Long Combo strategy using barrier options and its application in hedging against a price drop

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This paper presents hedging analysis against an underlying price drop by Long Combo strategy using barrier options. Application of hedging results in SPDR Gold Shares is performed as well. New approach of hedging analysis based on the profit functions is introduced. The main theoretical contribution is the derivation of profit functions in analytical form from secured position by Long Combo strategy constructed by barrier options. The practical application of the proposed hedging strategies in SPDR Gold Shares and the comparison of obtained hedging alternatives is the main practical benefit.

Keywords: *hedging, Long Combo strategy, barrier options, SPDR Gold Shares*

Introduction

High level of interconnection and integration in the global economy, dynamics of price movement and other trends in the last decades have led to raising requirements on optimal and effective market risk management so-called risk from price movement. Appropriate way of efficient risk management is hedging by options and options strategies. The main idea of hedging is to add new asset or assets (usually derivatives) to risky asset in order to create new portfolio, so-called hedging portfolio, hedged against an unfavourable price movement. (Carol, 2008), (Chorafas, 2008), (Zmeškal, 2004) By hedging trades we preventatively eliminate the sources of risk to avoid them completely (perfect hedge) or partially (quantile/shortfall hedge) (Tichý, 2004). It is the so-called offensive approach to risk. Basically it is the protection of existing asset or portfolio of asset. According to Tichý (2009), hedging is distinguished in a wider sense - hedging against financial risk and in the narrow sense - hedging against an unfavourable market price movement. Further on we will be using hedging in its narrow sense.

Option strategies using vanilla options can be found in many papers for example (Cohen, 2005), (Fontanills, 2005), (Frederick, 2007), (Smith, 2008), (Šoltés V., 2002), (Thomsett, 2008). In option strategy analysis the same approach is usually used. Way or ways of the given strategy formation are described, plot of payoff function is depicted, sometimes the payoff at expiration time is presented. Another approach, first used by Šoltés, is based on finding of profit functions in their analytical form. This approach allows to find the optimal algorithm for option trading strategy in case of strategies with more ways of formation as well as exactly expressing secured positions in hedging against an unfavourable price movement of an underlying asset. Many authors use this approach of analyzing (Amaitiek, Bálint, Rešovský, 2010), (Šoltés M., 2006), (Šoltés V., 2001), (Šoltés M., 2010a, 2010b), (Šoltés V., Šoltésová, 2003), (Šoltés V., 2003), (Šoltés V., 2005), (Šoltés V., Šoltés M., 2005a, 2005b), (Šoltés V., Amaitiek, 2010, 2011). The most option strategies is probably presented in the work (Šoltés V., 2002).

First aim of the theoretical and analytical section is derive profit functions in analytical form for ways of well-known Long Combo (LC) strategy using barrier options. LC strategy is suitable for hedging against a price drop. Second aim of this section is to propose hedging possibilities by Long Combo strategy using barrier options, to derive profit functions from secured position and to make their detail analysis. The work (Šoltés V., Šoltés M., 2005b) is dealing with the usage of this strategy in hedging against price drop using vanilla options. The main conclusions of this work are: using the Long Combo strategy created with vanilla options we can hedge a constant price of an underlying asset in case of price drop; it is possible to construct this strategy as a zero-cost strategy; the disadvantage of the given strategy is the impossibility to participate in the price increase, as the hedging cost.

The main aim of the practical section is to apply hedging by Long Combo strategy created by barrier options in SPDR Gold Shares. We use call and put option data on these shares with expiration time January 2013. Based on the Black-Sholes model (Black, Sholes, 1973), we calculate particular barrier option prices

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in statistical programme R. The classic version of the model is not designed for barrier options, but by its modification Merton (1973) derived the first formula for calculation of down and knock-out call European type option price. By analogy, Cox and Rubinstein (1985) derived the formula for calculation of up and knock-out call option price and later Rubinstein and Reiner (1991) and Haug (1998) applied Black-Sholes-Merton formula for every basic types of European type barrier options.

The fundamental method is analysis. We analyse individual ways of Long Combo strategy formation using barrier options and derive profit functions for each way of this strategy formation. Furthermore, these functions are analysed and way are selected, that are suitable for hedging against a price drop of an underlying asset. These ways are used in the formation of hedging possibilities. Based on analysis, conclusions are formulated. They can facilitate the investor selection process of appropriate real underlying price hedging strategy and simplify the comparison of hedging possibilities. Another used method is comparison. We evaluate profit/loss of proposed secured positions relative to unsecured position applied in a portfolio consisting of SPDR Gold Shares. In the end, the obtained results are compared.

Barrier options

Barrier options are a more complex financial instrument requiring better knowledge of the option issue. In this chapter we discuss more complicated terminology based on the understanding of basic option terms. Barrier options (category exotic options/subcategory path-dependent options[3]) have a barrier level in the form of marginal underlying spot price. Exceeding the barrier during the option life means option activation (knock-in), respectively deactivation (knock-out). Barrier level can be over or below the underlying spot price at the time of contract conclusion. Hence barrier options are linked to the condition, which must be satisfied for using the barrier options. On the other hand, it expires as worthless. (Kolb, 1999), (Taleb, 1997), (Weert, 2008)

As we have mentioned, barrier option can be knock-in or knock-out, down or up. Based on this, four basic types of barrier options are:

- up and knock-in (UI) call/put option – is activated, if an underlying price during the life of an option increases above upper barrier U;
- down and knock-in (DI) call/put option – is activated, if an underlying price during the life of an option decreases below lower barrier D;
- up and knock-out (UO) call/put option – is deactivated, if an underlying price during the life of an option increases above upper barrier;
- down and knock-out (DO) call/put option – is deactivated, if an underlying price during the life of an option decreases below lower barrier.

By analogy with vanilla options, barrier option can be call or put, European or American type. Barrier monitoring can be realized in discrete time, "i.e." in specified time intervals or continuously, "i.e." continuous monitoring of a price. Discrete monitoring is using in price movement monitoring of shares and commodities, continuous monitoring in exchange and interest rates.

Barrier option are cheaper in comparison to vanilla options due to the activation condition of knock-in respectively deactivation condition of knock-out barrier option. It can be proven that the sum of knock-in call (put) option premium and knock-out call (put) option premium is vanilla call (put) option premium, if options are for the same underlying asset and have the same strike price, time to expiration and barrier. This relation between barrier and vanilla option premiums with the same parameters is known as knock-in/out parity.

Derivation of profit function for particular ways of Long Combo strategy formation using barrier options

It is known, that Long Combo strategy is formed by a combination of Long Put position and Short Call position, "i.e." by buying *n* put options with strike price X_1, premium p_{1B}^0 per option (the buyer pays the seller for a right to sell an particular underlying asset), and at the same time by selling *n* call options with strike price X_2, premium c_{2S}^0 per option (the seller obtains from the buyer for an obligation to sell an particular asset), where $X_1 < X_2$ and the options have the same expiration time *T* and are for the same underlying asset.

If we put on barrier conditions (up and knock-in, up and knock-out, down and knock-in, down and knock-out) to vanilla options, we get barrier options, which are cheaper in the comparison to vanilla options with

[3] For path-dependent options (barrier, asian, lookback and forward-start options) the price of an underlying asset during the entire life of an option is important. It is monitoring marginal, maximum, minimum spot price and various averages of spot price during the life of the option.

the same parameters because of the activation (deactivation) condition, which raises the risk of not exercising the barrier option at the time of expiration.

In this section we will derive the profit function at the time of expiration for every ways of Long Combo strategy creation using European type barrier options assuming a zero rebate. There are 16 ways of this strategy creation by buying put barrier option and at the same time by selling call barrier options. We will based on the work (Kundríková, 2008), where functions of profit for 8 basic types of barrier options are derived.

Firstly we form Long Combo strategy by buying *n* down and knock-in put options with lower strike price X_1, premium p_{1BDI}^0 per option, barrier level *D,* and at the same time by selling *n* up and knock-in call options with higher strike price X_2, premium c_{2SUI}^0 per option, barrier level *U.*

Buying of down and knock-in put option gives a right to sell a particular underlying asset at strike price X_1 at time *T,* if the option is activated, that is, the price of the underlying asset exceeds predetermined lower barrier *D* from above during the life of the option (the option is activated, if it only touch the barrier)[4], which the following condition represents

$$\min_{0 \le t \le T} \left(S_t \right) \le D. \tag{1}$$

It is obvious, down and knock-in/out (up and knock-in/out) options have the barrier below (over) the actual underlying spot price S_0 at the time of the option contract conclusion.

As mentioned earlier, the buyer of down and knock-in put option have an obligation to pay an option premium p_{1BDI}^0 per option[5] at the time of contract conclusion, "i.e." at time 0. The profit function is expressed at the time of maturity. If time to maturity *t* is shorter than one year, it is used simple interest

$$p_{1BDI} = p_{1BDI}^0 \left(1 + r.t\right). \tag{2}$$

If time to maturity is longer than one year, the premium is calculating using the formula for compound interest

$$p_{1BDI} = p_{1BDI}^0 \left(1 + r\right)^t , \tag{3}$$

where *r* is nominal interest rate. In other profit function of this work we will be using the option premium adjusted by the time value of the money according to (2), respectively (3).

Let us denote underlying spot price at expiration time S_T, the profit function from buying *n* down and knock-in put option is

$$P\left(S_T\right) = \begin{cases} -n\left(S_T - X_1 + p_{1BDI}\right) & \text{if } \min\limits_{0 \le t \le T}\left(S_t\right) \le D \wedge S_T < X_1, \\ -np_{1BDI} & \text{if } \min\limits_{0 \le t \le T}\left(S_t\right) > D \wedge S_T < X_1, \\ -np_{1BDI} & \text{if } S_T \ge X_1. \end{cases} \tag{4}$$

At selling up and knock-in call option we have obligated to sell the given underlying asset at strike price X_2 at time *T,* if the option is activated, which is expressed by the condition

$$\max_{0 \le t \le T} \left(S_T \right) \ge U. \tag{5}$$

In this case the profit function is

$$P\left(S_T\right) = \begin{cases} nc_{2SUI} & \text{if } S_T < X_2, \\ -n\left(S_T - X_2 - c_{2SUI}\right) & \text{if } \max\limits_{0 \le t \le T}\left(S_t\right) \ge U \wedge S_T \ge X_2, \\ nc_{2SUI} & \text{if } \max\limits_{0 \le t \le T}\left(S_t\right) < U \wedge S_T \ge X_2. \end{cases} \tag{6}$$

Because $X_1 < X_2$, then we get the profit function from complex LC strategy by the sum of functions (4) and (6).

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[4] Every followed function in this work is built on this assumption.
[5] Subscript B (Buy) means, that we buy the given option. Subscript S (Sell) is used, if we suppose to negotiate a selling position.

$$P(S_T)=\begin{cases} -n\left(S_T - X_1 + p_{1BDI} - c_{2SUI}\right) & if \min\limits_{0\le t\le T}\left(S_t\right)\le D \;\; S_T < X_1, \\ n\left(c_{2SUI} - p_{1BDI}\right) & if \min\limits_{0\le t\le T}\left(S_t\right) > D \;\; S_T < X_1, \\ n\left(c_{2SUI} - p_{1BDI}\right) & if\; X_1 \le S_T < X_2, \\ -n\left(S_T - X_2 + p_{1BDI} - c_{2SUI}\right) & if \max\limits_{0\le t\le T}\left(S_t\right)\ge U \;\; S_T \ge X_2, \\ n\left(c_{2SUI} - p_{1BDi}\right) & if \max\limits_{0\le t\le T}\left(S_t\right) < U \;\; S_T \ge X_2. \end{cases} \tag{7}$$

If the following condition is satisfied

$$c^0_{2SUI} - p^0_{1BDI} \ge 0, \tag{8}$$

then no additional cost are needed and difference between the premium from selling call option and the premium from buying put option is our profit.

Fig. 1 depicts the function of profit from Long Combo by buying DI put option and at the same time by selling UI call option. If options are activated during the life of the options, the profit function is similar to the profit function from Long Combo strategy by vanilla options (see Šoltés V. – Šoltésová, 2003a). The profit function for the Long Combo strategy offers more alternatives for profit development depending on the underlying price movement.

Source: Own design

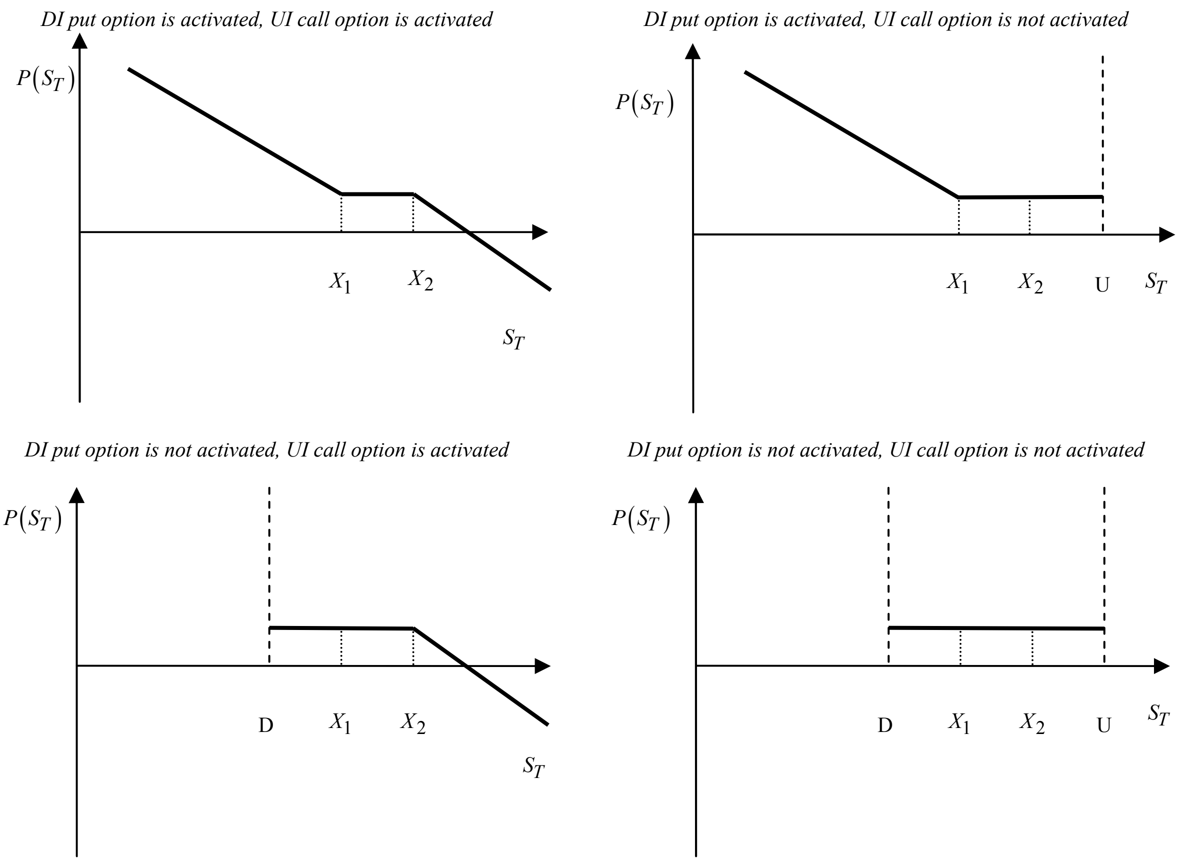

Fig. 1. Plot of the profit function from LC strategy by buying DI put option and by selling UI call option for possible underlying price scenarios.

If the spot price at expiration time is higher than break-even point (BEP) and UI call option is activated, this strategy produces a loss. Otherwise is profitable. BEP is $X_2 - p_{1BDI} + c_{2SUI}$.

Profit functions from Long Combo strategy using barrier options are different in barrier conditions. Substituting corresponding barrier conditions to generalized profit function we get the profit function of the given method of Long Combo strategy formation (Tab. 1), therefore it can be derived the generalized profit function, which has the following form

$$P(S_T) = \begin{cases} -n(S_T - X_1 + p_{1B} - c_{2S}) & \text{if } 1 \text{ is satisfied } S_T < X_1, \\ n(c_{2S} - p_{1B}) & \text{if } 2 \text{ is satisfied } S_T < X_1, \\ n(c_{2S} - p_{1B}) & \text{if } X_1 \le S_T < X_2, \\ -n(S_T - X_2 + p_{1B} - c_{2S}) & \text{if } 3 \text{ is satisfied } S_T \ge X_2, \\ n(c_{2S} - p_{1B}) & \text{if } 4 \text{ is satisfied } S_T \ge X_2. \end{cases} \quad (9)$$

Tab. 1. Barrier conditions for particular method of Long Combo strategy formation.

	Way of Long Combo strategy formation	Barrier condition 1	Barrier condition 2	Barrier condition 3	Barrier condition 4
1.	buying DI put option and selling UI call option	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$
2.	buying DI put option and selling UO call option	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$
3.	buying DI put option and selling DI call option	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$
4.	buying DI put option and selling DO call option	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$
5.	buying DO put option and selling UI call option	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$
6.	buying DO put option and selling UO call option	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$
7.	buying DO put option and selling DI call option	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$
8.	buying DO put option and selling DO call option	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$
9.	buying UO put option and selling UI call option	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$
10.	buying UO put option and selling UO call option	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$
11.	buying UO put option and selling DI call option	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$
12.	buying UO put option and selling DO call option	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$
13.	buying UI put option and selling UI call option	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$
14.	buying UI put option and selling UO call option	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) < U$	$\max_{0\le t\le T}(S_t) \ge U$
15.	buying UI put option and selling DI call option	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$	$\min_{0\le t\le T}(S_t) \le D$	$\min_{0\le t\le T}(S_t) > D$
16.	buying UI put option and selling DO call option	$\max_{0\le t\le T}(S_t) \ge U$	$\max_{0\le t\le T}(S_t) < U$	$\min_{0\le t\le T}(S_t) > D$	$\min_{0\le t\le T}(S_t) \le D$

Source: Own design.

Next section is dealing with these ways of LC strategy formation using barrier options, which should be used to hedge against an underlying price drop.

Hedging analysis against a price drop by LC strategy using barrier options

Let us assume a simple portfolio consisting from *n* pieces of risky underlying asset. Let us suppose that at time *T* in the future we want to buy *n* pieces of underlying asset, but we are afraid of price drop.

The profit function from unsecured position in the portfolio at time *T* is

$$P(S_T) = n S_T, \quad (10)$$

where S_T is underlying spot price at time *T*. The lower the spot price is, the lower profit we receive from selling of the given asset.

Therefore we have decided to hedge against the price drop of the underlying asset. Our analysis is based on the following definition of hedging. Hedging is a method to minimize a risk from an unfavourable price movement on financial markets. Because of the positive correlation between risk and profit, the reduction of risk (potential loss) would also reduce potential profit. It should be noted that using of hedging transactions does not mean avoiding a loss, however hedging the price at a particular acceptable interval.

Hedging analysis assumptions formulated on based of theoretical knowledge:

- Shares, bonds, exchange rates, interest rates, commodities can be risky asset.
- Hedging is realised by Long Combo strategy using barrier options.
- On financial market are barrier options for underlying assets which we want to hedge.
- There are real-traded barrier options with various life times, barrier levels and strike prices.
- It is possible to buy or sell an unlimited barrier options amount, including fractions.
- Barrier options are European type.
- Barrier is constant, determinated by the price of the given underlying asset.
- We use profit functions in analytical form.
- Rebate is not considered.
- Transaction costs and tax payments are neglect.
- Risk-free interest rate is constant during the life of option contract.
- Created hedging portfolio is static.

Logically, 4 ways of Long Combo strategy formation using barrier options are suitable in hedging against a price drop. Concretely, there are ways created by down and knock-in put options. If we hedge using the other described ways, we can end up in the so called unprotected scenarios, where the price of the underlying asset is not protected in case of a price drop. The next section is focused on the analysis of hedging portfolios by mentioned 4 ways of LC strategy formation.

I. One possibility is hedging by buying *n* down and knock-in put options with a lower strike price X_1, premium p^0_{1BDI} per option, barrier level *D* $\left(D < X_1\right)$, and at the same time by selling *n* up and knock-in call options with a higher strike price X_2, premium c^0_{2SUI} per option, barrier level *U* $\left(U > X_2\right)$. If we want to hedge a price of an underlying asset at a particular date in future, then we form Long Combo strategy by European type barrier options with expiration at identical date.
The profit function from secured position in portfolio is obtained as the sum of the profit function from LC strategy (7) and profit function from unsecured position in portfolio (10)

$$ZP_I\left(S_T\right) = \begin{cases} n\left(X_1 - p_{1BDI} + c_{2SUI}\right) & if \min\limits_{0\le t\le T}\left(S_t\right) \le D \wedge S_T < X_1, \\ n\left(S_T - p_{1BDI} + c_{2SUI}\right) & if \min\limits_{0\le t\le T}\left(S_t\right) > D \wedge S_T < X_1, \\ n\left(S_T - p_{1BDI} + c_{2SUI}\right) & if\ X_1 \le S_T < X_2, \\ n\left(X_2 - p_{1BDI} + c_{2SUI}\right) & if \max\limits_{0\le t\le T}\left(S_t\right) \ge U \wedge S_T \ge X_2, \\ n\left(S_T - p_{1BDI} + c_{2SUI}\right) & if \max\limits_{0\le t\le T}\left(S_t\right) < U \wedge S_T \ge X_2. \end{cases} \quad (11)$$

By analyzing of the profit function from secured position we have the statements:

- If an underlying price during time to maturity decreases below lower barrier and increases above upper barrier, hedging is similar to hedging by Long Combo strategy created by vanilla options. In this case, we hedge the price from interval $\left\langle X_1,\ X_2\right\rangle$. If the price increases over *U*, we cannot participate in the price increase, which is a fee for hedging.
- If an underlying price does not drop below the lower barrier and is below X_1 at the time of option expiration, we hedge lower selling price. In this case, the minimum price is limited by lower barrier. It is balanced by cheaper put barrier option premium compared to put vanilla option premium.
- If an underlying price does not increase above higher barrier level and is above higher strike price X_2 at the time of expiration, then we cannot participate in price growth, but the maximum price is limited by barrier *U.* Option premium c^0_{2SUI} from selling up and knock-in call option is lower compared to option premium c^0_{2S} from selling vanilla call option.

II. Another possibility is hedging by buying *n* down and knock-in put option with strike price X_1, premium p^0_{1BDI} per option, barrier *D* $\left(D < X_1\right)$, and at the same time by selling *n* up and knock-out call option with strike price X_2, premium c^0_{2SUO} per option, barrier *U* $\left(U > X_2\right)$. The profit function from secured position is

$$ZP_{II}(S_T)=\begin{cases} n(X_1 - p_{1BDI} + c_{2SUO}) & if \min_{0\le t\le T}(S_t)\le D \wedge S_T < X_1, \\ n(S_T - p_{1BDI} + c_{2SUO}) & if \min_{0\le t\le T}(S_t) > D \wedge S_T < X_1, \\ n(S_T - p_{1BDI} + c_{2SUO}) & if\ X_1 \le S_T < X_2, \\ n(X_2 - p_{1BDI} + c_{2SUO}) & if \max_{0\le t\le T}(S_t) < U \wedge S_T \ge X_2, \\ n(S_T - p_{1BDI} + c_{2SUO}) & if \max_{0\le t\le T}(S_t) \ge U \wedge S_T \ge X_2. \end{cases} \quad (12)$$

By analyzing (12) we can deduce following conclusions:

- If down and knock-in put option is activated, "i.e." if the underlying price drops or touches barrier *D,* then it is hedged from below by lower strike price X_1.
- If it is not activated, then it is hedged from below by lower barrier.
- If up and knock-out call option is deactivated, "i.e." $\max_{0\le t\le T}(S_t)\ge U$ and spot price is above X_2 at time to expiration, then we can fully participate in price growth.
- If up and knock-out call option is not deactivated and spot price is above X_2 at time to expiration, then we hedge the price at the higher strike price.

III. Let us use Long Combo strategy constructed by buying *n* down and knock-in put options, and by selling *n* down and knock-in call option with the same lower barrier *D* $(D < X_1 < X_2)$. As in the previous cases, we easily derive the profit function from secured position

$$ZP_{III}(S_T)=\begin{cases} n(X_1 - p_{1BDI} + c_{2SDI}) & if \min_{0\le t\le T}(S_t)\le D \wedge S_T < X_1, \\ n(S_T - p_{1BDI} + c_{2SDI}) & if \min_{0\le t\le T}(S_t) > D \wedge S_T < X_1, \\ n(S_T - p_{1BDI} + c_{2SDI}) & if\ X_1 \le S_T < X_2, \\ n(X_2 - p_{1BDI} + c_{2SDI}) & if \min_{0\le t\le T}(S_t) \le D \wedge S_T \ge X_2, \\ n(S_T - p_{1BDI} + c_{2SDI}) & if \min_{0\le t\le T}(S_t) > D \wedge S_T \ge X_2. \end{cases} \quad (13)$$

Results of the analysis:

- If barrier level is reached, then hedging is similar to hedging by vanilla options.
- If barrier level is not reached, then we fully participate in price growth.
- Price cannot drop below the lower barrier.

This hedging case is special interesting for investors, because in case of a small drop (above lower barrier *D*) and following random growth, the investor can fully participate in price growth. It protects investor from the bursting of the bubble and allows them to participate in possible further bubble increasing.

IV. Finally, let us hedge by buying *n* down and knock-in put option, and by selling *n* down and knock-out call option with the same barrier *D* in regard to this formula $(D < X_1 < X_2)$. In this case, the profit function from secured position by Long Combo strategy is

$$ZP_{IV}(S_T)=\begin{cases} n(X_1 - p_{1BDI} + c_{2SDO}) & if \min_{0\le t\le T}(S_t)\le D \wedge S_T < X_1, \\ n(S_T - p_{1BDI} + c_{2SDO}) & if \min_{0\le t\le T}(S_t) > D \wedge S_T < X_1, \\ n(S_T - p_{1BDI} + c_{2SDO}) & if\ X_1 \le S_T < X_2, \\ n(X_2 - p_{1BDI} + c_{2SDO}) & if \min_{0\le t\le T}(S_t) > D \wedge S_T \ge X_2, \\ n(S_T - p_{1BDI} + c_{2SDO}) & if \min_{0\le t\le T}(S_t) \le D \wedge S_T \ge X_2. \end{cases} \quad (14)$$

The analyse of profit function has resulted into following statements:

- If barrier is reached, then we hedge against a price drop and we can participate in the price growth.
- The price cannot be lower than the lower barrier.
- If barrier is not reached and the price of underlying asset is above X_2 at expiration time, then we have hedged the price at higher strike price.

This case is also interesting for investors, because in case of barrier reaching (greater decrease), we are hedged with strike price X_1 and we can fully participate in possible price growth.

Application in SPDR Gold Shares

On 9 August 2011, the shares of SPDR Gold Shares were traded at the NYSE Arc, Inc. at 172.64 USD per share. Let us suppose that we own a portfolio consisting of one share (let us consider by the most simple portfolio construction due to the simplification of result interpretation). We want to hedge a price from the selling of the given share to the certain future date at the certain amount by Long Combo strategy using barrier options.

Barrier options are traded on OTC market, where real-traded barrier option data are not published. Therefore we calculate option premiums for selected barriers and various strike prices using vanilla call and put option data on SPDR Gold Shares (see Tab. II).

Tab. I. Call and put options on SPDR Gold Shares from 09/08/2011 with expiration date on 18/01/2013 (values in USD).

CALL				PUT		
BID	**MID**	**ASK**	**X**	**BID**	**MID**	**ASK**
32.90	33.50	34.10	150	10.20	10.70	11.15
30.20	30.70	31.20	155	12.20	12.60	13.00
27.20	28.05	28.90	160	14.35	14.75	15.15
17.10	18.48	19.85	185	28.60	29.28	29.95
15.60	16.70	17.80	190	32.05	33.50	34.95
13.10	13.58	14.05	200	39.40	41.00	42.60

Source: data from http://finance.yahoo.com, own design.

Black-Sholes model for stocks without dividends with the following parameters: type of option (DI/DO/UI/UO CALL/PUT), actual underlying spot price, strike price, barrier level, time to maturity, rebate = 0, risk-free interest rate = cost of carry rate, implied volatility of underlying asset, is used to calculate barrier option prices. Expiration time is calculated using the European standard 30E/360. The risk-free rate is the U.S. Treasury rate, which was at 0.24 % for the given time to expiry (source: Bloomberg). Calculations we realize in the statistical program R.

Calculated prices of selected barrier options in USD are listed in the tables Tab. 3, 4 and V here.

Tab. II DI put option prices at various strike prices and barrier levels with expiration date on 18/01/2013.

X/D-I	145	150	155
150	10.69		
155	12.57	12.59	
160	14.65	14.72	14.74

Source: Own calculations in R.

Tab. 4. UI and UO call option prices at various strike prices and barrier levels with expiration date on 18/01/2013.

X/U-I	200	210	215	X/U-O	200	210	215
185	18.44	18.28	18.12	185	0.04	0.20	0.36
190	16.69	16.60	16.49	190	0.01	0.10	0.21
200		13.56	13.53	200		0.02	0.05

Source: Own calculation in R.

Tab. 5. DI and DO call option prices at various strike prices and barrier levels with expiration date on 18/01/2013.

X/D-I	145	150	160	X/D-O	145	150	160
185	3.09	4.63	9.27	**185**	15.39	13.85	9.21
190	2.62	4.00	8.16	**190**	14.08	12.70	8.54
200	1.88	2.94	6.31	**200**	11.70	10.64	7.27

Source: Own calculation in R.

In the next section, 6 hedging alternatives using data from Tab. II. till Tab. V. are proposed. Furthermore, the profit functions from secured position in the portfolio are compared to the profit function from unsecured position. The given profit function interval is evaluated as profitable, if the profit from secured position using Long Combo strategy is higher than the profit from unsecured position. On the contrary, if the profit from secured position is lower than the profit from unsecured position, the given profit function interval of the particular hedging alternative is evaluated as unprofitable.

In the case of first 3 alternatives, let us suppose, that we want to hedge against more than 10 % share price drop, “i.e.” below 155 USD.

1. We buy put option with the strike price 155, and at the same time we sell call option with the strike price 200. In this case, the profit function from secured position in the portfolio is

$$ZP_1\left(S_T\right)=\begin{cases}155.98 & \text{if } S_T<155,\\ S_T+0.98 & \text{if } 155\le S_T<200,\\ 200.98 & \text{if } S_T\ge 200.\end{cases} \tag{15}$$

Plot of the function is depicted in Fig. 2.

Source: Own design.

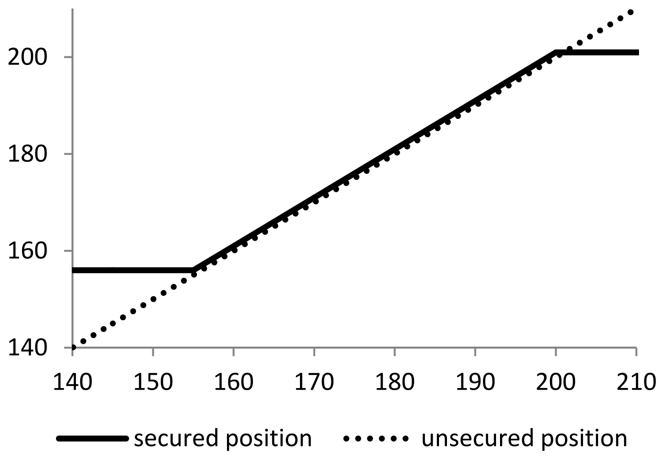

Fig. 2. Plot of the profit function from secured position in the portfolio.

If $S_T<155$, then this alternative produces profit from the interval $\left(0.98,155.98\right\rangle$ which increases with the spot price drop. If the spot price is from the interval $\left\langle 155,200\right)$, then we hedge the constant amount of profit equal to 0.98. If the spot price is from the interval $\left\langle 200,200.98\right)$, then we have the profit from the interval $\left(0,0.98\right\rangle$ decreasing with the spot price growth. If $S_T\ge 200.98$, then this alternative is unprofitable because of impossibility to participate in the price growth. In this case, the loss increases with the spot price growth. Minimum price is 0 and the maximum price is unlimited.

2. Let us create hedging portfolio by buying DI put option with the strike price 160, lower barrier 155, and at the same time by selling UI call option with the higher strike price 190, upper barrier 200. The profit function from secured position in the portfolio is expressed by formula (16) and plot of this profit function is depicted in Fig. 3.

$$ZP_2\left(S_T\right)=\begin{cases}161.85 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)\le 155\wedge S_T<160,\\ S_T+1.85 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)>155\wedge S_T<160,\\ S_T+1.85 & \text{if } 160\le S_T<190,\\ 191.85 & \text{if } \max\limits_{0\le t\le T}\left(S_t\right)\ge 200\wedge S_T\ge 190,\\ S_T+1.85 & \text{if } \max\limits_{0\le t\le T}\left(S_t\right)<200\wedge S_T\ge 190.\end{cases} \tag{16}$$

If, then this alternative produces profit from the interval which increases with the spot price drop. If the spot price is from the interval, then we have the profit from the interval increasing with the price drop or the constant amount of profit equal to 1.85. If, then we hedge the constant amount of profit equal to 1.85. If the spot price is from the interval, then we have the constant amount of profit equal to 1.85 or the profit from the interval increasing with the price drop.

Source: Own design.

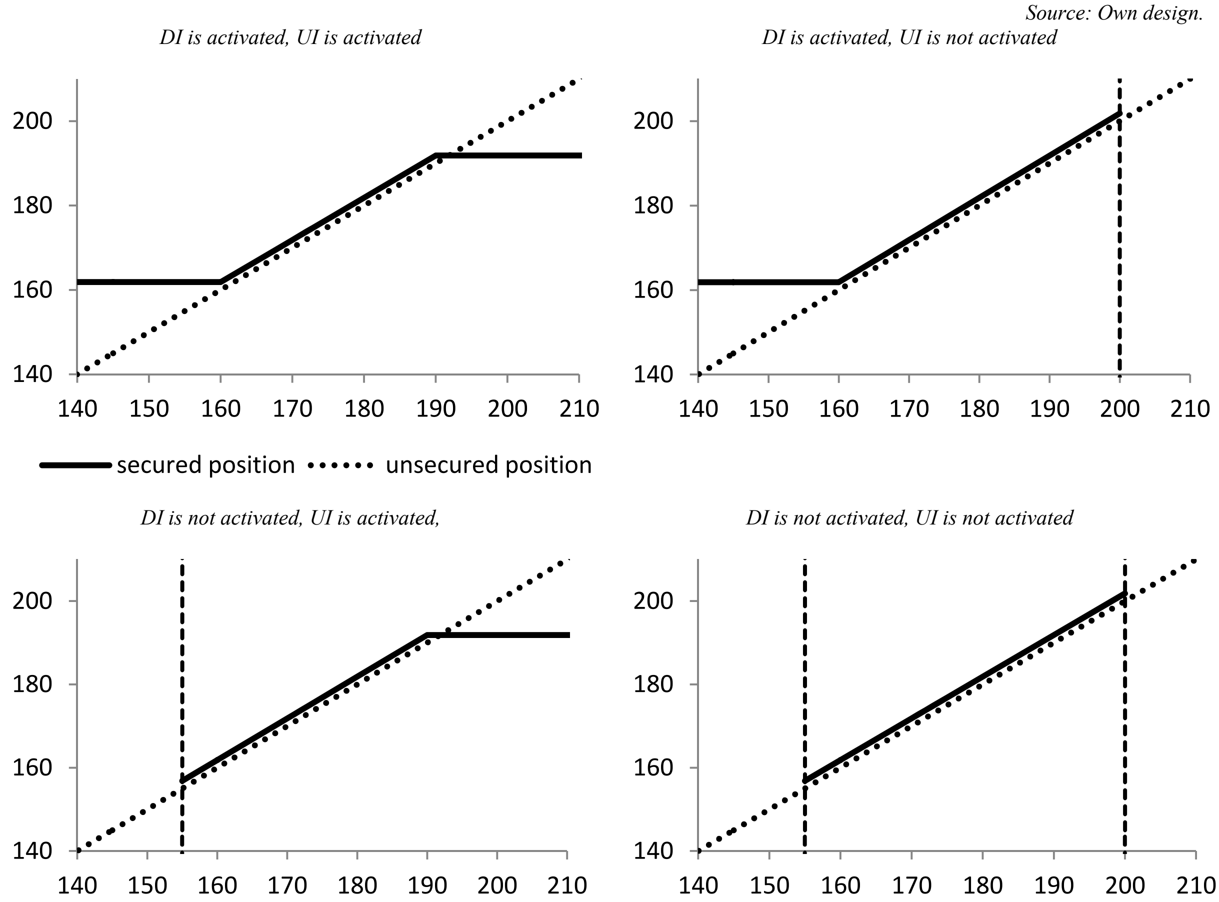

Fig. 3. Plot of the profit function from secured position in the portfolio.

If the spot price is from the interval $(191.85, 200)$, then we get the constant amount of profit equal to 1.85 or the loss from the interval $(0, 8.15)$ increasing with the price growth. If $S_T \geq 200$, then this alternative is unprofitable because of impossibility to participate in the price growth. The loss increases with the spot price growth. Minimum price is 8.15 and maximum price is unlimited.

3. Let us form the strategy by buying DI put option with the same parameters as the previous alternative, and at the same time by selling UO call option with strike price 190 and barrier level 200. The profit function from secured position is

$$ZP_3(S_T) = \begin{cases} 145.27 & \textit{if } \min\limits_{0 \leq t \leq T}(S_t) \leq 155 \wedge S_T < 160, \\ S_T - 14.73 & \textit{if } \min\limits_{0 \leq t \leq T}(S_t) > 155 \wedge S_T < 160, \\ S_T - 14.73 & \textit{if } 160 \leq S_T < 190, \\ 175.27 & \textit{if } \max\limits_{0 \leq t \leq T}(S_t) < 200 \wedge S_T \geq 190, \\ S_T - 14.73 & \textit{if } \max\limits_{0 \leq t \leq T}(S_t) \geq 200 \wedge S_T \geq 190. \end{cases} \tag{17}$$

Plot of the function is depicted in Fig. 4.

If, then this alternative produces profit from the interval increasing with the spot price drop. If, then we have the loss from the interval increasing with the price growth. If, then we have the loss from the interval increasing with the price growth or the constant amount of loss equal to 14.73. If the spot price is from the interval, then we have the constant amount of loss equal to 14.73. If the spot price is from the interval, then we have the constant amount of loss equal to 14.73 or the loss from the interval increasing with the price growth. If, then we have the constant amount of loss equal to 14.73. In this case, we can participate in the price increase.

Source: Own design.

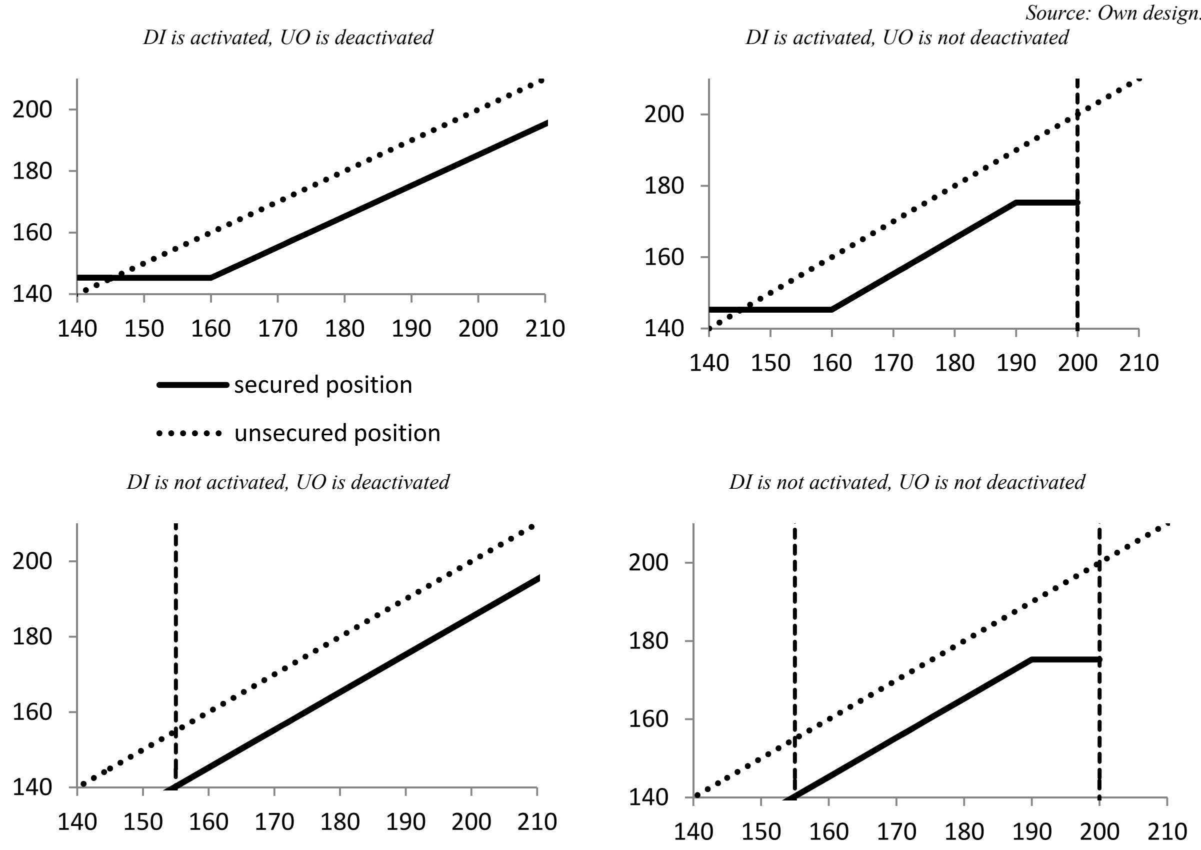

Fig. 4. Plot of the profit function from secured position in the portfolio.

Let us assume, that we want to hedge a minimum price at 150 USD, "i.e." we want to hedge against more than 13 % spot price drop.

4. We buy put option with strike price 150, and at the same time we sell call option with strike price 190. In this case, the profit function from secured position in portfolio is

$$ZP_4\left(S_T\right)=\begin{cases}156 & \text{if } S_T<150,\\ S_T+6 & \text{if } 150\leq S_T<190,\\ 196 & \text{if } S_T\geq 190.\end{cases} \tag{18}$$

Plot of the function is depicted in Fig. 5.

Source: Own design.

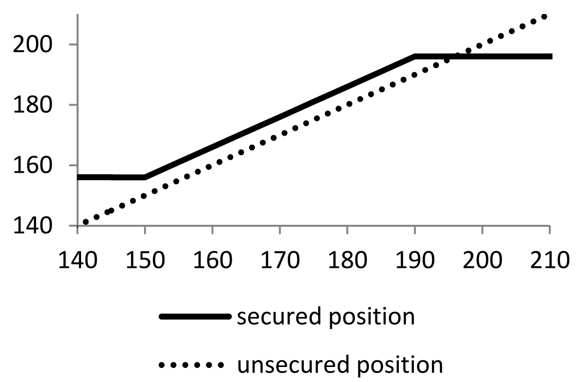

Fig. 5. Plot of the profit function from secured position in the portfolio.

5. Let buy DI put option with lower strike price 155 and barrier level 150, and at the same time sell DI call option with the same lower barrier and strike price 190. In this case, we have the profit function from secured position expressed by the formula (19) and plot of the given profit function is in Fig. 6.

$$ZP_5\left(S_T\right)=\begin{cases}146.41 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)\le 150 \wedge S_T<155,\\ S_T-8.59 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)>150 \wedge S_T<155,\\ S_T-8.59 & \text{if } 155\le S_T<190,\\ 181.41 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)\le 150 \wedge S_T\ge 190,\\ S_T-8.59 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)>150 \wedge S_T\ge 190.\end{cases} \qquad (19)$$

Source: Own design.

If the barrier is exceeded during the option live — *If the barrier is not exceeded*

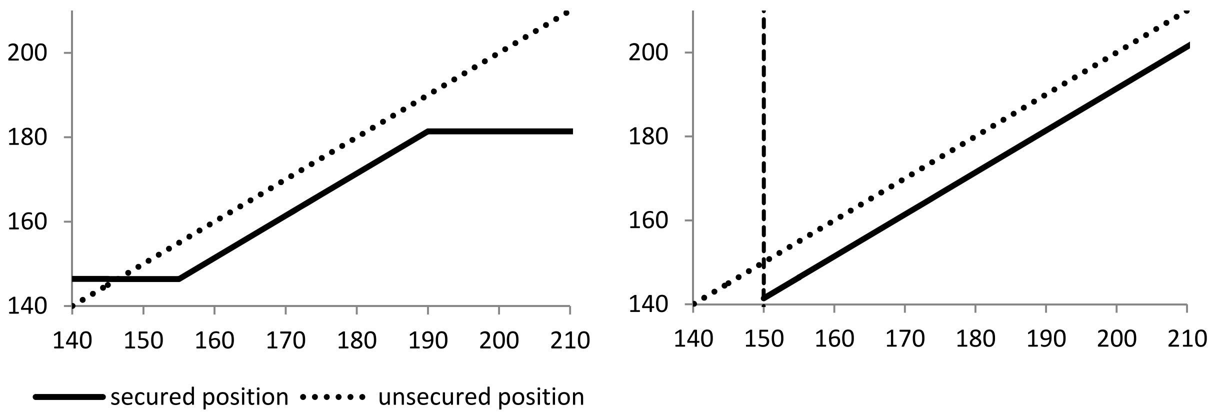

Fig. 6. Plot of the profit function from secured position in the portfolio.

6. And finally, we form the Long Combo strategy by DO call option, other parameters are the same as at the hedging alternative No. 5. In this case the profit function is

$$ZP_6\left(S_T\right)=\begin{cases}155.11 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)\le 150 \wedge S_T<155,\\ S_T+0.11 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)>150 \wedge S_T<155,\\ S_T+0.11 & \text{if } 155\le S_T<190,\\ 190.11 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)>150 \wedge S_T\ge 190,\\ S_T+0.11 & \text{if } \min\limits_{0\le t\le T}\left(S_t\right)\le 150 \wedge S_T\ge 190.\end{cases} \qquad (20)$$

Plot of the function is depicted in Fig. 7.

Source: Own design.

If the barrier is exceeded — *If the barrier is not exceeded*

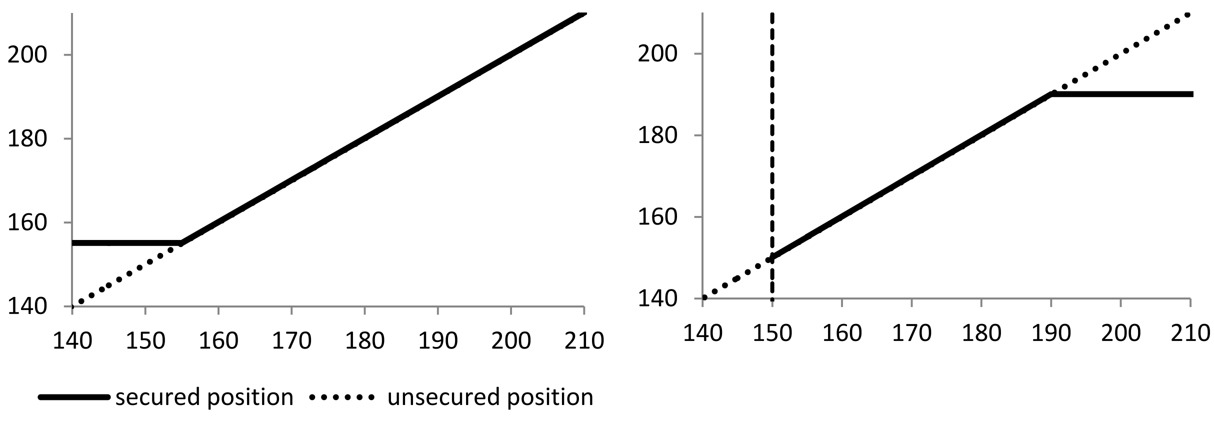

Fig. 7. Plot of the profit function from secured position in the portfolio.

Profit/loss comparison of the proposed hedging alternatives at different intervals and barrier conditions are shown in Tab. 6 (comparison of 1st, 2nd and 3rd alternative) and Tab. 7 (comparison of 4th, 5th and 6th alternative) (see Appendix 1).

Appendix 1

Tab. 6. Profit/loss and minimum/maximum/constant profit/loss for 1., 2. and 3. hedging alternative. Profit decreases with spot price growth at expiration time, loss increases with the spot price growth. In the course of profit/loss min means minimum profit/loss, max means maximum profit/loss.

Spot price interval at expiration	Barrier conditions	1. alternative			2. alternative			3. alternative		
		profit/loss	min	max	profit/loss	min	max	profit/loss	min	max
$S_T \le 145.27$	$\min_{0 \le t \le T}(S_t) \le 155$	**profit**	10.71	155.98	**profit**	16.58	145.27	profit	0	145.27
$145.27 \le S_T \le 155$	$\min_{0 \le t \le T}(S_t) \le 155$	**profit**	0.98	10.71	**profit**	6.85	16.58	loss	0	9.73
$155 \le S_T \le 160$	$\min_{0 \le t \le T}(S_t) \le 155$	**constant profit**	0.98	0.98	**profit**	1.85	6.85	loss	9.73	14.73
	$\min_{0 \le t \le T}(S_t) \ge 155$	**constant profit**	0.98	0.98	**constant profit**	1.85	1.85	constant loss	14.73	14.73
$160 \le S_T \le 190$		**constant profit**	0.98	0.98	**constant profit**	1.85	1.85	constant loss	14.73	14.73
$190 \le S_T \le 191.85$	$\max_{0 \le t \le T}(S_t) \ge 200$	**constant profit**	0.98	0.98	**profit**	0	1.85	constant loss	14.73	14.73
	$\max_{0 \le t \le T}(S_t) \le 200$	**constant profit**	0.98	0.98	**constant profit**	1.85	1.85	loss	14.73	16.58
$191.85 \le S_T \le 200$	$\max_{0 \le t \le T}(S_t) \ge 200$	**constant profit**	0.98	0.98	**loss**	0	8.15	constant loss	14.73	14.73
	$\max_{0 \le t \le T}(S_t) \le 200$	**constant profit**	0.98	0.98	**constant profit**	1.85	1.85	loss	16.58	24.73
$200 \le S_T \le 200.98$	$\max_{0 \le t \le T}(S_t) \ge 200$	**profit**	0	0.98	**loss**	8.15	9.13	constant loss	14.73	14.73
$S_T \ge 200.98$	$\max_{0 \le t \le T}(S_t) \ge 200$	**loss**	0	infinity	**loss**	9.13	infinity	constant loss	14.73	14.73

Source: Own calculation.

Tab. 7. Profit/loss and minimum/maximum/constant profit/loss for 4., 5. and 6. hedging alternative. Profit decreases with spot price growth at expiration time, loss increases with the spot price growth. In the course of profit/loss min means minimum profit/loss, max means maximum profit/loss.

Spot price interval at expiration	**Barrier conditions**	**4. alternative**			**5. alternative**			**6. alternative**		
		profit/loss	**min**	**max**	**profit/loss**	**min**	**max**	**profit/loss**	**min**	**max**
$S_T \leq 146.41$	$\min_{0 \leq t \leq T}(S_t) \leq 150$	**profit**	9.59	156	**profit**	0	146.41	profit	8.70	146.41
$146.41 \leq S_T \leq 150$	$\min_{0 \leq t \leq T}(S_t) \leq 150$	**profit**	6	9.59	**loss**	0	3.59	profit	5.11	8.70
$150 \leq S_T \leq 155$	$\min_{0 \leq t \leq T}(S_t) \leq 150$	**constant profit**	6	6	**loss**	3.59	8.59	profit	0.11	5.11
	$\min_{0 \leq t \leq T}(S_t) \geq 150$	**constant profit**	6	6	**constant loss**	8.59	8.59	constant profit	0.11	0.11
$155 \leq S_T \leq 190$		**constant profit**	6	6	**constant loss**	8.59	8.59	constant profit	0.11	0.11
$190 \leq S_T \leq 190.11$	$\min_{0 \leq t \leq T}(S_t) \geq 150$	**profit**	5.89	6	**constant loss**	8.59	8.59	profit	0	0.11
	$\min_{0 \leq t \leq T}(S_t) \leq 150$	**profit**	5.89	6	**loss**	8.59	8.70	constant profit	0.11	0.11
$190.11 \leq S_T \leq 196$	$\min_{0 \leq t \leq T}(S_t) \geq 150$	**profit**	0	5.89	**constant loss**	8.59	8.59	loss	0	5.89
	$\min_{0 \leq t \leq T}(S_t) \leq 150$	**profit**	0	5.89	**loss**	8.70	159	constant profit	0.11	0.11
$S_T \geq 196$	$\min_{0 \leq t \leq T}(S_t) \geq 150$	**loss**	0	infinity	**constant loss**	8.59	8.59	loss	5.89	infinity
	$\min_{0 \leq t \leq T}(S_t) \geq 150.$	**loss**	0	infinity	**loss**	14.59	infinity	constant profit	0.11	0.11

Source: Own calculation.

Conclusion

The first aim of this work was to derive the profit functions for ways of Long Combo strategy formation using barrier options. Complex description and analysis of Long Combo strategy using barrier options based on analytical expression of profit functions is not to be found in any contemporary literature. In practice, derived profit functions can be applied to the real underlying assets, and thus simplifying the investor decision-making process considering expectations about future underlying price movement. On the other hand, it can also help in comparison with other trading strategies and in construction of hedging option strategies.

The second aim was to create a hedging portfolio consisting of one risky asset type and barrier options, to derive profit function from secured position in the given portfolio. Based on the obtained theoretical knowledge, we analyzed these hedging possibilities. We came to the following conclusions:

- There are 4 hedging possibilities for Long Combo strategy using barrier options.
- Hedging using LC with barrier options expands hedging opportunities in different directions, thereby it offers more alternatives for price hedging, allowing to secure only the most likely unfavourable future price movement scenarios, "i.e." allows adaptation to hedger's specific individual requirements.
- Hedging by Long Combo strategy using barrier options enables us to minimize risk from the price drop and also to profit from the opposite underlying price movement.
- By hedging using Long Combo strategy with vanilla options we can hedge the minimum and maximum selling price.
- The advantage of hedging against a price drop using the Long Combo strategy with the barrier and vanilla options is the possibility of its formation at zero initial cost.

The third aim was to apply Long Combo strategy to hedging of SPDR Gold Shares and to compare the proposed hedging alternatives.

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