Capital circulation disruptions and company default

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The paper concerns selection of input variables for discriminant analysis and logistic regression applications as regards default prediction for companies in the Czech Republic. The selection has been based on the assumption that a company default is caused by its capital flow disruptions. These disruptions are reflected by company balance sheet and income statement, that can provide for the establishment of ratio and index indicators. The model based on quadratic discriminant function displayed the most significant classification ability.

Keywords: *Multivariate statistics, company default, capital circulation disruption, discriminant analysis, logistic regression*

Introduction

Currently, the Czech Republic, namely the North Moravian Region, has been facing economic, social, and political problems from the plan of closing operations of the deep mining colliery of Paskov. Difficult mining geology and decreasing coal prices have constituted reasons for action. Both these factors have caused the Paskov colliery operations to be in the deep red. The colliery closure would result in losing about 2.5 thousand jobs per se, and would imply losses of thousands of jobs at supply house companies. The difficult situation is aggravated by the fact that the colliery of Paskov operates in the region of the highest unemployment rate in the Czech Republic.

The closure notice aroused strong protest and criticism among the colliery staff and attempts are made to negotiate the controversy. There is still some time left before commencing the planned action, which might provide for finding of solutions that would ensure social stability and political peace. Also other countries face similar problems as regards decreasing demand for minerals (Csikósová et al. 2013).

An obvious exigency of such situation is the ability to predict if a specific company can continue in operation or will have to file for bankruptcy.

The prediction of future economic development of companies, which especially concerns bankruptcy predictions, is very important regarding commercial activities between companies, their investment or acquisition activities, and so on. There are many expert references to the matter, of which synopses are of the primary concern (Balcaen and Ooghle 2006; Hol et al. 2002; Morris 1997). The work of Balcaen and Ooghle identifies many issues to be dealt with concerning prediction of future economic performance of companies. They start by defining the terms of failure or financial distress, going through specification of bankruptcy causes, right to the evaluation of individual prediction methods.

If the firms investigated are classified as successful (activities continue) or bankrupt (activities terminate), we can eliminate the problem of defining the term "failure." Nevertheless, we must agree with the opinion, quote: "Some companies may have filed for bankruptcy in order to get rid of their debts and restart their activities with a clean sheet," (Balcaen and Ooghe 2006).

It is relatively easy to choose a method of predicting companies' future economic performance. For decades, the methods of discriminant analysis and logistic regression dominated (Aziz and Dar 2006). Since the nineties of the past century, the artificial intelligence methods, especially neuron networks, have been gaining prominence, as their prediction accuracy can outdo statistical methods (Tseng and Chi 2005; Zhang et al. 1999).

Nevertheless, all the methods mentioned need specific variables, the selection of which poses the most difficult issue of the matter, as there is little agreement on which of the variables is the best choice. It can be assumed that it is related to what can cause a company default or bankruptcy models implied.

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Bankruptcy causes and models

The activities of a company can be terminated by "an unexpected event, such as natural disaster" (Balcaen and Ooghe 2006). Nevertheless, out-and-out, bankruptcy marks the end of a longer process. The bankruptcy can be caused by many factors: "High interest rates, recession squeezed profits, heavy debt burdens, government regulation, nature of operations" (Charitou et al. 2004); "Increased production cost, inability of management to control a larger organization, industry and economic trends, technical obsoleteness" (Tseng and Chi 2005); "General and immediate environment, management/entrepreneur characteristics, corporate policy, company characteristics" (Wu 2010); "Lack of sufficient cash flows from operating activities" (Charitou et al. 2004). Many nonaccounting or qualitative failure indicators are given by Balcaen and Ooghe (2006).

Reflecting on the bankruptcy causes, we can think about a definite model of the process that leads to default. It is especially the lack of such model that may be the reason for disagreement as regards the right choice of default factors (Hol et al. 2002; Morris, 1997). Charitou et al. (2004) and Tang and Chi (2005) maintain that "…the main limitation of the present study and almost of previous failure prediction studies is the lack of a sound theoretical framework…."

On the other hand, it is necessary to note that many such models exist. They can be basically structured as follows (Morris 1997; Aziz and Dar 2006; Laitinen and Kankaanpää 1999):

Normative theories:

- disequilibrium models of corporate failure,
- financial models of corporate failure,
- agency models of corporate failure, and
- management theories of corporate failure.

Positive theories:

- univariate models,
- multivariate models,
- iterative models,
- early warning studies, and
- case study research.

Lízal (2002) mentions three models: neoclassical model, financial model, and corporate governance model. Another way of the model structuring distinguishes statistical models, expert systems, and theoretical models (Aziz and Dar, 2006). Nevertheless, it is necessary to pay attention to applicability of theoretical approaches per se, quote: "Some of the models derived from such theories are essentially descriptive, many others are not really operational in an economic context since they make unrealistic assumptions, while some of them were found to yield similar or even inferior results applying the so called ad hoc financial ratio models" (Morris, 1997).

Finding a compromise in the debate over strictly theoretical models on the one hand and empirical approaches on the other could provide a solution to the problem.

Gaining agreement on causes and models of default might be of great importance, because they imply the choice of input variables for predicting future economic performance of a company. "There seems to be no consensus on the superior variables…" (Balcaen and Ooghe 2006). "The choice of variables is usually pragmatic" (Aziz et al. 1988), or "based on popularity" (Altman 1968) or "frequent appearance in the literature" (Gilbert et al. 1990). Another reason may be that "ratios performed well or were used in one of the previous studies" (Beaver 1967; Norton and Smith 1979; Tam and Kiang 1992; Pindado and Rodrigues 2004) or based on the intuition of researchers (Charitou et al. 2004). A time dimension of the input variables, that is, their change and development over time, is very often underestimated (Balcaen and Ooghe 2006).

It can be assumed that a compromise between theory and empirical approaches is implied in a company capital circulation.

Capital circulation disruptions and company default

From an economic point of view, the activities of a company can be described as the circulation of capital (Mital et al. 2007; Dvořáček et al. 2008). Initially, an entrepreneur must provide for the necessary capital by borrowing from either a bank or other sources (stockholders or profits from other projects, etc). The input capital provision serves for fixed assets realization (purchases of land, buildings, machines, etc.), current assets (raw materials and other inputs), and hiring a workforce. As based on the consumption and wear-and-tear of production factors, products are made and sold, and revenue was created. Part of this revenue is returned to the lending institution, investors, or owners, and part is used for refreshing consumed current assets, pays for workforce, and redressing the balance of fixed assets. Capital can be also expanded from internal or external resources so that the process of assets' rejuvenation may accelerate on a higher level. The refreshment of assets and paying the workforce must be continuous; otherwise, a reduction of reproduction follows—both assets and workforce are becoming smaller in size, number, and intensity, of which, the process soon reaches its limits. If this happens, an extra capital of one's own or from other independent capital resources must be provided for. Nevertheless, this provision is not always at hand or possible. A capital deficiency may lead to default.

The failure of the capital provision for operations of a company is principally caused by the following:

- A major increase in company assets and workforce that is not accompanied by revenue increases (company grows too fast), and
- A capital circulation disruption caused by customers' weak demand, decreases of account receivables, production difficulties per se, difficulties in ensuring production process inputs.

The best manifestation of lacking capital provisions is the company's inability to pay debts. In this situation, the peril of default is imminent, as "the inability to pay its financial obligations as they mature causes failure of firms" (Beaver, 1967; Charitou et al., 2004; Smith and Winakor, 1935).

The firms solve the problem of lacking capital in many ways. Nevertheless, all solutions are reflected by the figures of the balance sheet and income statement. The principal ways of solving the problem are given in Table 1.

Tab. 1. Solving problem of capital deficiency and implications.

Provision		Balance sheet and income statement figures
(i.)	Inside capital extra deposit	Increase of equities
(ii.)	Outside capital extra deposit	Increase of indebtedness
(iii.)	Delay of debt payment	Increase of quick liabilities
(iv.)	Limiting production inputs	Decrease of current assets
(v.)	Limiting production (and consequently sales)	Decrease of receipts or receivables
(vi.)	Limiting reproduction of fixed assets	Decrease of fixed assets
(vii.)	Sale of fixed/current assets	Decrease of total assets
(viii.)	Staff layoff	Decrease of personnel costs

The changes of balance sheet and income statement figures imply changes of the related indicators‒financial ratios. As regards Table 1 data structure, these ratio indicators can be specified ad follows:

(i): Increase of equities: equity capital/total assets or equity capital index,
(ii): increase of indebtedness: loan capital/total assets or loan capital/total assets index,
(iii): increase of indebtedness as regards current liabilities: current liabilities/total assets or current liabilities index,
(iv): decrease of current assets: current assets index or current assets/total assets,
(v): decrease of sales: sales index or
decrease of total assets utilization: sales/total assets or
change of current liquid assets proportion: current liquid assets/current assets or receivables/current assets,
(vi): decrease of fixed assets volume: fixed assets/total assets or
decrease of fixed assets development: fixed assets index,
(vii): decrease of total assets volume: total assets index,
(viii): decrease of wages and salaries: personnel costs index.

As based on experience with reviewing basic accountancy documents of bankrupt companies, some financial ratios can be excluded ex ante. This especially concerns the following:

- Indicators based on an equity capital index imply retained income in its structure, the fact of which leads to equity negative figures if long term in the red operations is the case. Then, the index calculation related to deficit expansions of two consecutive periods, implying an equity decline has the same development‒from the mathematical point of view‒as if both profit and equity of the same period increased.

- Indicators based on personnel costs, as these costs are influenced by workforce numbers, average wage, and labour productivity developments.

The choice of appropriate ratios can be supported empirically by investigating average values of specific ratios regarding successful and bankrupt firms. The following table gives such averages for 93 successful and 93 default firms in the Czech Republic. All these firms will be characterized in what follows.

Tab. 2. Averages for successful and default companies.

Indicator	Successful firms	Default firms	Statistical significance*
Equity capital/total assets	0.61	-1.35	S
Loan capital/total assets	0.38	2.31	S
Loan capital /total assets index	0.87	1.67	S
Current liabilities/total assets	0.19	1.77	S
Current assets index	1.19	0.74	S
Sales index	1.42	0.86	S
Sales/fixed assets	8.59	212.13	-
Current liquid assets/current assets	0.34	0.09	S
Fixed assets/total assets	0.44	0.41	N
Fixed assets index	1.79	0.76	S
Total assets index	1.11	0.75	S
Receivables/current assets	0.50	0.66	S
Sales/total assets	1.43	1.59	N
Current assets/total assets	0.54	0.58	N

*S... Statistically significant, $p = 0.05$
N... Statistically insignificant, $p = 0.05$; (pair, t-test)*

The tests of statistical significance were performed for the averages of successful and default firms, which eliminated the following indicators from Table 2.

- Fixed assets/total assets, sales/total assets, current assets/total assets‒the indicators were eliminated because of statistically insignificant differences of relative indicator averages between the files of successful and default firms;
- Sales/fixed assets‒the indicator was not subjected to statistically significant investigation because its values for both successful and default firms showed excessive variance, which meant that variance coefficients indicated excessive inhomogeneity of both files. As regards especially default companies, the reason is in the decreased reproduction of fixed assets, their depreciation and sale, which in concurrence with decreasing receipts leads to extremely high values of the indicator.
- Fixed assets index‒the indicator was eliminated because some firms had reported zero figures of fixed assets before they were declared bankrupt.

The following indicators seem to have an advantage of application:
(i) equity capital/total assets, (ii) loan capital index, (iii) quick liabilities/total assets, (iv) current assets index, (v) sales index, (vi) current liquid assets/current assets, (vii) total assets index, and (viii) receivables/current assets.

Predicting a company default

In analogy to application of statistical methods for predicting default of firms, which had been employed by other investigations, two files were created. One of them comprised successful companies that showed retained positive earnings trends and continued in their activities. The other file consisted of bankrupt companies that ceased to exist in their original form. Each file comprised 93 firms.

The time span of the database has been 14 years, 1996–2010. All selected firms were only from the Czech Republic, aiming at easy comparison of the financial statement data with an identical provision. The majority of the firms investigated were from the production sector. Only a few of them operated in the services sector.

The files of 93 successful and 93 bankrupt companies were assembled as follows: initially, the bankrupt companies were selected. The bankrupt firms in time, t, were chosen at random from the list of bankrupt companies provided by the Business Register of the Czech Republic. A prerequisite was that all basic financial statement data were Internet accessible. The date of issue of these statements must have been 12 months before their default notice at the longest, and in time, t-1, at the end of the year. We also recorded the financial statement data a year before, that is, the time, t-2. The successful firms in time, t, were selected on their then current and recent economic performances. Internet business registers served the purpose. Those successful firms were preferred whose size was similar to their bankrupt counterparts. The firm's total assets provided by their financial statements were the measure of their size. The date of issue of the financial statements must not have been older than 12 months before our investigation, time, t-1, at the end of the year. We also recorded the financial statement data a year before, that is, time, t-2.

This information provided for the input data of calculating relevant indexes of the companies' economic situation, namely, the indicator indexes as indicator values in time, (t-1)/(t-2).

Discriminant analysis and logistic regression application

Regarding both files, the basic financial statement data provided for the input variables of the statistical method applications. In the form of ratios and indexes, these variables were subject of discriminant analysis. A correct discriminant analysis application implies fulfilling an assumption of multivariate normal distribution. The testing of a multivariate normal distribution is difficult because even if individual random vector items of all discriminators show univariate normality, the joint probability density does not necessarily have a multivariate normal distribution. A practical verification of the discriminant vector multivariate normality can take advantage of this fact. First, the univariate normality of individual discriminators is tested. If at least a single discriminator cannot show its univariate normality, it is obvious that any multivariate normality also cannot be proved, and the testing is terminated. If all discriminators show normality, it is necessary to continue in the testing of the multivariate normality.

To examine the univariate normality of each discriminator, concerning files of both successful and default companies, they were tested for skewness and kurtosis combinations. The testing results led to the conclusion that not a single discriminator is of normal distribution, and consequently, the multivariate normal data distribution cannot exist. An analysis of the individual discriminator Rankit plots attested to this fact. The testing results are not surprising because it is rather a rule than an exception that economic data deviate from normality. Most investigation works do not test normality and assume their models to be robust enough to provide for rational approximations even without fulfilling the assumption of normal distribution of input data.

It followed that the data were purified by excluding outliers. The objective was to approximate a normal distribution. The outliers were identified using two methods–univariate and multivariate. The former consisted in the application of inner fences on individual discriminator values, and the latter was based on the application of the Mahalanobis distance of individual multivariate data from the mean value. The outlier exclusion resulted in reducing the original files of 93 successful and 93 default companies so that the inner fencing and Mahalanobis distance applications provided for 76 successful and 74 bankrupt companies, and 88 successful and 85 bankrupt companies, respectively.

Linear discrimination application for 2 classes

First, the linear discriminant analysis was applied.

The point of departure is in the knowledge of input matrixes $\boldsymbol{X}_1 = (\boldsymbol{x}_{11} \quad \boldsymbol{x}_{12} \cdots \boldsymbol{x}_{1m})$ of size, $n_1 \times m$, for the first class of successful companies, and $\boldsymbol{X}_2 = (\boldsymbol{x}_{21} \quad \boldsymbol{x}_{22} \cdots \boldsymbol{x}_{2m})$ of size $n_2 \times m$, for the second class of default companies,

where

$\boldsymbol{x}_{1j}$ are n_1 – size vectors making for columns of the matrix, $\boldsymbol{X}_1$, and

$\boldsymbol{x}_{2j}$ are n_2 – size vectors making for columns of the matrix $\boldsymbol{X}_2$ ($j = 1,\ldots,m$).

Each line of the matrix, $\boldsymbol{X}_1$, consists of discriminators of successful companies; each line of the matrix, $\boldsymbol{X}_2$, consists of discriminators of default companies. Each column of the matrix, $\boldsymbol{X}_1$, consists of discriminators of all successful companies; each column of the matrix, $\boldsymbol{X}_2$, consists of discriminators of all default companies.

The linear function coefficients, $L(\boldsymbol{x}) = \boldsymbol{a}^T \boldsymbol{x}$, can be estimated from

$$\hat{\boldsymbol{a}} = (\bar{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_2)\, \boldsymbol{S}^{-1}, \tag{1}$$

where

$\bar{\boldsymbol{x}}_1$ or $\bar{\boldsymbol{x}}_2$ are sample averages of successful or bankrupt companies,

$\boldsymbol{S}^{-1}$ is an inversion matrix to the common covariance matrix,

$$\boldsymbol{S} = \frac{(n_1 - 1)\boldsymbol{S}_1 + (n_2 - 1)\boldsymbol{S}_2}{n_1 + n_2 - 2} \quad \text{and} \tag{2}$$

$\boldsymbol{S}_1$ or $\boldsymbol{S}_2$ is a sample covariance matrix for successful or bankrupt companies.

Furthermore, an invariable needs to be calculated:

$$\hat{b} = -\frac{1}{2}\hat{\boldsymbol{a}}^T(\bar{\boldsymbol{x}}_1 + \bar{\boldsymbol{x}}_2) \, . \tag{3}$$

Whether a firm, which has been characterized by discriminators, $\boldsymbol{x}_0$, is classified successful or bankrupt, this is decided by the inequality $\hat{\boldsymbol{a}}^T \boldsymbol{x}_0 + \hat{b} > 0$ or $\hat{\boldsymbol{a}}^T \boldsymbol{x}_0 + \hat{b} \le 0$, respectively.

The classification results of the linear discrimination are presented in Table 3.

Tab. 3. Classification success of original vis-à-vis reduced files (linear discrimination).

	Classification success [%]		
	Successful firms	**Default firms**	**Total**
Original file (93 successful firms + 93 default firms)	87.10	89.25	88.17
File reduced by inner fencing (76 successful firms + 74 default firms)	100.00	90.54	95.33
File reduced by Mahalanobis distance (88 successful firms + 85 default firms)	95.45	94.12	94.80

The table shows that the efforts to approximate data multivariate normality are fruitful as they could substantially improve classification abilities of the linear discriminant function. The best overall classification results were attained by reduction of the individual discriminator values taking advantage of inner fencing. Concerning multivariate normality, such data reduction proved to be the most successful.

Quadratic discrimination application for 2 classes

Regarding both the original and reduced files of successful and default companies, the testing for their covariance matrix identity followed. The testing resulted in rejecting the hypotheses of covariance matrix identity. This fact indicates usage of quadratic discriminant analysis for the discriminant function estimation. Dimitras et al. (1996) refer to usage of linear instead of quadratic discriminate analysis as to shortcomings of discriminant analysis applications if covariance matrixes are not identical. The applications of the quadratic discriminant analyses are not very frequent, which may be caused by a relatively complicated calculation of the quadratic function coefficients and their substantially larger number in comparison to the number of the linear discriminant function coefficients. For an analysis of eight discriminators, the quadratic discrimination implies calculation of 44 coefficients. The linear discrimination would ask for only 8 coefficients.

The quadratic discriminant function can be expressed as

$$Q(\boldsymbol{x}) = \boldsymbol{x}^T \boldsymbol{G}\boldsymbol{x} + \boldsymbol{h}^T \boldsymbol{x} + C \, , \tag{4}$$

where

$$\boldsymbol{G} = \frac{1}{2}\left(\boldsymbol{S}_2^{-1} - \boldsymbol{S}_1^{-1}\right) , \tag{5}$$

$$\boldsymbol{h}^T = \bar{\boldsymbol{x}}_1 \boldsymbol{S}_1^{-1} - \bar{\boldsymbol{x}}_2 \boldsymbol{S}_2^{-1} \, , \tag{6}$$

$$C = 0{,}5 \ln \frac{|\boldsymbol{S}_2|}{|\boldsymbol{S}_1|} - 0{,}5\left(\bar{\boldsymbol{x}}_1^T \boldsymbol{S}_1^{-1} \bar{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_2^T \boldsymbol{S}_2^{-1} \bar{\boldsymbol{x}}_2\right), \tag{7}$$

$\boldsymbol{S}_1^{-1}$ and $\boldsymbol{S}_2^{-1}$ are inverse matrixes for sample covariance matrixes, $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$.

If a company of the discriminator value, $\boldsymbol{x_0}$, is classified, and if a quadratic inequality,

$$\boldsymbol{x_0}^T\boldsymbol{G}\boldsymbol{x_0} + \boldsymbol{h}^T\boldsymbol{x_0} + \boldsymbol{C} > 0$$, is valid,

then the company is classified as successful. If an inverse inequality,

$$\boldsymbol{x_0}^T\boldsymbol{G}\boldsymbol{x_0} + \boldsymbol{h}^T\boldsymbol{x_0} + \boldsymbol{C} \leq 0$$, is valid,

then the company is classified as bankrupt.

The classification that resulted from applications of the quadratic discriminant function is given in Table 4.

Tab. 4. Classification success of original vis-à-vis reduced files (quadratic discrimination).

	Classification success [%]		
	Successful firms	**Default firms**	**Total**
Original file (93 successful firms + 93 default firms)	97.85	84.95	91.40
File reduced by inner fencing (76 successful firms + 74 default firms)	100.00	100.00	100.00
File reduced by Mahalanobis distance (88 successful firms + 85 default firms)	98.86	96.47	97.69

The table makes it obvious that the best classification results followed from those files of successful and default companies that had been reduced by inner fencing. The quadratic discriminant function based on data of such reduction could classify both successful and bankrupt companies with 100 % rate of success.

The classification success of linear vis-à-vis quadratic discriminant function is shown in Tables 3 and 4, which further leads to a conclusion that models based on quadratic discriminant functions provide for better classification results. Nevertheless, the classification results can be distorted by an occurrence of calculating discriminant functions by utilizing the entire files and also classifying the files in their entirety.

Logistic regression application

The method of logistic regression was applied to the same set of variables. The results of classification by logistic regression are presented in Table 5.

Tab. 5. Classification success of original vis-à-vis reduced files (logistic regression).

	Classification success [%]		
	Successful firms	**Default firms**	**Total**
Original file (93 successful firms + 93 default firms)	97.85	95.7	96.77
File reduced by inner fencing (76 successful firms + 74 default firms)	97.37	100.00	98.67
File reduced by Mahalanobis distance (88 successful firms + 85 default firms)	98.86	98.85	98.84

It follows from the comparison of classification successes of quadratic discrimination and logistic regression that the best results were attained if data files had been reduced by inner fencing and then classified by the quadratic discriminant function.

Discussion

The setting of 8 discriminators, which were used for discriminant analysis applications, was oriented by the idea that capital circulation disruptions can be the cause of company default. The original financial documentation data were taken into account, changes of which reflected disrupted capital flows, and also, the results of statistical significance assessments were considered concerning differences of the input variable mean values between the files of successful and default companies.

The idea of different ratios between successful and default companies relates back to the beginning of ratio utilizations (Beaver 1967).

It was valid for all variables of application that the values of statistical significance were on the significance level, p = 0.01 with the exception of sales index with a significance level, p, of 0.05.

As such, it is warranted to assume that the selection of ratios based on concrete causes of default (capital flow disruptions) and the statistical significance of differences of input variable mean values between successful and bankrupt companies were the cause of varying classification accuracy.

If accuracies of the linear and quadratic discriminations are compared, it is obvious that models based on the quadratic discriminant function perform better than the linear ones. The classification accuracy increases if normality or at least multivariate normality of data distribution is attempted. The results of the logistic regression are given in Table 5 and attest to the fact that the classification accuracy cannot be influenced by adjusting the distribution of data. It is logical because logistic regression does not assume multivariate normality of input data distribution. Let us compare classification accuracies of the quadratic discrimination and logistic regression. If logistic regression does not assume multivariate normality of data distribution, we can compare classification accuracy of the quadratic discriminant function deduced from the files reduced by inner fencing with the accuracy of the logistic regression function calculated from the original files of successful and default businesses. The former function's discrimination ability was 100 %, concerning both successful and bankrupt firms. The latter's success rate was 96.77 %.

To verify better discrimination abilities of the quadratic function, the method of cross validation was applied (Refaeilzadeh et al. 2009). The data were divided into two parts, at which point, one part (training set) served the training purposes–model development, and the other part (testing set) provided for the validation itself. The cross-validation fundamental type is that of k-fold cross-validation. The data are divided into subsets, k, which have the same number of items. Then, k number of iterations is performed so that each validation utilizes various data subsets, and the resting subsets, k-1, are used for the model development. The results of individual iterations are average ascertained.

To predict accuracies of classification using quadratic discrimination and logistic regression, the method of 3-fold cross validation was used. As regards quadratic discrimination, the files of 76 successful and 74 bankrupt companies were divided into three parts (structuring of successful firms: 25 + 25 + 26; structuring of default firms: 25 + 25 + 24). As regards logistic regression, the files of 93 successful and 93 bankrupt firms were also divided into three parts, each of which consisted of 31 successful and 31 bankrupt firms.

The resulting average values of the three iterations are given in Table 6.

Tab. 6. Company classification success.

	Classification success [%]		
	Successful firms	**Default firms**	**Total**
Quadratic discrimination	96.10	98.67	97.33
Logistic regression	96.77	94.62	95.70

The application of the cross-validation method attests to better classification accuracies of models that have been based on the quadratic discriminant function.

The idea of the capital circulation disruption as a possible cause of bankruptcy, which has been addressed previously, can be qualified as belonging to a theory based on cash flow analyses. There is no universal concord with the idea of the cash flow–based indicators. Some authors consider them important in efforts of predicting failure of companies (Aziz and Lawson 1989; Gilbert et al. 1990); some acknowledge their importance as regards longer periods (Henebry 1996). The opposing views have been reviewed by Charitou et al. (2004). It is possible to assume that this might be simply caused by methodical inconsistencies of the cash flow indicators–see its definition by Aziz and Lawson (1989), Beaver (1967), Charitou et al. (2004). Nevertheless, our input variables do not imply cash flow indicators.

Conclusion

Various models and theoretical approaches have been applied in efforts of predicting failure of firms, and there is no agreement regarding which theory might be the best option. From the point of view of application frequency, the methods of discriminant analysis and logistic regression dominate. There is also no concord with the best choice of input variables.

This investigation has been based on the idea that default can be caused by a disrupted circulation of capital. From an economic point of view, the functioning of a company is based on the capital circulation. If this circulation is disrupted, the company tries to solve the problem, which finds its reflection in the financial statements issued–balance sheet and income statement. We have taken financial ratios implied in these statements to provide for input variables of statistical method applications. Apart from the financial ratios, financial indexes relating to two periods preceding the default (or the data search for non-failure firms) also were used. This could solve the problem of zero time implication deficiency of statistical models (Balcaen and Ooghe 2006). A set of 8 input variables was created from possible ratios and indexes that reflected disruptions of capital flows, with respect to statistical significance of input variable mean differences between successful and default businesses. The input set of variables for 186 firms was first subjected to discriminant analysis. The data were purified by elimination of outliers, and linear and quadratic discrimination was applied to them. The accuracy of classification of firms of known status was better as regards the application of the quadratic discrimination. The comparison of the classification results of the quadratic discrimination and logistic regression followed. Here again, the quadratic discrimination showed higher classification accuracies, which was also endorsed by the application of the cross validation method.

The input variables applied for our investigation are not the only ones that reflect disruptions of capital circulation. This field is open to further investigative effort.

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