Surveying of inaccessible rock faces and volume calculation of the irregular solids using robotic total station

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In the present-day geodetic practice, innovative technologies are increasingly gaining ground, but they also bring new possibilities for solving various geodetic tasks. It is mainly the ability to convey quick and precise surveying and recording of the spatial data which is advantageous of these technologies, making them more efficient in contrast with the traditional surveying methods.

Surveying of irregular bodies' surfaces in order to determine their volume can be performed on different levels of detail. This relates to the number of detailed survey points that capture the surface curve of the three-dimensional bodies, while also being connected with time requirements for their surveying.

Since the number of detailed survey points directly affects the precision of the volume determination, it is crucial – in order for optimizing the land survey – to evaluate the impact of the number of detailed survey points on volume changes when calculating the cubature of three-dimensional bodies. It is desirable to achieve such results where the selected number of detailed survey points ensures neglectable inaccuracies in comparison with the real values of the measured volume, together with significantly lower time requirements.

Key words: *Robotic total stations, terrestrial laser scanning, volume calculation*

Introduction

When surveying, the surveyor encounters the issue how to safely survey inaccessible rock faces, quarries and similar objects. There are several methodologies for surveying the surface profile of inaccessible rock faces that require proper selection of instrumentation and equipment. Individual methodologies differ in the principle, way of surveying, accuracy, level of the surveyed area approximation, method of data processing, and also varying requirements on surveying as well as computing devices. Mainly non-contact surveying systems – prismless and robotic total stations, photogrammetric cameras, terrestrial and aerial laser scanners find their application for hazardous conditions of a movement in such terrain (Pukanská et al., 2008).

This paper is focused on methods of surveying by using robotic total station equipped with a reflectorless distance meter (without use of surveying prism) and with a feature of scanning. Electronic total stations with a function of scanning are quite common instruments these days. Similarly as with electronic total stations without this function, the methodology is based on the principle of spatial polar method for determining the position of detailed survey points. The difference between these two types of instruments dwells in the method of spatial data collection (surveying of detailed points). Whilst in the first case detailed points are being surveyed and recorded manually by morphologic distribution, with this type of instrument detailed points are recorded automatically in a square or possibly a rectangular grid, depending on the setup of survey details. The advantage of this methodology is fully automated process of surveying, and the disadvantages are both acquisition costs of this class of instrumentation and more strict requirements on the computer software, whereas these instruments are able to collect several tens of thousands of spatial points in a relatively short time (Rákay, 2013).

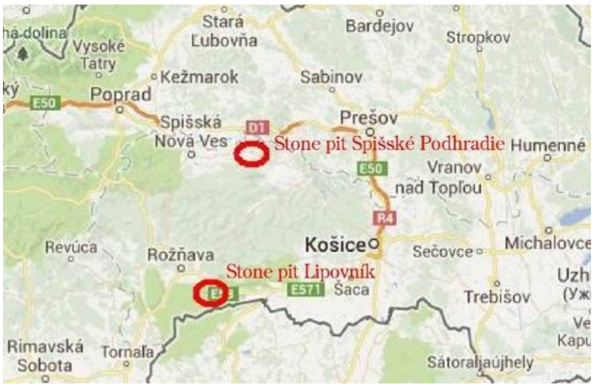

Fig. 1. Lipovník and Spišské Podhradie quarries. A map of broader relations.

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Surveying of areas of interest

Irregular surfaces of rock faces that are present in Lipovník and Spišské Podhradie quarries (Fig. 1) were selected as testing locations for surveying and subsequent analysis of calculations of volumes of irregular bodies.

Realisation of surveying in the Lipovník quarry

The rock wall of the first storey of the Lipovník quarry was the subject of surveying. The rock wall was surveyed using robotic total station Trimble VX Spatial Station from two survey stations (5001 and 5002), positions of which was determined by the RTK method based on the GNSS technology (using Leica GPS900CS). The accuracy of spatial position of the surveyed point (Single 3D point accuracy using Trimble VX Spatial Station) in the scanning mode is 10 mm, as declared by the manufacturer. The accuracy of coordinates determined by RTK method (RMS accuracy with real time kinematic – RTK using Leica GPS900CS) is 10 mm + 1ppm in a horizontal direction and 20 mm + 1ppm in vertical direction, as declared by the manufacturer. A geodetic connection of points of the oriented aligning base 5001 – 5002 was realized by Slovak spatial observation system – SK*POS* (in analogy to (Bartoš et al., 2011; Gašinec et al., 2012)). Positions of the instrument survey stations, as well as range of surveying from individual stations are illustrated in Fig. 2.

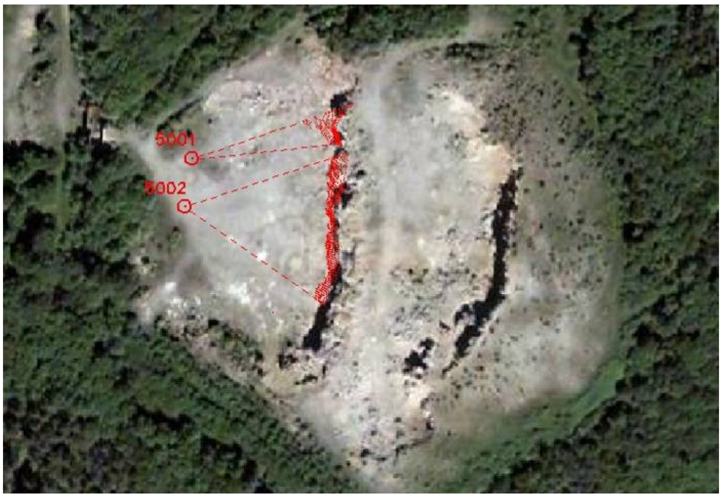

Fig. 1. Positions of the instrument survey stations at the Lipovník quarry.

The purpose of this survey was surveying of the quarry wall surface using a set of points arranged into a regular grid for determining the influence of density (number) of points on calculation of changes in the irregular body volume, and hence what is the impact of particularity when capturing irregular surface of the quarry wall on calculation of cubature of the body bounded by this surface.

Because of the influence of central projection there is a deformation of a square grid of points defined by the grid of reference plane in its projection to the surface of surveyed object, so the survey of quarry wall was realized from 2 survey stations (5001, 5002) in order to eliminate this effect as much as possible (Bartoš et al., 2011; Gašinec et al., 2012).

Subject of testing, the surface of the rock wall of quarry, was divided into six (6) parts (Fig. 3) of equal acreage (100 m^2). The quarry wall was surveyed using a square grid of points with a grid of 10x10 cm. The testing surface No.1 was located about 63m from the survey station No. 5001 and the average distance of surfaces No. 2 to No. 6 from the survey station No. 5002 was between 68 and 81 metres (Rákay, 2013).

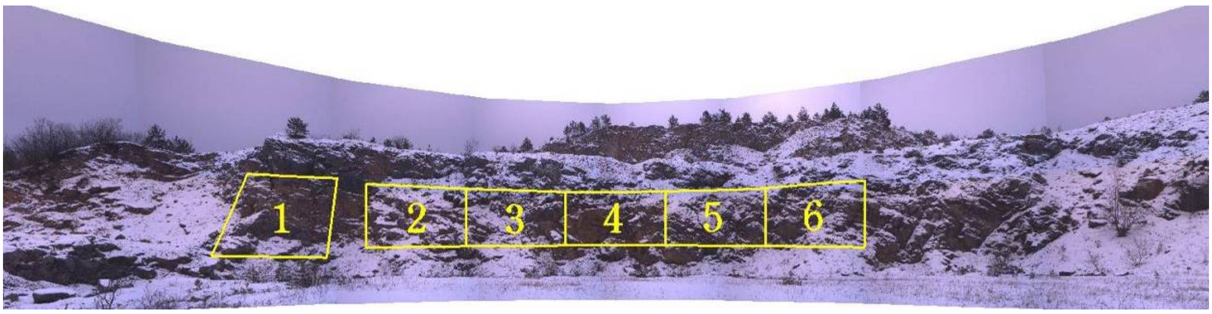

Fig. 2. Arrangement of tested surfaces No. 1 to No. 6 on the first level of the rock wall in the Lipovník quarry.

Realisation of surveying in the Spišské Podhradie quarry

The rock wall at locations where mining is still being performed was the subject of the survey. Irregular surface of the walls was surveyed similarly as in the Lipovník quarry, using robotic total station Trimble VX Spatial Station from two survey stations (5003 and 5004), positions of which was determined by the RTK method based on the GNSS technology (using Leica GPS900 CS). Positions of the instrument survey stations, as well as range of surveying from individual stations are illustrated in Fig. 4.

Fig. 3. Positions of the instrument survey stations at the Spišské Podhradie quarry.

Surfaces of the rock walls of the quarry that had been divided into four (4) parts of equal acreage (100 m^2) were the subject of the survey. The quarry wall was surveyed using a square grid of points with a grid of 10x10 cm. Parts No. 7 and No. 8 (Fig. 5) were located about 87m from the survey station No. 5003 and parts No. 9 and No.10 were surveyed from the survey station No. 5004 with the average distance of surveyed surface from the survey station of about 55 metres. Parts No. 9 and No. 10 were created with approximately 35 % overlap due to insufficient amount of survey data and in order to preserve the same surface area of all tested surfaces (100 m^2) (Rákay, 2013).

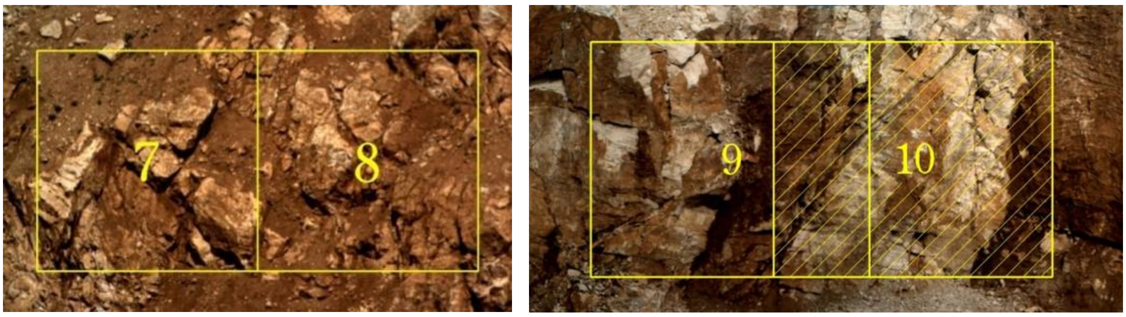

Fig. 4. Arrangement of tested surfaces No. 7 to No. 10 on the rock wall in the Spišské Podhradie quarry.

Processing of surveying results

Description of real objects or phenomena tends to be difficult because of their complexity, therefore models, i.e. simplified representation of reality, are formed, the purpose of which is to examine and explain the basic principles (Sabolová et al., 2012). To perfectly project a rough topographic surface it is necessary to survey a large number of detailed survey points. The Tab. 1 shows the number of points corresponding to a point grid with a grid size of 10x10 cm to 100x100 cm. The Fig. 6 shows that together with the requirement of increasing detail of an irregular surface survey (i.e. getting closer to represent its real shape) also the number of required detailed survey points exponentially increases.

Highly realistic terrain relief image is obtained by detailed surveying that though closely relates to higher demands of the spatial data collection along with higher requirements on the computer hardware for processing the survey results. In terms of optimization of the entire surveying process it is important to find such a level of surveying detail that would meet the required precision criteria at as small time losses as possible.

Sets of data with a grid size of 20x20 cm, 30x30 cm to 100x100 cm, which were subsequently processed in the RealWorks Survey software, were created by a selection of individual points for every single part of the quarry wall surface from a set of points with a grid size of 10x10 cm, within the survey of wall surfaces of the Lipovník and Spišské Podhradie quarries and their division into testing surfaces No. 1 to 10. Approximating surfaces represented by networks of triangles (Fig. 8) were created from sets of points of the square grid (Fig. 7).

Tab. 1 Numerical representation of dependence of the number of detailed survey points on the square grid size

Dimension of the points network grid mesh [cm]	**Number of detailed points in the area of 100 m²**
10x10	10 201
20x20	2 601
30x30	1 225
40x40	676
50x50	441
60x60	324
70x70	256
80x80	196
90x90	169
100x100	121

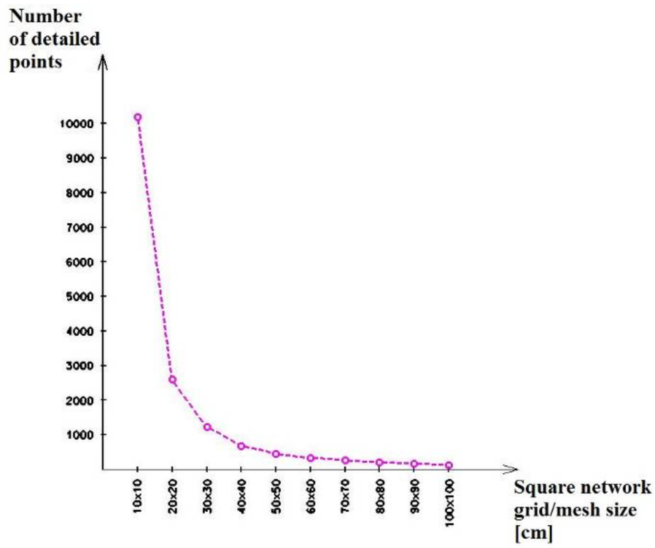

Fig. 5. Dependence of the number of detailed survey points on the square grid size.

Fig. 6. Set of points of the surface No.6 with a density of 10x10 cm and 100x100 cm [3].

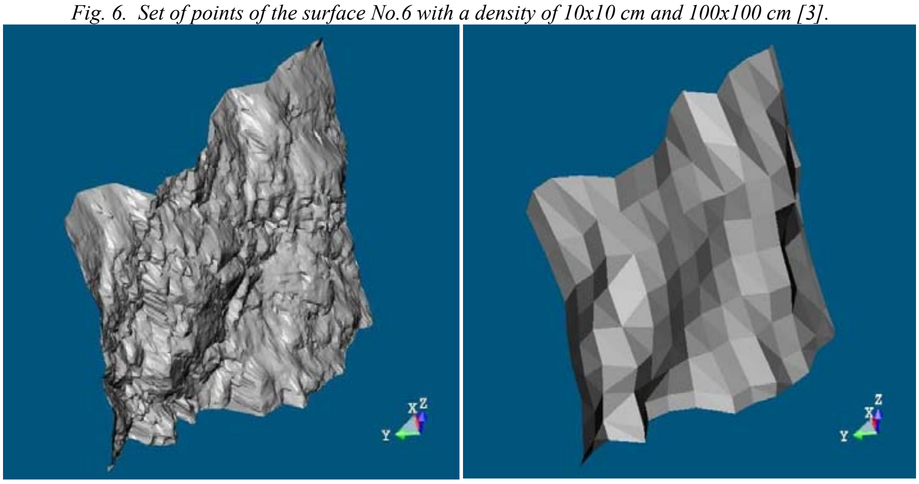

Fig. 7. Approximation of surface No.6 by a network of triangles with a density of points of 10x10 cm and 100x100 cm, respectively [3].

Calculation of the cubature

A level of approximation of the surface that bounds these bodies affects the determination of irregular bodies volume in a decisive extent. Whereas the very rock face surface does not present an irregular body, changes in the volume were obtained as difference in calculated volumes versus the reference plane located about 3 m behind individual surface approximations. In Fig. 9, the irregular surface is shown in red colour, the plane for a volume comparison is shown in green colour, and the projection is defined in pale green (almost transparent) colour. In this specific case, the volume was calculated between the red irregular surface and its orthogonal projection to the green (reference).

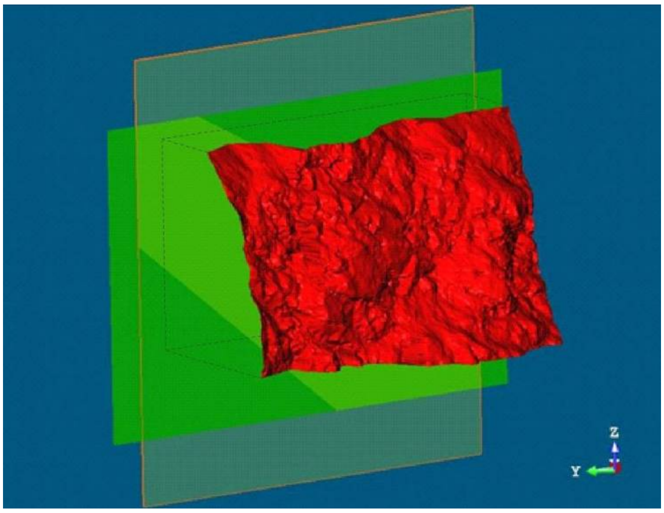

Fig. 8. Calculating the volume between an irregular surface and the reference plane. .

Results of the volume calculations and subsequent volume changes are presented in the following tables (Tab. 2 and Tab. 3) for surfaces No. 1 to No. 10.

Tab. 2. The values of volumes and volume changes calculated for solid bodies bounded by surfaces No. 1 to No. 5.

	Surface No. 1		Surface No. 2		Surface No. 3		Surface No. 4		Surface No. 5	
Network dimension [cm]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]
10x10	466.93	0.00	720.80	0.00	356.08	0.00	364.69	0.00	613.80	0.00
20x20	463.57	3.36	717.82	2.97	354.73	1.35	363.49	1.21	612.47	1.33
30x30	464.57	2.36	718.83	1.97	355.50	0.58	363.13	1.56	612.34	1.46
40x40	460.68	6.25	716.37	4.43	354.53	1.54	362.89	1.80	610.44	3.36
50x50	462.88	4.05	715.68	5.12	353.96	2.12	362.52	2.17	611.40	2.40
60x60	459.89	7.04	712.49	8.30	355.89	0.19	363.43	1.27	612.19	1.61
70x70	461.48	5.45	712.92	7.88	352.78	3.30	363.44	1.25	610.83	2.97
80x80	459.58	7.36	710.38	10.41	350.65	5.43	361.87	2.82	608.69	5.10
90x90	456.31	10.62	715.04	5.75	356.62	-0.54	360.05	4.64	610.36	3.44
100x100	458.64	8.29	706.69	14.10	353.16	2.92	360.02	4.68	608.88	4.91

Tab. 3. The values of volumes and volume changes calculated for solid bodies bounded by surfaces No. 6 to No. 10.

	Surface No. 6		Surface No. 7		Surface No. 8		Surface No. 9		Surface No. 10	
Network dimension [cm]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]	**Volume** [m³]	**Change in volume** [m³]
10x10	571.77	0.00	430.72	0.00	647.85	0.00	592.39	0.00	571.79	0.00
20x20	572.37	-0.59	430.51	0.21	646.11	1.74	595.49	-3.10	572.68	-0.88
30x30	573.75	-1.98	431.49	-0.77	648.29	-0.44	595.24	-2.84	572.09	-0.30
40x40	572.71	-0.94	431.11	-0.39	645.95	1.91	595.38	-2.98	573.10	-1.31
50x50	572.66	-0.88	432.34	-1.62	649.06	-1.20	595.32	-2.93	573.04	-1.25
60x60	573.58	-1.81	431.26	-0.54	646.80	1.05	595.64	-3.25	573.03	-1.24
70x70	573.21	-1.43	429.69	1.02	650.38	-2.52	595.96	-3.57	572.98	-1.18
80x80	571.71	0.06	428.20	2.52	646.64	1.21	594.34	-1.95	573.15	-1.36
90x90	570.69	1.09	428.71	2.00	649.29	-1.44	594.43	-2.04	572.70	-0.91
100x100	569.97	1.80	428.38	2.34	647.27	0.59	595.14	-2.75	573.77	-1.98

Graphical interpretation of results

Searching for and studying dependencies between two or more statistical characteristics (variables) is one of the basic tasks of mathematical statistics with wide use in technical and social sciences. A regression analysis deals with these problems. Its main task is to represent this dependence using a regression function, which subsequently allows to predict values of dependent variable based on values of one or more independent variables. Simple regression deals with perceiving the course of dependency of a dependent variable y (values of which are stated as y_i) on the independent variable x (values stated as x_i). The very first step in the process is to collect the data, i.e. pairs of corresponding values (x_i , y_i), where i = 0, 1, ..., n. The next step is to plot the points [x_i , y_i] (where i = 0, 1, …, n) into a rectangular system of coordinates. Thus we can obtain a set of points from which, in most cases, we are able to presume what type the curve that most properly reflects dependence of y over x would be.

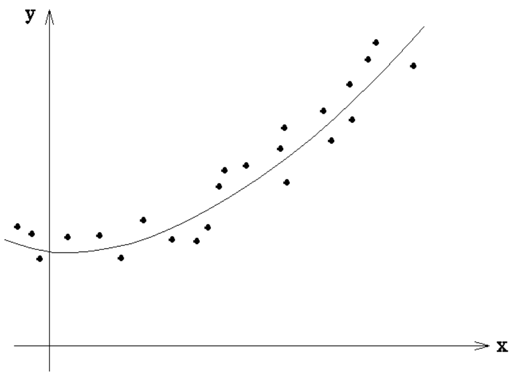

Fig. 9 Simple regression

The purpose of regression (equalisation) of argument y is to find such regression function $\tilde{y} = f(x)$, which would most appropriately expresses dependence of y on x. Graphically, it means to find such a regression line (curve, straight line), which will be placed most closely to the individual points [x_i , y_i].

Selection of an appropriate type of regression model based on a plotted dot diagram is not always a simple task. Several typical courses can be drawn up (polynomial types, power types, exponential types, hyperbolic types, logistic types...), and subsequently recalculated to which extent this found regression curve represent the dependence of y on x.

The dot diagram of obtained results shows the trend of direct dependency therefore it can be described by a polynomial of the 1st degree, i.e. a straight line:

$$\tilde{y} = a_0 + a_1 x \; . \tag{1}$$

As an optimal expressing of a theoretical relation $y = f(x)$ can be considered one that satisfies the minimum condition of objective function Φ, created on the principle of the least squares method (LSM). It is the search for the minimum of the sum of squares of deviations $v_i = y_i - \tilde{y}_i$.

$$\sum_{i=1}^{n} v_i^2 = \sum_{i=1}^{n} (y_i - \tilde{y}_i)^2 = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2 = \Phi(a_0, a_1) \cdot \tag{2}$$

Hence we get the so-called criterion function of $m+1$ unknown quantities. It is a non-decreasing and non-negative function, so its stationary point is its minimum. Therefore, first partial derivations according to all a_k (k = 0, 1, ..., m) are set equal to 0. Subsequently, we get a system of two equations that can be solved using a matrix algebra in such a way, that we firstly create matrices:

$$\underset{(n,1)}{X} = \begin{pmatrix} 1 & x_0 \\ . & . \\ . & . \\ . & . \\ 1 & x_n \end{pmatrix}, \quad \underset{(n,1)}{Y} = \begin{pmatrix} y_0 \\ . \\ . \\ . \\ y_n \end{pmatrix}, \quad \underset{(2,1)}{A} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}. \tag{3}$$

Afterwards, the system of normal equations can be defined as:

$$(X'.X).A = X'.Y \; . \tag{4}$$

Hence, the matrix of unknown parameters A can be defined:

$$\underset{(2,1)}{A} = \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = (\underset{(2,n)}{X'} . \underset{(n,2)}{X})^{-1} . \underset{(2,n)}{X'} . \underset{(n,1)}{Y} \tag{5}$$

and found values of a_0, a_1 are substituted into the equation (1) (Turčan, et al., 2002).

Evaluation of achieved results

The Fig. 11 graphically illustrates the dependence of the cubature change of volumetric elements created byorthogonal projections of tested surfaces No. 1 to 10 on the area of the reference plane placed about 3 m behind the individual surfaces. Vertical axis represents volume change in m^3, the horizontal axis represents the size of intervals of the square grid network of the distribution of detailed survey points on the rock wall surfaces, representing the course of irregular surfaces No. 1 to 10.

Using the regression function $\tilde{y} = 0{,}07023 + 0{,}10376\,x$ (calculated as the average of individual regression functions calculated separately for each surface - see Tab. 4), the Fig. 11 graphically illustrates average changes in the calculated volume of these types of irregular bodies depending on the level of detail of measurements. As it follows from Tab. 2 and Tab. 3, the volume element calculated at a density of distribution of detailed survey points in a grid of 10x10cm, whose value was considered as reference due to the largest number of detailed survey points, usually has the largest numerical value of the volume. Therefore, the numerical value of volume elements generally decreases gradually with increasing interval of square grid to 20x20 cm, 30x30 cm to 100x100 cm. As illustrated in the Fig. 11, it is clear that the average change in calculated volumes between scanning in a level of detail 10x10 cm and 100x100 cm is about 4 m^3 over the area of 100 m^2.

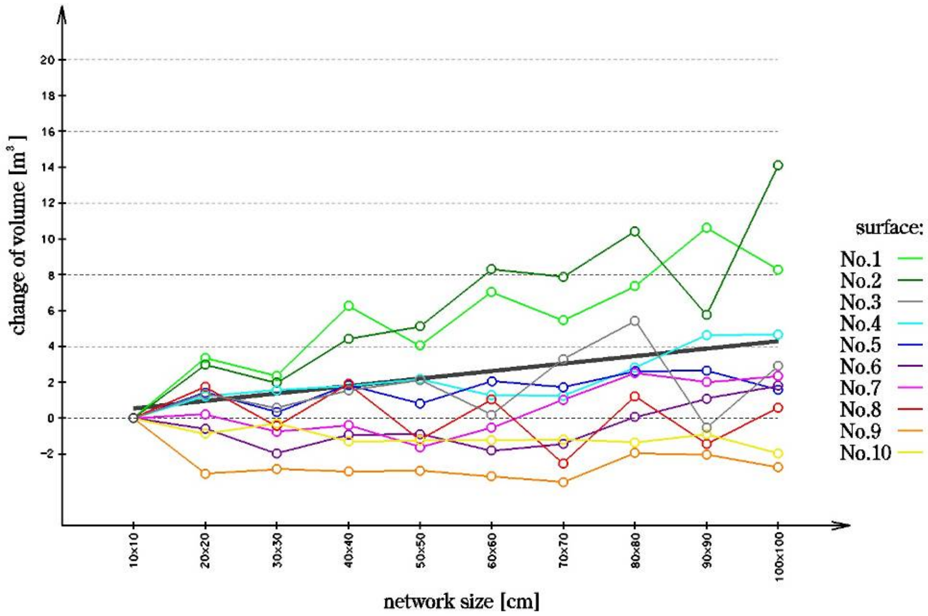

Fig. 10. The regression function y = 0,07023 + 0,10376.x reflecting an average dependence of the volume change on the size of the square grid network representing the surface of the rock wall surfaces.

Tab. 4. Coefficients of the regression function $y = a_0 + a_1x$ for the illustration of dependence of the volume change on the size of the square grid network representing the surface of irregular bodies.

Coefficients of regression function	Surface										
	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8	No.9	No.10	average
a_0	0,1457	-0,3226	0,1671	-0,0944	0,1662	-0,8299	0,6839	-0,3737	1,0218	0,1381	**0,0702**
a_1	0,4531	0,3063	0,0614	0,1058	0,0533	0,0540	-0,0838	0,0299	0,0226	0,0347	**0,1037**

Conclusion

Determination of volume of irregular bodies (objects) is a common task mainly in quarrying of industrial minerals, in mining geology and resources calculation, geological research, design and construction of various mining and engineering works and objects, mainly transportation and water-management objects, where volume of real or imaginary bodies (objects) bounded by regular or irregular areas has to be determined. The overall accuracy of volume determination is conditioned predominantly by surveying and processing work errors. The approximation error has the greatest influence on the accuracy of volume determination.

The accuracy of volume determination depends on the following main factors:

- quantity and accuracy of geodetically defined points representing the surface course of the irregular body,
- mathematical expression of the irregular body surface,

- a method of irregular body geometrization, i.e. its decomposition to a set of various types of elementary, geometric bodies, as well as the use of method for volume calculation.

In conclusion, the accuracy of volume determination of irregular bodies is dependent on the degree of exploitation of surveying, modelling and computational methods capable of approximating the irregular body and thus represent its actual, real shape. Of course, this assumes an optimal investment of energy and resources (also financial) into the achievement of results with required accuracy. In regard to the financial aspect, which is nowadays often one of the decisive factors, it is crucial to be well-informed about the characteristics and applicability of individual methods, as well as about their requirements for the quality of input data. By optimizing these factors (adequate density of the surveyed points, suitable selection of the modelling method and compliance with its marginal conditions), we are able to ensure that the acquired results will have a desired significance and the price paid for their acquisition is economically favourable.

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