Analysis of a simulation of missing satellite observations in the deformation network

Slavomír Labant[1] and Gabriel Weiss[1]

The present paper covers the issue of missing measurement data between two or more actually located epochs within a deformation network. Real measurements were attained from experimental locating of the 3D geodetic deformation network applying the global navigation satellite system (GNSS) to the points around the upper reservoir of water transfer power station Čierny Váh. Contrary to the other terrestrial technologies, the advantage of using GNSS is seen in the fact that the technology is not influenced by uncertainties of low-level layers of atmosphere and individual visibilities between points falling within the screened area are unnecessary. In the process of resolving, simulated and subsequently generated were missing observation data between two epochs. Simulation intermediate results were subsequently analysed, graphically illustrated and verified was at what time point the shift of an unstable point occurred.

Introduction

Time series analysis presents one of the most significant applications of statistical methods in various branches. If some data are missing in an empirical time series they can be replaced by empirical data simulation. Simulation of the missing empirical data is then subjected to the time series analysis. The modelled time series is verified by regression and correlation analysis with the possibility of subsequent prognoses of the time series further development.

Time series (TS) is a sequence of materially and spatially comparable data, positively from the point of time arranged in the past –to– present TS $\{x_i, t_i\}_n$. TS is minimally two-dimensional (planar) selection of data, in which a random variable is explicitly bound to time and/or constitutes with the time in which it emerged an inseparable pair (triplet, etc.). TS present, in majority, waveforms of working quantities of a host of technologies: seismic, geophysical, biological, meteorological records, records on pollution of soil, water, air, demographical data, etc. Hence, the TS analysis is a set of methods applied to describing past behaviour of the TS, possibly for forecasting its future behaviour.

Selection of processing space

Provided that the only factor of dynamics is time, the principal one-dimensional TS model can be entered as a function of the modelled indicator in the form of

$$y_t = f(t, I_t), \tag{1}$$

where: I_t is value of the random component in time $t = 1,2,...,n$. Its modelling can be approached by applying the traditional (formal) model, the Box-Jenkins methodology or spectral analysis.

With multi-dimensional models it is anticipated that development of the indicator being observed does not influence only the time factor but other factors as well $x_1, x_2,...,x_n$. Such a multi-dimensional model [5] can be entered in the form

$$y_t = f(t, x_1, x_2,...,x_n, I_t). \tag{2}$$

The Box-Jenkins methodology has the random constituent that can be made up of dependent random quantities for the TS model construing. Modelling of TS using the spectral analysis utilises Fourier series.

The time series traditional model is based on breakdown of the time series to four components:

T_t - Trend – long-term tendency, changes in average behaviour of the TS,

C_t - Cyclic – around the trend fluctuations with varying lengths of cycle, with varying speed of changes within the rise and fall phases of TS values,

S_t - Seasonal – periodic component, regularly reoccurring within a specific season (cycles),

I_t - Random (irregular), which remains in the time series even once T_t, C_t and S_t components have been removed; made up of random influences; has no observable nature.

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The model shape can be:

Additive: $Y_t = T_t + C_t + S_t + I_t$ - most frequently used in the practice, (3)

Multiplicative: $Y_t = T_t \cdot C_t \cdot S_t \cdot I_t$, $(\log Y_t = \log T_t + \log C_t + \log S_t + \log I_t)$, (4)

where Y_t - are expected, theoretical values of (dependent variable Y) TS.

The purpose of TS decomposition is to estimate the basic TS trend and to foretell the TS data by at least a single time step into the future. [2].
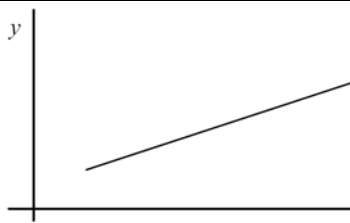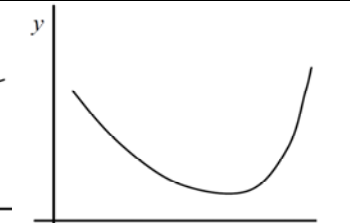
The trend component analysis

Purpose of the TS analysis is seen especially in the tendency of its, i.e. of the trend component, analysis. The TS trend component can be obtained by various means, especially by:

- simple diagrammatic (subjective) methods such as balancing of fluctuations, method of cycles averaging, sum of the areas of triangles method,
- analytical formulation of the trend using trend functions – the entire time series is equalised at once, by a single function,
- adaptive approaches – equalisation materializes gradually along shorter periods, e.g. using exponential smoothing, the moving averages method, etc.

Analytic expression of the trend using trend functions

In the practice most frequently used trend functions for both analysis and prognosis of time series are for example: linear, parabolic (Tab. 1), exponential, logistic trend etc.. The most often method of estimating the trend function parameters is the least squares method (LSM), which can be utilised in case that the selected trend function is linear at its parameters. Its advantage is simplicity and minimising of the residual component variance. It is directly appropriate for both linear and parabolic functions. In case of the exponential function this has to be linearized so that it would become linear from the point of parameters. [8]

Tab. 1. Analytic expression of the trend using trend functions:

For constant trend it holds that:	For linear trend it holds that:	For quadratic trend it holds that:
$Tr_t = a_0 = y$, $t = 1,2,...,n$,	$Tr_t = a_0 + a_1 t$, $t = 1,2,...,n$,	(if $y_{t+2} - 2y_{t+1} + y_t \div$ const.)
	a_0, a_1 - calculation as at LRM	$Tr_t = a_0 + a_1 t + a_2 t^2$, $t = 1,2,...,n$,
		a_0, a_1, a_2 - calculation of the 2nd degree polynomial
Forecast: $Y_T = a_0$	Forecast: $Y_T = a_0 + a_1 T$	Forecast: $Y_T = a_0 + a_1 T + a_2 T^2$

The appropriate trend function type can be selected based on the time series past development analysis applying the interpolation criteria:

- Factual/subject analysis of the examined phenomenon – if it is an increasing or decreasing function, if there is an inflection point or if in concern is an infinitely increasing function, an infinitely decreasing function, etc.
- Time series diagram analysis – danger of visual selection dwells in its subjectivity, whilst the diagram shape is extremely dependant on the scale used.
- Calculation of values of the trend function "success rates" – minimal sum of squares of residua, etc..
- Fisher characteristics of adequacy.
- Analysis of the time series differences. [6].

TS modelling methods are based on the "future follows from the past" principle (in relatively stable environments), and thus are suitable also for forecasting, predicting, prognoses – data extrapolation:

- Forecast – from the TS historical data calculated (objective) datum for the subsequent, future time step, usually derived from a short time series.
- Prediction – from historical data of an appropriately long TS estimated datum for the next time step.
- Prognosis – from long-term TS historical data estimated datum for a distant time horizon.

Quality TS extrapolation (forecast, prediction, prognosis) depends heavily on its type, and it seems to be most appropriate to utilise extrapolation criteria. Simulation rests in that separated from the analysed TS are several final values, calculated from the rest of them is the trend and its appropriateness is assessed based on how well it manages to extrapolate the unused part of the series. Suitable for extrapolating are only some $50-60\%$ of models [6]. Inequality coefficients most often present the rate of the model prognostic quality.

Adaptive approaches to expressing the TS trend component

Available are two concepts, namely the moving averages method and exponential smoothing.

Exponential smoothing or degradation of historical data decreases weights of outdated data in a specific time interval. "Fresher" values will be of more profound impact upon the analysed TS future development than "older" values. It seems to be highly appropriate to introduce weights that will be inversely related to the observation "age". At exponential smoothing, the weights present exponential function of their ageing. [2].

The moving averages method resides in that that sequence of the original empirical observation is replaced by a series of moving averages. Most frequently used is moving average of 5, 7 or 9 values (at times also more), whereas with odd number of values the calculated moving average can be assigned directly to the middle member of contemplated values.

Under the assumption that appropriate to apply on moving part of a balanced time series is the linear trend, calculated are simple moving averages

$$\bar{y}_t = \frac{1}{m}\sum_{i=-p}^{p}y_{t+i} = \frac{y_{t-p} + y_{t-p-1} + \ldots + y_{t+p}}{m};$$ (5)

if it is more appropriate to use the parabolic trend, calculated are weighted moving averages in the form of

$$\bar{y}_t = \frac{1}{g}\sum_{i=-p}^{p}w_i \cdot y_{t+i}, \quad w_i = g \cdot W_i, \quad \sum_{i=-p}^{p}W_i = 1, \quad W_i = W_{-i}.$$ (6)

Used then are the following systems of weights W_i for:

- Five-member weighted moving average $(-2,3,6,3,-2)/8$, (7)
- Seven-member weighted moving average..$(-2,3,6,7,6,3,-2)/21$, (8)
- Nine-member weighted moving average $(-21,14,39,54,59,54,39,14,-21)/231$. (9)

The subjective problem that stays when calculating moving averages is the series. Most frequently opted for are averages of the lowest series. Shown in Tab. 2 are forecasting moving averages – for the case that calculated is being the anticipated value for a single forward time step. [8] [9]

Tab. 2. Weights of forecasting moving averages.

Number of TS values	Forecasting weights- 1st series	Forecasting weights- 2nd series
3	$(-2,1,4)/3$	
5	$(-4,-1,2,5,8)/10$	$(3,-3,-4,0,9)/5$
7	$(-2,-1,0,1,2,3,4)/7$	$(3,-1,-3,-3,-1,3,9)/7$
9	$(-8,-5-2,1,4,7,10,13,16)/36$	$(14,0,-9,-13,-12,-6,5,21,42)/42$
11	$(-10,-7,-4,-1,2,5,8,11,14,17,20)/55$	$(45,9,-17,-33,-39,-35,-21,3,37,81,135)/165$

Regression and correlation analysis

In scientific studies, quantitatively are often evaluated two or more quantities expressed by functional relations for 2D: $y = f(x)$, or for 3D: $z = g(x,y)$ etc. Regression analysis deals with determining the type of relation among investigated correlated (dependent) variables. Correlation analysis, on the other hand, involves determining strength (tightness) of the relation amongst variables. Examined variables are empirical values x_i, y_i and the equalising curve $y = f(x)$ is continuous, and passes among points of the empiric polygon. Distances between points and the curve $y = f(x)$ present residua (regression errors) ε_i. Result of the regression analysis if functional dependency between variables x_i, y_i characterised by the equalising curve optimally approximated to the empirical polygon [1]. Thus, in concern is selection of an appropriate regression model type, and subsequently calculation of the best estimate of its parameters. Utilised for the purpose are multiple methods, the most universal one of them being the LSM, result of which

is the so-called regression curve. The overall result will be a series of paired values that are taken for the measurement of a 2D quantity. At the regression analysis values of one variable are taken for the measurement of continuous random value y for given values of the other variable x. Linear regression can be thus defined as evaluation of quantitative relations by the use of linear function.

Linear regression model – LRM

Empirical polygon (point diagram) of values x_i, y_i shows the tendency of direct dependency, and hence can be described by the 1st degree polynomial (straight line). The straight line represents an abscissa whereas its validity is restricted by the branch of values being processed x_i and y_i.

Relation between x_i and y_i can be "bidirectional", i.e. trend value $\hat{Y}_i$ may be dependant on x_i and/or trend value $\hat{X}_i$ may be dependant on y_i. Calculation of parameters a_1, a_0, and b_1, b_0, respectively, as the so-called their best estimates by use of the LSM is based on the pertinent shape of the target function:

Type „$\hat{Y}_i$ on x_i": $\hat{Y} = \hat{a}_0 + \hat{a}_1 x$, Type „$\hat{X}_i$ on y_i": $\hat{X} = \hat{b}_0 + \hat{b}_1 y$,

$$\begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix} = \hat{a} = (X^T X)^{-1}(X^T y), \qquad \begin{pmatrix} \hat{b}_0 \\ \hat{b}_1 \end{pmatrix} = \hat{b} = (Y^T Y)^{-1}(Y^T x), \qquad (10)$$

where $X = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix}, y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix},$ where $Y = \begin{pmatrix} 1 & y_1 \\ 1 & y_2 \\ \vdots & \vdots \\ 1 & y_n \end{pmatrix}, x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$ (11)

Appropriateness of compensations of values x_i, y_i using the LRM is determined based on the correlation characteristic:

- overall variance s_{ove}^2: $\quad s_y^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(y_i - \bar{y})^2, \qquad s_x^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(x_i - \bar{x})^2,$ (12)

- residual variance s_{res}^2: $\quad s_{y,x}^2 = \dfrac{1}{n-2}\sum\limits_{i=1}^{n}(y_i - \hat{Y}_i)^2, \qquad s_{x,y}^2 = \dfrac{1}{n-2}\sum\limits_{i=1}^{n}(x_i - \hat{X}_i)^2,$ (13)

- theoretical variance s_{the}^2: $s_{\hat{Y}}^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(\hat{Y}_i - \bar{y})^2,\ at\ \bar{Y} \equiv \bar{y},\ \ s_{\hat{X}}^2 = \dfrac{1}{n}\sum\limits_{i=1}^{n}(\hat{X}_i - \bar{x})^2,\ at\ \bar{X} \equiv \bar{x}.$ (14)

Approximately holding for appropriateness of by LRM processed data is the variance equation:
$$s_{ove}^2 \cong s_{res}^2 + s_{the}^2 \qquad (15)$$

Evaluation of the regression model quality (functional dependence = 1, functional independence = 0):

Determination index: $I_{xy}^2 = \dfrac{s_{teo}^2}{s_{cel}^2};$ (16)

Fisher's adequacy characteristics: $F_{ad} = \dfrac{s_{cel}^2}{s_{rez}^2}$, ($s_{res}^2$ should be as small as possible) (17)

Pearson's correlation coefficient: $r = \dfrac{\sum\limits_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n s_x s_y},$ (18)

whilst if $r > 0\,(<0)$ the relation is positive (negative).

The LRM of both types constitute so/called pooled regression models, and together they form so-called correlation scissors. The more the scissors are "open" the smaller is r, and the functional dependence is lower, and vice versa. If $r = \pm 1$, the more functional the statistical significance becomes. Considered for proper regression model is if $|r| > 0{,}7\ (0{,}8)$. [2]

Nonlinear regression models – NLRM

Not always is selection of proper NLRM types easy. At times, nonlinear regression model can be by introducing functional relations among regression coefficients transformed into a linear model. The most frequently used types are: polynomial (up to 4-5 degree), power (potential), exponential, hyperbolic, logistic types, etc.. At selecting the most appropriate NLRM type form amongst the recalculated possible ones it is most appropriate to adhere to minimal sum of orders of evaluations (at identical overall variance) [1]:

- minimal residual variance,
- minimal theoretical variance,

- max correlation index value,
- min number of specified parameters, etc..

Experimental deformation network surveying and evaluation of the GNSS signals

Experimental surveying was performed in the deformation network (DN) located in the vicinity of the upper reservoir of water transfer power station (WTPS) Čierny Váh, in the area of the Low Tatras National Park. Performed in the locality were observations using the GNSS technology and by static method using single-band GPS Sokkia Stratus receivers. [4]

Terrestrial measurements cannot be performed whereas high forest cover precludes visibility between pairs of the network points. By the manufacturer stated precision for the static method of GNSS technology is in position of $5 mm + 1 \cdot D_{ij} ppm$ and elevation $10\ mm + 1 \cdot D_{ij} ppm$. Observations of DM consisting of 7 points were performed in 4/2004 and 7/2008, respectively, with sufficient time interval (

Fig. *1*). Used in course of performing observations was the static method of determining position of all points of the network (5001 to 5007). Point 5001 was selected as the reference point of the network, whereas it was stabilised in the terrain of maximum size and flatness. The network was made up using 11 vectors (Fig. 1). Signals were in individual points of the network received within the time interval of 1 to 7 hours (7 hours – reference point 5001).

Originally, the observed data were treated by post-processing in the software environment of Spectrum Survey, in the geocentric WGS-84 system of coordinates. The results were geographical coordinates φ, λ and elevations h of individual network points. Imported into the Trimble Geomatic Office software for the post-processing adjustment of the network were observations files in the RINEX 2.11 format from two reference stations SKPOS (SKLM and GANP). RINEX files provided for change of the system of coordinates in ETRS-89, which accounts for movement of the Eurasian lithospheric plate, and hence also shift in relation to coordinate axes of the WGS-84 system.

Used for processing was the Gauss-Markov model of full rank. Epochs 2004 and 2008 were adjusted both separately and applying bivariate methods using various weights [7]:

1. According to recommendation of the technology manufacturer: $5 mm + 1 \cdot D_{ij}\ ppm$,
2. Seeding the covariance matrix from Spectrum Survey, where $s_0^2 = RMS$,
3. Seeding the covariance matrix from Spectrum Survey, where $s_0^2 = 1$,
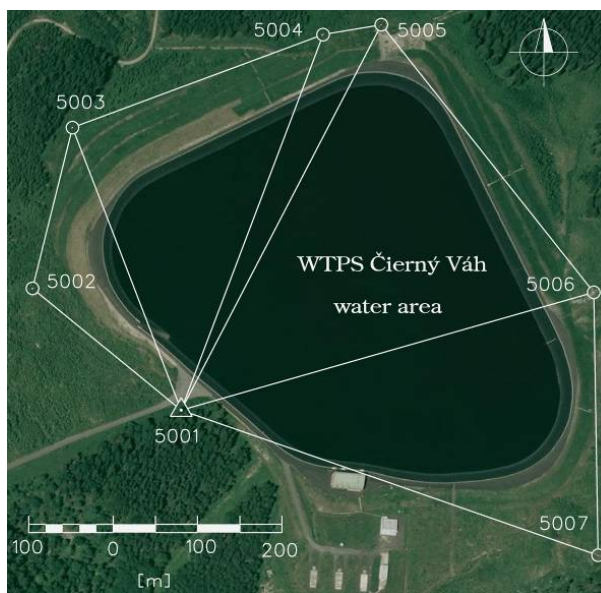4. Weights of observations were estimated using the MINQUE method.

Fig. 1. *Structure of 3D network in the location WTPS Čierny Váh.*

From the point of standard deviations, the most precise proved to be the solution using covariance matrices from the Spectrum Survey. With the MINQUE method [3], the weights were estimated based on by the manufacturer recommended precision, whilst numeric results of processing proved that applying of MINQUE method was an adequate substitute of laborious inputting of covariance matrices from the Spectrum Survey.

At investigating variations between epochs there are several millimetre differences in estimates of adjusted coordinates of points when different measurement weights are used. Yet, all of the processing

performed pointed to a shift of point 5005 (coordinates differences of the point 5005: 1. 25,18 mm, 2. 25,59 mm, 25,59 mm, 4. 24,95 mm – from adjust with using various weights). Revealed differences at other points were due to accumulation of measurement errors.

Real time simulation

Real time simulation presents investigating movement of a point between epochs 4/2004 and 7/2008. Observations that were between the epochs missing were obtained by simulation in the MATLAB software environment. At processing GNSS observations and testing of the deformation vector $\Delta\hat{C} = {}^{7/2008}\hat{C} - {}^{4/2004}\hat{C}$ of network it was established that a fluctuation in point 5005 occurred between epochs 2004 and 2008. That is why was the point selected for more in detail examination using simulation. The purpose of simulation is to find out in which epoch could the methods used record movement of the point for the first time and to illustrate how was the point moving during the monitoring period.

52 months passed between epochs 4/2004 and 7/2008, respectively, when were observations of the network performed. Chosen according to the number of months was also the number of simulated observations between the two epochs, i.e. the simulation consisted of 52 observations. The number was opted for so that met was the condition of a wide selection numerosity, where $n > 30$, and that the simulation graphical presentation was sufficiently transparent along all axes of coordinates, planes and in the 3D space.

The prerequisite of a successful simulation is that movement of point 5005 is in space straightforward, regular, and that during the observation period the point would move from " ${}^{4/2004}5005$ " to point " ${}^{7/2008}5005$ ". To simulate real time it proved to be necessary to obtain the observation vector that is loaded with random errors that continuously influence each of the measurements.

Simulation of the observation vector loaded with random errors

Simulation of real observations resides in loading by random errors that cannot be by any means eliminated. Random error Δ_i is the error that at an identical measured quantity, method, measurement conditions and thoroughness of measurement randomly attains various quantities and signs. Mean value of the principal set of random errors with normal distribution is $E(\Delta) = 0$. Random (stochastic) errors are in the process of measuring occurring spontaneously, individually, and are not subject to any natural relations.

Their properties are established based on empirically established qualities of random errors:

- probability of occurrence of a positive or negative error is identical,
- small errors are more probable than major errors; probability of the error occurrence is a function of their magnitude,
- errors that exceed certain limit are taken for gross errors, and no errors are occurring beyond certain boundary. [1].

Used during simulating random errors in the observation vector ${}^{4/2004}L$ (L from epoch 4/2004) performed in MatLab environment was the "normrnd" command, which out of the input data (observations) generates input observations of the subsequent epoch that are loaded with normally distributed random errors. The input data can be a scalar, vector (our case) or a matrix. Value of the random error is influenced by standard deviation the size of which is defined. The input data are the observation vector ${}^{4/2004}L$ and the standard deviation $\sigma[m]$. Used at simulating was the standard deviation that had been specified by the manufacturer to be of $\sigma = 5mm$. Input vector ${}^{4/2004}L$ and by errors loaded simulated vector from epoch ${}^{5/2004}L$ (the 1st simulated epoch) are shown in Tab. 3 .

Simulated observation vector ${}^{5/2004}L$ was used in bivariate adjustment of the network in epoch 5/2004. Whereas the shift occurred at point 5005, investigated

Tab. 3 Vector of real and simulated observations

Observation vector		${}^{4/2004}L$ [m]	${}^{5/2004}L$ [m]	dL [mm]
5001 5002	ΔX	-38,650	-38,6489	-1,1
	ΔY	-210,806	-210,8047	-1,3
	ΔZ	78,159	78,1651	-6,1
5001 5003	ΔX	-205,608	-205,6094	1,4
	ΔY	-234,000	-234,0007	0,7
	ΔZ	183,508	183,5016	6,4
5001 5004	ΔX	-379,841	-379,8493	8,3
	ΔY	10,425	10,4215	3,5
	ΔZ	287,820	287,8214	-1,4
5001 5005	ΔX	-411,409	-411,4117	2,7
	ΔY	71,426	71,4193	6,7
	ΔZ	302,107	302,1124	-5,4
5001 5006	ΔX	-285,825	-285,8286	3,6
	ΔY	405,734	405,7339	0,1
	ΔZ	110,219	110,219	0,0
5001 5007	ΔX	-74,740	-74,7412	1,2
	ΔY	508,855	508,857	-2,0
	ΔZ	-95,332	-95,3333	1,3
5002 5003	ΔX	-166,978	-166,9863	8,3
	ΔY	-23,200	-23,2051	5,1
	ΔZ	105,385	105,3862	-1,2
5003 5004	ΔX	-174,199	-174,2053	6,3
	ΔY	244,44	244,4383	1,7
	ΔZ	104,319	104,3143	4,7
5004 5005	ΔX	-31,584	-31,5899	5,9
	ΔY	60,983	60,9779	5,1
	ΔZ	14,281	14,2790	2,0
5005 5006	ΔX	125,589	125,5899	-0,9
	ΔY	334,316	334,3154	0,6
	ΔZ	-191,873	-191,8677	-5,3
5006 5007	ΔX	211,088	211,0868	1,2
	ΔY	103,129	103,1214	7,6
	ΔZ	-205,581	-205,5810	0,0

coordinated of the point were gradually depicted as scattered charts in the directions of axes X, Y and Z, which resulted in three time series (Fig. 2). Each chart contains the output of actual testing. If the deformation vector $d\hat{C}$ test for point 5005 demonstrated movement of the point, position of the point is plotted in the chart as a red asterisk, and in the contrary case as a blue dot.

Decomposition of time series was confirmed by simulation of straightforward movement of point 5005 between epochs 4/2004 and 7/2008. LSM presents the most frequently method of estimating parameters of trend functions, and the method is applicable in case that the selected trend function is linear in its parameters. Satisfactory for the given time series are linear trends for which it holds that $Tr_t = a_o + a_1 t$, $t = 1, 2, .., n$.

Verification of the TS trend proved that the given trend has high "rate of success" of the trend function, i.e. minimal sum of squares of the residues (min square error of the estimate). Plotted for graphical illustration of the observation vector loaded with the random error that reflects effect of random errors of the ±5 mm interval were the charts of residues illustrated in Fig. 3.
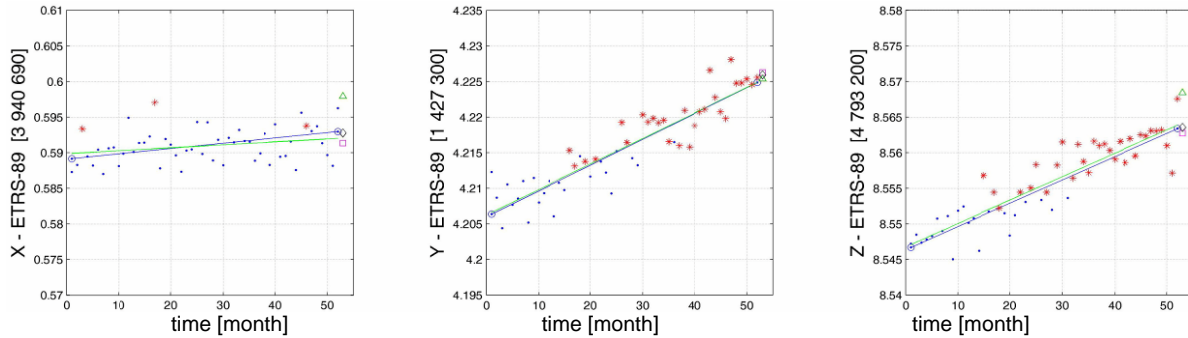

Fig. 2. Illustration of the simulation along individual coordinate axes.
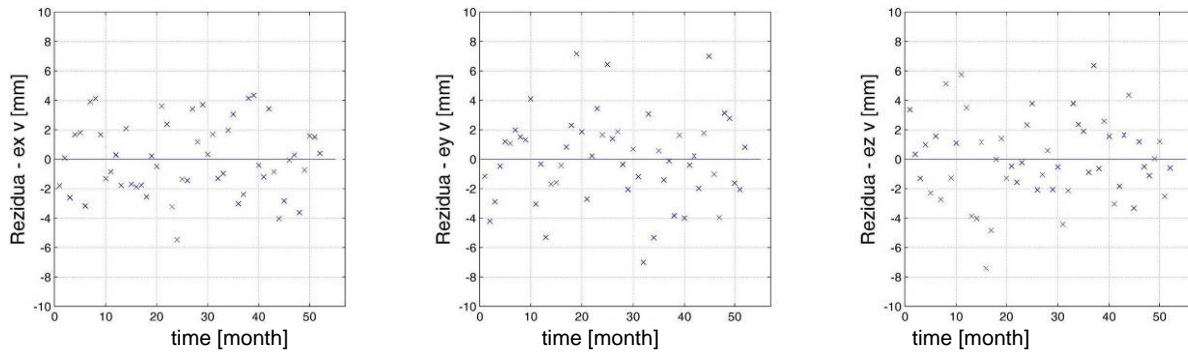

Fig. 3. Charts of residues in individual coordinate axes.

By using the moving averages method elaborated was a forecast of the following position of the point in the future. Used for 3, 7, 11 values of the time series from Tab. 1 were the 1st order forecast weights. A breakdown of forecasts is shown in Tab. 4.

Tab. 4 Summary of the time series forecast

The 1st order forecast weights according to:	Point coordinates 5005		
	X [3 940 69∗,∗∗∗∗ m]	Y [1 427 30∗,∗∗∗∗ m]	Z [4 793 20∗,∗∗∗∗ m]
3 values of TS	0,5998	4,2182	8,5697
7 values of TS	0,5945	4,2245	8,5711
11 values of TS	0,5943	4,2250	8,5675

According to the number of presumed values illustrated in the graphs (Fig. 4) are forecasts of: three numbers - green △, seven numbers - pink □ and eleven numbers - black ◊. Demonstrated in Fig. 4 are details from Fig. 2 in the order from left to right: axis X, axis Y and axis Z.
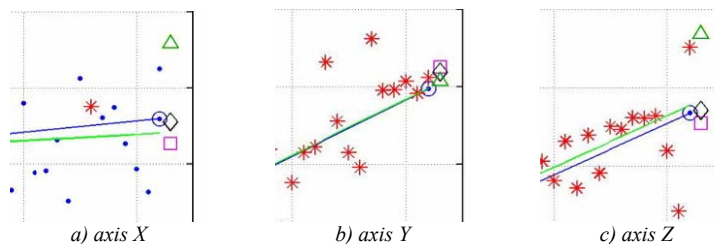

a) axis X *b) axis Y* *c) axis Z*

Fig. 4. Forecasted next position of point 5005 for individual axes.

The forecast that most closely reflects linear trend of simulation also most closely corresponds with the forecast of 7 and 11 TS values.

Examined was also mutual correlation between coordinates of point 5005. In the *YZ* plane, where the tests certify the most profound movement, also the correlation is highest. In the other two cases are correlation scissors slightly opening, which point to a lower level of correlation (Fig. 5). Tests of the 1*D* deformation vector $d\hat{C}$ confirm shift of point 5005, whilst there was no movement evident along axis *X*. Hence, the trend of movement of point 5005 along axis *X* is not the same as is the case along axes *Y* and *Z*, where the shift is comparable. Shift of point 5005 can be demonstrated by statistical processing already from the 25th simulation epoch (Fig. 2, Fig. 5).
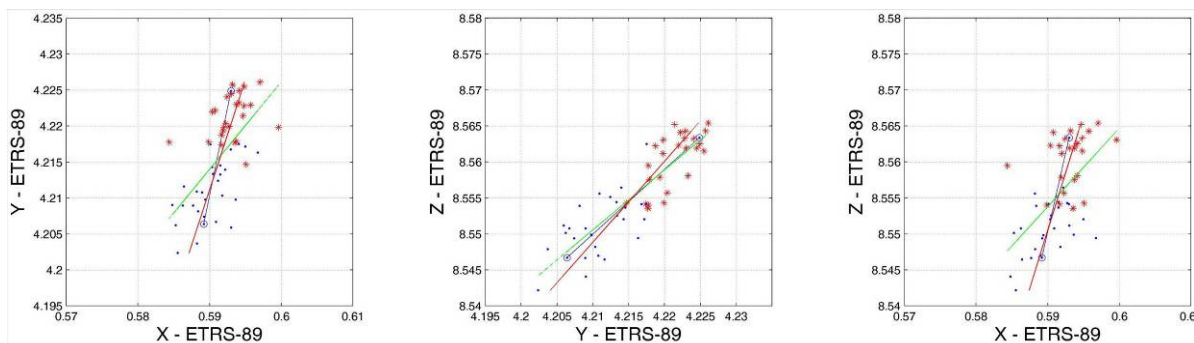

Fig. 5. Correlation of individual constituents of movement of point 5005 in simulation.

3D visualisation of point 5005 (Fig. 6) in both epochs with absolute confidence ellipsoids and positions of points 5005 from individual simulation epochs during the observation period (52 months).

Conclusion

Deformation analysis demonstrated in the period of 2004 to 2008 travel of point 5005, and this was used for further analysing using simulations. Missing observations were obtained via simulation in MatLab programming environment. Anticipated was even straightforward shift, whilst observations were loaded with random errors. In between epochs 2004 and 2008, gradually determined were 52 positions of the point.

Examined by use of time series were the time points when the methods commenced to demonstrate shifts of the point, which provided the possibility to forecast the following point position. Simulation results proved that usage of time series was appropriate for determining movement of the point among epochs, and in addition they demonstrated relevance of the real-time measurement for early revealing of deformations and timely commencement of remedial activities.
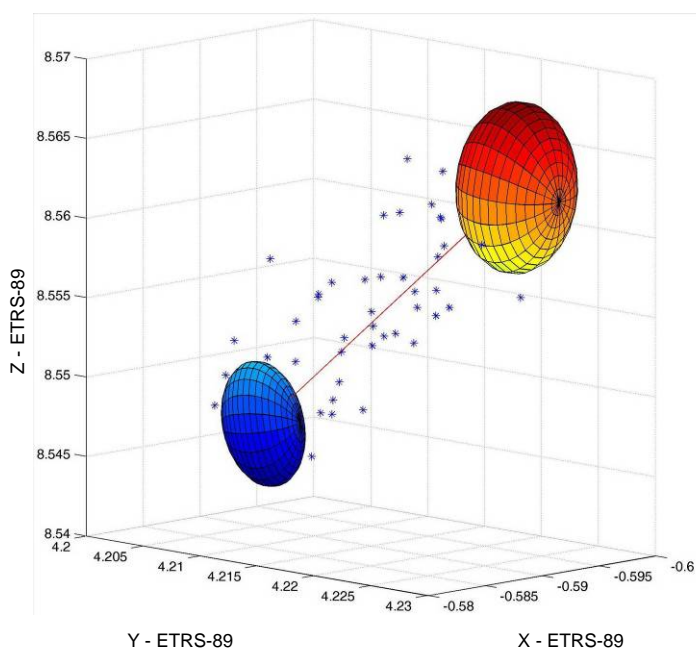

Fig. 6. Graphical visualisation of point 5005 point movements since epoch 4/2004 till epoch 7/2008.

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