.010	3.43	4.07	4.07	3.39
6		ΔZ	183.516	183.516	0	26.87	128.56	12.19	32.94	183.518	183.518	183.518	183.518	1.80	2.26	2.26	1.90
7	5001	ΔX	-379.835	-379.835	0	28.94	50.35	4.78	56.59	-379.843	-379.843	-379.843	-379.843	-7.89	-8.32	-8.32	-7.65
8	5004	ΔY	10.424	10.424	0	25.10	24.61	2.33	27.29	10.429	10.430	10.430	10.429	5.37	5.80	5.80	5.47
9		ΔZ	287.840	287.840	0	27.96	104.64	9.92	32.94	287.830	287.828	287.828	287.831	-9.53	*-12.08*	*-12.08*	-9.24
10	5001	ΔX	-411.424	-411.424	0	29.28	49.62	4.71	56.59	-411.413	-411.414	-411.414	-411.413	*10.98*	*10.41*	*10.41*	*10.5*
11	5005	ΔY	71.432	71.432	0	25.72	29.68	2.81	27.29	71.430	71.429	71.429	71.430	-2.14	-2.64	-2.64	-1.97
12		ΔZ	302.125	302.125	0	28.11	91.61	8.69	32.94	302.127	302.124	302.124	302.127	2.40	-1.43	-1.43	2.37
13	5001	ΔX	-285.836	-285.836	0	27.94	55.56	5.27	56.59	-285.838	-285.838	-285.838	-285.838	-1.65	-1.93	-1.93	-1.79
14	5006	ΔY	405.728	405.728	0	29.22	32.07	3.04	27.29	405.723	405.723	405.723	405.724	-4.51	-5.46	-5.46	-4.39
15		ΔZ	110.230	110.230	0	26.11	88.32	8.38	32.94	110.230	110.228	110.228	110.230	0.35	-1.69	-1.69	0.35
16	5001	ΔX	-74.748	-74.748	0	25.75	30.02	2.85	56.59	-74.747	-74.747	-74.747	-74.747	1.15	1.48	1.48	1.10
17	5007	ΔY	508.852	508.852	0	30.35	19.56	1.85	27.29	508.854	508.854	508.854	508.854	1.88	1.76	1.76	1.81
18		ΔZ	-95.337	-95.337	0	25.96	51.10	4.85	32.94	-95.335	-95.336	-95.336	-95.335	1.64	0.93	0.93	1.67
19	5002	ΔX	-166.984	-166.987	3	26.70	34.27	3.25	56.59	-166.986	-166.986	-166.986	-166.986	-1.81	-1.73	-1.73	-1.73
20	5003	ΔY	-23.197	-23.209	12	25.23	15.51	1.47	27.29	-23.201	-23.200	-23.200	-23.201	-4.13	-2.79	-2.79	-4.31
21		ΔZ	105.378	105.382	-4	26.07	42.11	3.99	32.94	105.381	105.380	105.380	105.381	2.92	2.33	2.33	2.95
22	5003	ΔX	-174.208	-174.203	-5	26.77	23.67	2.24	56.59	-174.210	-174.209	-174.209	-174.210	-2.35	-1.32	-1.32	-2.19
23	5004	ΔY	244.440	244.437	3	27.50	11.70	1.11	27.29	244.439	244.439	244.439	244.439	-1.06	-1.27	-1.27	-0.92
24		ΔZ	104.308	104.324	-16	26.05	30.75	2.92	32.94	104.313	104.310	104.310	104.313	4.67	1.66	1.66	4.85
25	5004	ΔX	-31.561	-31.589	28	25.32	57.59	5.46	56.59	-31.570	-31.570	-31.570	-31.571	-9.13	-9.28	-9.28	-9.83
26	5005	ΔY	60.996	61.008	-12	25.61	35.77	3.39	27.29	61.000	61.000	61.000	61.001	4.49	3.56	3.56	4.56
27		ΔZ	14.301	14.285	16	25.14	58.53	5.55	32.94	14.297	14.296	14.296	14.297	-4.07	-5.35	-5.35	-4.39
28	5005	ΔX	125.575	125.588	-13	26.27	47.11	4.47	56.59	125.575	125.576	125.576	125.576	0.38	0.66	0.66	0.69
29	5006	ΔY	334.291	334.296	-5	28.46	23.03	2.18	27.29	334.294	334.293	334.293	334.294	2.63	2.17	2.17	2.58
30		ΔZ	-191.895	-191.895	0	26.96	58.07	5.51	32.94	-191.897	-191.895	-191.895	-191.897	-2.06	-0.25	-0.25	-2.02
31	5006	ΔX	211.092	211.088	4	27.16	13.06	1.24	56.59	211.091	211.091	211.091	211.091	-1.21	-0.59	-0.59	-1.10
32	5007	ΔY	103.132	103.124	8	26.04	8.86	0.84	27.29	103.130	103.131	103.131	103.130	-1.61	-0.77	-0.77	-1.81
33		ΔZ	-205.564	-205.567	3	27.10	25.16	2.39	32.94	-205.566	-205.564	-205.564	-205.566	-1.71	-0.39	-0.39	-1.67

	vector		L	L°	dL	$s_{\hat{L}}$ [mm]				r				Pope - τ_{KRIT} = 3.132							
			[m]	[m]	[mm]	1	2	3	4	1	2	3	4	1		2		3		4	
1	5001 5002	ΔX	-38.645	-38.645	0	4.79	4.88	4.88	5.91	0.37	0.41	0.41	0.38	0.47	+	0.79	+	0.79	+	0.37	+
2		ΔY	-210.804	-210.804	0	4.86	3.70	3.70	4.11	0.39	0.46	0.46	0.38	1.14	+	1.51	+	1.51	+	1.33	+
3		ΔZ	78.134	78.134	0	4.79	5.89	5.89	4.51	0.38	0.43	0.43	0.38	0.77	+	0.77	+	0.77	+	0.83	+
4	5001 5003	ΔX	-205.632	-205.632	0	4.26	4.45	4.45	5.17	0.53	0.69	0.69	0.53	0.12	+	0.28	+	0.28	+	0.08	+
5		ΔY	-234.013	-234.013	0	4.26	3.36	3.36	3.59	0.54	0.72	0.72	0.53	0.75	+	0.75	+	0.75	+	0.89	+
6		ΔZ	183.516	183.516	0	4.23	5.51	5.51	3.94	0.53	0.72	0.72	0.53	0.40	+	0.24	+	0.24	+	0.46	+
7	5001 5004	ΔX	-379.835	-379.835	0	4.22	4.31	4.31	5.05	0.57	0.56	0.56	0.55	1.63	+	1.59	+	1.59	+	1.37	+
8		ΔY	10.424	10.424	0	4.09	3.18	3.18	3.51	0.53	0.54	0.54	0.55	1.23	+	1.65	+	1.65	+	1.41	+
9		ΔZ	287.840	287.840	0	4.17	5.47	5.47	3.86	0.57	0.68	0.68	0.55	2.01	+	1.49	+	1.49	+	2.17	+
10	5001 5005	ΔX	-411.424	-411.424	0	4.23	4.36	4.36	5.05	0.57	0.58	0.58	0.55	*2.24*	+	*2.03*	+	*2.03*	+	*1.89*	+
11		ΔY	71.432	71.432	0	4.13	3.34	3.34	3.51	0.54	0.58	0.58	0.55	0.48	+	0.66	+	0.66	+	0.51	+
12		ΔZ	302.125	302.125	0	4.18	5.33	5.33	3.86	0.57	0.66	0.66	0.55	0.50	+	0.19	+	0.19	+	0.56	+
13	5001 5006	ΔX	-285.836	-285.836	0	4.29	4.09	4.09	5.17	0.54	0.67	0.67	0.53	0.35	+	0.33	+	0.33	+	0.33	+
14		ΔY	405.728	405.728	0	4.38	3.16	3.16	3.59	0.54	0.65	0.65	0.53	0.95	+	1.24	+	1.24	+	1.16	+
15		ΔZ	110.230	110.230	0	4.23	5.18	5.18	3.94	0.52	0.67	0.67	0.53	0.08	+	0.23	+	0.23	+	0.08	+
16	5001 5007	ΔX	-74.748	-74.748	0	4.82	4.05	4.05	5.91	0.37	0.40	0.40	0.38	0.31	+	0.45	+	0.45	+	0.24	+
17		ΔY	508.852	508.852	0	5.06	3.21	3.21	4.11	0.41	0.42	0.42	0.38	0.45	+	0.64	+	0.64	+	0.56	+
18		ΔZ	-95.337	-95.337	0	4.82	5.24	5.24	4.51	0.37	0.41	0.41	0.38	0.44	+	0.21	+	0.21	+	0.47	+
19	5002 5003	ΔX	-166.984	-166.987	3	4.84	4.64	4.64	5.91	0.39	0.31	0.31	0.38	0.47	+	0.55	+	0.55	+	0.37	+
20		ΔY	-23.197	-23.209	12	4.79	3.25	3.25	4.11	0.37	0.26	0.26	0.38	1.14	+	1.46	+	1.46	+	1.33	+
21		ΔZ	105.378	105.382	-4	4.80	5.32	5.32	4.51	0.38	0.27	0.27	0.38	0.77	+	0.73	+	0.73	+	0.83	+
22	5003 5004	ΔX	-174.208	-174.203	-5	4.66	4.00	4.00	5.64	0.43	0.25	0.25	0.44	0.58	+	0.55	+	0.55	+	0.44	+
23		ΔY	244.440	244.437	3	4.66	2.86	2.86	3.92	0.45	0.24	0.24	0.44	0.25	+	0.80	+	0.80	+	0.27	+
24		ΔZ	104.308	104.324	-16	4.61	4.66	4.66	4.31	0.43	0.21	0.21	0.44	1.17	+	0.66	+	0.66	+	1.28	+
25	5004 5005	ΔX	-31.561	-31.589	28	4.59	5.03	5.03	5.61	0.42	0.48	0.48	0.44	2.35	+	1.77	+	1.77	+	1.96	+
26		ΔY	60.996	61.008	-12	4.54	3.83	3.83	3.89	0.44	0.53	0.53	0.44	1.12	+	0.84	+	0.84	+	1.31	+
27		ΔZ	14.301	14.285	16	4.56	5.64	5.64	4.28	0.42	0.35	0.35	0.44	1.04	+	1.15	+	1.15	+	1.15	+
28	5005 5006	ΔX	125.575	125.588	-13	4.65	4.77	4.77	5.64	0.42	0.47	0.47	0.44	0.09	+	0.15	+	0.15	+	0.14	+
29		ΔY	334.291	334.296	-5	4.74	3.52	3.52	3.92	0.45	0.41	0.41	0.44	0.62	+	0.74	+	0.74	+	0.75	+
30		ΔZ	-191.895	-191.895	0	4.65	5.62	5.62	4.31	0.44	0.40	0.40	0.44	0.50	+	0.06	+	0.06	+	0.53	+
31	5006 5007	ΔX	211.092	211.088	4	4.88	3.14	3.14	5.91	0.39	0.17	0.17	0.38	0.31	+	0.41	+	0.41	+	0.24	+
32		ΔY	103.132	103.124	8	4.91	2.56	2.56	4.11	0.35	0.19	0.19	0.38	0.45	+	0.62	+	0.62	+	0.56	+
33		ΔZ	-205.564	-205.567	3	4.86	4.28	4.28	4.51	0.39	0.20	0.20	0.38	0.44	+	0.18	+	0.18	+	0.47	+

On the whole, from the point of view of standard deviations, the highest accuracy is achieved with the use of covariance matrices with the RMS value as the a priori variance factor. An average standard deviation in the measurement is $s_{\hat{L}} = 3.96$ *mm.* These deviation values are most approximate to the method of applying $s_0^2 = 1$ model where average deviation is $\bar{s}_{\hat{L}} = 4.00$ *mm.* Both methods consider individual vectors in the directions of the axis of the chosen coordinate system separately.

When cofactors are used according to the recommendation of the manufacturer of receivers, all the measurements are assigned weights using the relation: $5\ mm + 1 \cdot D\ ppm$.

Accuracy in determining the vectors of observations depends on their length, thus the bigger the distance between the surveyed points the lower is the anticipated accuracy. Average deviation of the measured values obtained by numerical computation was $\bar{s}_{\hat{L}} = 4.57$ *mm.*

MINQUE is a mathematical method used for determination of the best estimate of the measured values. The average value of standard deviations determined using the MINQUE method is $\bar{s}_{\hat{L}} = 4.46$ *mm.* This method was used for independent assessment of cofactors (weights) in the direction of the axes X, Y and Z. Through application of this method, the highest weights were estimated and assigned to vectors in the direction of the axis Y, then X and the lowest in the direction of the axis Z.

Differences between average values are not significant, however, as Tab. 2 shows, numerical values for individual quantities observed when using different cofactors vary. Different values in corrections v, standard deviations of estimates of the measured values $s_{\hat{L}}$ as well as in redundancies of measurements r are given in Tab. 2. The table column Pope shows Pope Test, in particular, its testing criterion and the test result, where "+" shows that the observation component passed the test and "-" failed the test (erroneous value – exclude from measurement). Tab. 2 shows the observed values in the order $\Delta X_{i,j}$, $\Delta Y_{i,j}$, $\Delta Z_{i,j}$ for individual GPS vectors.

Dependence of coordinate estimation on the choice of cofactors

The values of the parameters related to the coordinates of the calculated network points depend on the choice of cofactors of the measured quantities. $\underset{(k,1)}{\hat{C}}$ - a vector of the coordinate estimates of the points surveyed is determined as (Weiss et al., 2008; Weiss et al., 2010):

$$\underset{(k,1)}{\hat{C}} = \underset{(k,1)}{C^\circ} + \underset{(k,1)}{d\hat{C}}, \tag{12}$$

where $d\hat{C}$ is a vector of estimates of the calculated coordinates most commonly in *mm*:

$$d\hat{C} = \left(A^T Q_L^{-1} A\right)^{-1} A^T Q_L^{-1}(L - L^\circ) = N^{-1} A^T Q_L^{-1} dL, \tag{13}$$

where A and dL are equal in all types of processing, differences in $d\hat{C}$ depend on the type of cofactors used $q_{Li,i+1}$. Similarly, standard deviations from estimates of the coordinates of the calculated points $s_{\hat{C}}$ obtained from the main diagonal of the covariance matrix:

$$\Sigma_{\hat{C}} = s_0^2 . Q_{\hat{C}}, \tag{14}$$

where the cofactor matrix is $Q_{\hat{C}} = \left(A^T . Q_L^{-1} . A\right)^{-1}$ and a unit a priori variance factor $s_0^2 = \dfrac{v^T . Q_L^{-1} . v}{n-k}$.

Tab. 3 gives approximate values of the coordinates of the points C° (in the order $X^\circ, Y^\circ, Z^\circ$), estimates of coordinate cofactors to approximate coordinates $d\hat{C}$ as well as estimates of the coordinates of the calculated points $\hat{C}$ with standard deviations of the estimates of coordinate points $s_{\hat{C}}$ of processing when choosing different types of cofactors.

As the GN is adjusted with reference point 5001, this reference point is not included in the adjustment and the values of standard deviations of coordinates of this reference point are $s_{\hat{X}}^2 = s_{\hat{Y}}^2 = s_{\hat{Z}}^2 = 0$.

Using standard deviations of estimates of the calculated point coordinates we can determine:

- mean space error s_{pi} and average space error $\bar{s}_p$ (Tab. 4):

$$s_{pi} = \sqrt{s_{\hat{X}i}^2 + s_{\hat{Y}i}^2 + s_{\hat{Z}i}^2}, \qquad \bar{s}_p = \frac{1}{b}\sum_{i=1}^{b} s_{pi}, \tag{15}$$

  - mean coordinate value $s_{\hat{X}\hat{Y}\hat{Z}i}$ and average coordinate error $\bar{s}_{\hat{X}\hat{Y}\hat{Z}}$ (Tab. 4):

$$s_{\hat{X}\hat{Y}\hat{Z}i} = \sqrt{\frac{s_{\hat{X}i}^2 + s_{\hat{Y}i}^2 + s_{\hat{Z}i}^2}{3}} = \frac{s_{pi}}{\sqrt{3}}, \qquad \bar{s}_{\hat{X}\hat{Y}\hat{Z}} = \frac{1}{k}\sum_{i=1}^{k} s_{\hat{X}\hat{Y}\hat{Z}i}, . \tag{16}$$

Tab. 3. Estimates of GMM coordinates of the GN adjustment; q: 1.) 5mm+1 D ppm, 2.) s_0^2=1, 3.) s_0^2=RSM, 4.) MINQUE.

Point number		C° [m]	$d\hat{C}$ [mm]				$s_{\hat{C}}$ [mm]				$\hat{C}$ [m]			
			1	2	3	4	1	2	3	4	1	2	3	4
5002	X	3 941 063.361	-1.73	-3.27	-3.27	-1.73	4.79	4.88	4.88	5.91	3 941 063.359	..3.358	..3.358	..3.359
	Y	1 427 021.991	-4.44	-5.14	-5.14	-4.31	4.86	3.70	3.70	4.11	1 427 021.987	..1.986	..1.986	..1.987
	Z	4 792 984.570	2.89	3.93	3.93	2.95	4.79	5.89	5.89	4.51	4 792 984.573	..4.574	..4.574	..4.573
5003	X	3 940 896.374	-0.54	-2.00	-2.00	-0.46	4.26	4.45	4.45	5.17	3 940 896.373	..6.372	..6.372	..6.374
	Y	1 426 998.782	3.43	4.07	4.07	3.39	4.26	3.36	3.36	3.59	1 426 998.785	..8.786	..8.786	..8.785
	Z	4 793 089.952	1.80	2.26	2.26	1.90	4.23	5.51	5.51	3.94	4 793 089.954	..9.954	..9.954	..9.954
5004	X	3 940 722.171	-7.89	-8.32	-8.32	-7.65	4.22	4.31	4.31	5.05	3 940 722.163	..2.163	..2.163	..2.163
	Y	1 427 243.219	5.37	5.80	5.80	5.47	4.09	3.18	3.18	3.51	1 427 243.224	..3.225	..3.225	..3.224
	Z	4 793 194.276	-9.53	*-12.08*	*-12.08*	-9.24	4.17	5.47	5.47	3.86	4 793 194.266	..4.264	..4.264	..4.267
5005	X	3 940 690.582	*10.98*	*10.41*	*10.41*	*10.52*	4.23	4.36	4.36	5.05	3 940 690.593	..0.592	..0.592	..0.593
	Y	1 427 304.227	-2.14	-2.64	-2.64	-1.97	4.13	3.34	3.34	3.51	1 427 304.225	..4.224	..4.224	..4.225
	Z	4 793 208.561	2.40	-1.43	-1.43	2.37	4.18	5.33	5.33	3.86	4 793 208.563	..8.560	..8.560	..8.563
5006	X	3 940 816.170	-1.65	-1.93	-1.93	-1.79	4.29	4.09	4.09	5.17	3 940 816.168	..6.168	..6.168	..6.168
	Y	1 427 638.523	-4.51	-5.46	-5.46	-4.39	4.38	3.16	3.16	3.59	1 427 638.518	..8.518	..8.518	..8.519
	Z	4 793 016.666	0.35	-1.69	-1.69	0.35	4.23	5.18	5.18	3.94	4 793 016.666	..6.664	..6.664	..6.666
5007	X	3 941 027.258	1.15	1.48	1.48	1.10	4.82	4.05	4.05	5.91	3 941 027.259	..7.259	..7.259	..7.259
	Y	1 427 741.647	1.88	1.76	1.76	1.81	5.06	3.21	3.21	4.11	1 427 741.649	..1.649	..1.649	..1.649
	Z	4 792 811.099	1.64	0.93	0.93	1.67	4.82	5.24	5.24	4.51	4 792 811.101	..1.100	..1.100	..1.101

When determining the estimates of coordinates of the surveyed points, the solution with cofactors obtained from a covariance matrix with provided estimates of coordinates of the calculated points with the lowest standard deviations .

Tab. 4. Mean and average coordinate and space errors; q: 1.) 5mm+1 D ppm, 2.) $s_0^2 = 1$, 3.) $s_0^2 = RSM$, 4.) MINQUE.

Point number	**Mean coordinate value** [mm]				**Mean space error** [mm]			
	1	**2**	**3**	**4**	**1**	**2**	**3**	**4**
5002	4.813	4.905	4.905	4.906	8.336	8.495	8.495	8.497
5003	4.252	4.522	4.522	4.288	7.366	7.832	7.832	7.427
5004	4.162	4.418	4.418	4.192	7.209	7.653	7.653	7.262
5005	4.182	4.420	4.420	4.192	7.244	7.656	7.656	7.262
5006	4.302	4.221	4.221	4.288	7.452	7.310	7.310	7.427
5007	4.904	4.245	4.245	4.906	8.493	7.353	7.353	8.497
	Average coordinate error [mm]				**Average space error** [mm]			
Network	**4.436**	**4.455**	**4.455**	**4.462**	**7.683**	**7.717**	**7.717**	**7.729**

Conclusion

Determination of the accuracy of points measured using GNSS technology based on a posteriori or empirical characteristics has proved to be advantageous making all the initial data from observations constantly available.

In the presented paper, the observed values were primarily processed in the software environment of Spectrum Survey. Using the software environment of Trimble Geomatic Office, coordinates of the observed points were transformed into ETRS-89 coordinate system, which also considers movement of the Euro-Asian lithospheric plate and thus the shift towards the global WGS-84 system.

Processing was performed using Gauss-Markov model of adjustment of observation equations where constrained adjustment was applied. Observations were processed and adjusted using 4 types of different cofactors (weights), namely according to the recommendations of the manufacturer of the satellite receivers $5\ mm + 1 \cdot D\ ppm$, using covariance matrices from Spectrum Survey, where $s_0^2 = RMS$ was used as a priori unit variance factor, and the constant $s_0^{\ 2} = 1$. In the fourth case, the cofactors (weights) of the observation components were also estimated by the MINQUE method. Application of covariance matrices from Spectrum Survey proved to be the most accurate solution in terms of standard deviations. When using the MINQUE method, weights were estimated based on the accuracy recommended by the manufacturer, where the numerical results of the processing showed the application of the MINQUE method as a suitable alternative to tedious entering of covariance matrices from the software environment of Spectrum Survey. The highest weights were estimated and assigned to the vectors in the direction of the axis Y, the X and the lowest in the direction of the axis Z using the MINQUE method.

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