Maximizing the net present value by applying an optimal cut-off grade for long-term planning of the copper open pits

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Cut-off grade optimization is one of the basic steps in the open pit planning and design and it is aimed to achieve maximum net present value (NPV). The basic algorithm for the cut-off grade optimization is Lane's algorithm, subsequently modified by different authors. Long-term planning of the surface mining operation involves consideration of all relevant parameters and factors (geology of ore

deposits, mining technology, mining and processing capacities, costs, metal prices, recoveries, etc.) to achieve the maximization of the value to be realized by the excavation and processing of mineral resources.

Using different methods and mathematical algorithms, which are implemented by the modern software tools, it is possible to manage costs and revenues, i.e. business economy.

The importance of cut-off grade optimization for long-term planning of exploitation at the copper open pits was illustrated using the case study on Kraku Bugaresku – Cementacija surface copper mine, that exploits two types of ore - sulphide and oxide.

The analysis results show that the selective mining of ore with a copper oxide grade ≤ 10 % in total copper and ore with a copper oxide > 10% in total copper achieve an increase in the NPV by 24.77 %, and cut-off grade optimization would achieve an additional increase in the NPV by 10.07 %.

Key words: open pit mining, long term planning, cut-off grade optimization, net present value maximization.

Introduction

Open pit mine design is a multistep process applied to determine a number of technical parameters and the design of a number of aspects of the operation.

In that process, cut-off grade is the most significant aspect, as it provides a basis for calculating the quantity of ore and waste in a given period.

The profit over time may be enhanced only by flow of high grade material to the processing plant.

This strategy supports the objective function and, depending upon the grade-tonnage distribution of the deposit, higher net present value (NPV) may be realized during earlier years to recover the initial investment [1], [2].

The long-term planning process is complex as solutions from one step affect all subsequent steps and the whole process becomes circular.

This is a large scale mathematical optimization problem, and therefore it is not surprising that the current software packages are not able to consolidate the problem in a single algorithm, despite the achieved level of development.

The reason for this problem has been simply paraphrased by Whittle: "the pit outline with the highest value cannot be determined until the block values are known. The block values are not known until the mining sequence is determined; and the mining sequence cannot be determined unless the pit outline is available" [3].

The most common approach to solving a problem is dividing it into sub-problems, i.e. stages of the process can be divided into sub-phases.

Figure 1 illustrates the various stages in an open pit mine design and their interactions.

Profitable exploitation of a mineral deposit requires considerable evaluation and planning. First, it must be determined what portion of the deposit is economical to mine (the mineable reserve) and which mining methods can be applied in the given circumstances.

The next step is to define the ultimate pit limit and mining schedule. Finally, the optimal cut-off grade is determined.

The goal of this effort is to determine the most profitable mining plan and the highest rate of return on invested funds. These operations are performed as part of a long-term planning.

In the last 40 years or more, there has been a wide range of applications of numerical methods in mining science. Today, with the application of geostatistics, 3D modeling, Lerchs - Grossmann's algorithm, Lane's algorithm and many other methods based on computer programs, it is possible to create better mining plans. The consequence of this is to achieve significantly better results in the exploitation of mineral deposits with a low grade of material in complex operating conditions [5], [6].

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Fig. 1. Steps in planning per circular analysis [4].

When planning the mining of copper ore, the final result depends on the perception of the whole technological process of copper obtaining as a final product. This procedure is carried out through three main processes, namely: the process of ore mining, the process of ore concentration and the process of metallurgical treatment (roasting of the ore and reduction of the obtained metal oxide are used for pyrometallurgical processes). The technological process of metallurgical treatment also includes the copper refining.

These processes are dependent on each other so that the output parameters of one process affect the parameters of the subsequent process.

Literature review

The optimum cut-off grade has been the subject of much research, as one of the most important parameters in the design of open pit mines because it directly affects the NPV. Kennet Lane is considered a pioneer in this field of mining science. The basis of Lane's work is the assumption that the future prices of mineral materials are deterministic. Lane in his seminal work (1964) presented an algorithm for cut-off optimization, which was later used as the basis by other authors in their further research [7]. Lane (1988) further develops his ideas, and, in solving the problem of cut-off grade optimization, takes into account the opportunity cost that arises as a result of the time delay of higher grade ore processing [8]. He explains that, if low quality material is processed today, it essentially means delaying the processing of any available higher quality ore to some later date. Since the longer these potential profits are deferred into the future the more they are discounted, then the act of processing marginal ore imposes an opportunity cost due to the time value of money. This opportunity cost can be minimized by operating at higher cut-off grades and processing higher quality material sooner. With this understanding, the cut-off grade optimization problem becomes a balancing act between reducing the opportunity cost against the wasting of the resource.

Cut-off grade optimization under deterministic price environments continues to be an area of active research. Yi et al. (1988) developed a method for solving the problem using Optimum Control Theory [9]. Whittle et al. (1995) proposed the use of opportunity costs in cut-off grade optimization in monometallic deposits by introducing two pseudo costs, called delay and change costs [10]. Cairns et al. (2003) presented a model to determine the impact of certain economic factors, such as net price and interest rate, on cut-off grade and grade distribution [11]. Minnitt (2004) applies Lane's theories on a simulated Witwatersrand-type gold deposit to illustrate the increase in the project's NPV that can be achieved [12]. Asad (2005) developed Lane's algorithm for cut-off grade optimization for an open pit mine with multiple metals and stockpiles [13]. Subsequently, he improved the algorithm by considering dynamic metal prices and cost escalation (2007) [14].

Another approach to solving the cut-off grade optimization problem is that the metal prices are viewed as stochastic, as opposed to Lane's approach where they are viewed as deterministic. One of the first works that took into account the price uncertainty in cut-off grade optimization was presented by Krautkraemer (1988) [15]. Cairns (1998) and Shinkuma (2000) investigated the problem of cut-off grade optimization in both cases: in the case of deterministic and stochastic prices. Although their work revealed some interesting insights, they fail to provide a method for determining the actual optimal cut-off grade strategy [16], [17]. Bascetin and Nieto (2007) modified Lane's algorithm by adding an optimization factor on the basis of the generalized reduced gradient algorithm, and presented a program to determine the maximum NPV [18]. Next, Osanloo, Gholamnejad and Karimi (2008) found environmental issues to be a crucial parameter in cut-off grade optimization. They improved Lane's algorithm on the basis of maximization of NPV at the same time minimizing the environmental

costs. They claimed that their algorithm is more effective in long-term production planning [19]. After this, Gholamnejad (2008) incorporated rehabilitation cost into cut-off grade optimization [20].

Methodology

Long-term production planning of an open pit mining operation is dependent upon several factors, and cutoff grade is the most significant of them. It provides a basis for the determination of the quantity of ore and waste in a given period. Cut-off grade is calculated by comparing the costs and benefits.

Cut-off grade is one of the most important technical and economic parameters which affect the economic efficiency and the social security in a mine enterprise.

Thereby, cut-off grade optimization is one of the fundamental processes for optimization and therefore a key parameter in the mining planning and design, production management and the foundation of the decision-making in mining investments.

The cut-off grade in optimization involves complex analysis and scientific approach, which is tightly related to economic management, applying the mathematical formulae and knowledge of software packages.

There are many approaches for the determination of cut-off grades, but most of the studies that are done in the last four decades show that the determination of cut-off grades with the objective of maximizing NPV is the most acceptable method, This method is also applied in this case study.

The objective function of the cut-off grade optimization model is to maximize the NPV of the operation subject to mining, processing and refining. For capacity constraints on mentioned processes this may be represented mathematically as follows:

$$MaxNPV = \sum_{n=1}^{N} \frac{P_n}{(1+\delta)^n}$$

$$Om_n \le M \forall n$$
(1)
(2)

$$Qm_n \le M \lor n \tag{2}$$

$$Qc_n \le C \forall n$$
 (3)

$$Qr_n \le R \forall n \tag{4}$$

where: n = period (year) indicator,

N = total life of operation (years),

P = profit (\$/year),

 δ = discount rate (%),

M = mining capacity (tons/year),

C = processing or milling capacity (tons/year),

R = refining capacity (tons /year),

Qm = quantity of material mined (tons/year),

$$Qc$$
 = quantity of ore processed (tons/year),,

Qr = quantity of concentrate refined (tons /year).

Taking into account all the costs and revenues in the mining, processing and smelting processing, profit is determined using the following equation:

$$P = \sum_{i=1}^{1} (p_i - r_i) \cdot Qr_i - m \cdot Qm - c \cdot Qc - f \cdot T$$
(5)

where: p = metal selling price (\$/ton of product)

r = refining cost (\$/ton of product)

m = mining cost (\$/ton of material mined)

c = processing or milling cost (\$/ton of ore)

f = administrative/ fixed cost (\$/year)

i = economic mineral indicator (copper, gold, silver, ...)

l = number of economic minerals in the ore.

The time T is defined by the limiting capacity of any of two stages of the mining operation, which introduces two cases depending upon the actual constraining capacity. Therefore, it changes as follows depending on whether a mine or mill is limiting the operation, respectively:

$$T = \frac{Q_m}{M} \tag{6}$$

$$T = \frac{Q_c}{C} \tag{7}$$

Depending upon the grade-tonnage distribution of the deposit, higher NPV may be realized during earlier years to recover the initial investment [21].

The policy of the optimal cut-off grade usually indicates a general decline in cut-off grade during exploitation time resulting in decrease of present value. One implication of this phenomenon is that the contents which is uneconomic for treatment in the early years can be cost-effectively treated in the later years of exploitation [22]. This is achieved by forming the stockpiles with low grade ore. The result of this is not only an increase the mine life, but also NPV.

The management of stockpiles of low grade ore is possible using the following two options [23], [24]:

- 1. The stockpile is utilized in parallel to the mining operation. This means that material is sent to the processing plant either from mine or stockpile. This decision is based on the overall economy/profitability of the operation.
- 2. The stockpile is utilized after the mine is exhausted. This simplifies the decision-making, since the stockpile acts as an additional portion of the deposit, where all available material is economical. However, high grade material in the stockpile is scheduled to be utilized earlier than the low grade material.

In this case study, the second case is chosen, whereby the stockpile is formed with an oxide ore, which has not been considered for further treatment.

The analysis conducted in this case study, in order to achieve the maximum NPV, was achieved by applying the following mathematical programming and optimization methods:

- 1. Methods of geostatistical modeling
- 2. Methods of long-term planning and optimization of an open pit
- 3. Methods of cut-off grade optimization.

The procedure was carried out through the following main steps:

- 1. The first step involved modeling, or development of a block model and determination of the quality of each block in the model using a database on the quality of the samples from drill holes. Interpolation is used to define the quality of all blocks. Modeling of deposits was done in the Gemcom Gems software.
- 2. After its creation, the block model was used for open pit optimization and schedule optimization. The process involved identifying combinations of controlled variables which maximize the value of the project in the context of a given set or range of assumptions. In this case study the open pit optimization was carried out in the Whittle, a software package that applies a modified Lerchs-Grossmann's algorithm in two steps. Software for generating nested pits and defining the pushbacks uses a pit parameterization technique (Revenue Factors), and a calculation is based on undiscounted cash values. In the next step, the simulation and DCF analysis were performed to obtain the highest NPV.
- 3. The next step was cut-off optimization, which was realized in software Whittle. The software uses a modification of the Lane's algorithm, such as it evaluates the whole NPV rather than the NPV of a single increment when optimizing the cut-offs for an increment. For each increment finds the cut-offs which maximizes the total NPV, including the pseudo costs. Thereby, the cut-offs for other increments remain constant. Iteration is carried out until NPV is not stabilized (Figure 2).



Fig. 2. Cut-off curve for simple increments (Whittle method).

According to the capacity which is limiting output, it can be noticed the three cases that lead to three optimum cut-off grades, which are called limiting economic cut-off grades [22]. These cases are:

- Mine limiting
- Mill limiting
- Refinery limiting

The capacities of the mining, processing, and refining stages limit the operation either independently or jointly. If a pair of stages is limiting the operation, then the output from each constraining stage must be balanced to utilize the maximum capacity of these stages.

In this case study, cut-off optimization was carried out in the case of mining and processing capacity limits. The mine and processing plant will be in balance when the quantity of ore per unit of material mined equals the ratio *C/M*. In that case, pseudo costs (delay and change costs) should be added to the processing costs. Pseudo costs behave as time costs [22]. Due to the economic conditions immutable change costs are zero. The delay cost increases the cut-off. As the resource is used up, the NPV of the remainder of the resource tends to fall, and is zero when no further resource remains. Since the delay costs are dependent on the remaining NPV, they too tend to fall. In general, therefore, optimized cut-offs start high and progressively decrease throughout the life of the project.

The basic equation for limiting economic cut-off grades is:

$$v = \sum_{i=1}^{l} (p_i - r_i) \cdot Qr_i - m \cdot Qm - c \cdot Qc - (f + F) \cdot T$$
(8)

The term

$$F = \delta \cdot V^* - \frac{dV^*}{dT} \tag{9}$$

represents an additional time cost, where V* is the optimum present value.

Substituting equations (6) and (7) into (8) yields the basic equations for limiting economic cut-off grades:

$$v_m = \sum_{i=1}^{l} \left(p_i - r_i \right) \cdot Qr_i - m \cdot Qm - c \cdot Qc - \left(f + F \right) \cdot \frac{Q_m}{M}$$
(10)

$$v_c = \sum_{i=1}^{l} (p_i - r_i) \cdot Qr_i - m \cdot Qm - c \cdot Qc - (f + F) \cdot \frac{Q_c}{C}$$
(11)

In expression (10) only the terms $(p_i - r_i) \cdot Qr_i$ and $c \cdot Qc$ vary with cut-off g. Thus, the mining limiting cut-off grade, becomes:

$$g_m = \frac{c}{(p-r) \cdot y} \tag{12}$$

When mill has a bottleneck and hence delays the operation, the opportunity cost is distributed per ton of ore processed. The cut-off grade is chosen such that in addition to processing and refining costs, it pays the opportunity cost of not receiving the future cash flows. Now, the optimum cut-off with the processing limiting g_c is given by

$$g_c = \frac{c + (f+F)/Qc}{(p-r) \cdot y}$$
(13)

where: $\binom{f+F}{Qc}$ is the opportunity cost distributed per ton of ore processed and y represents recovery or

the proportion of valuable product recovered from the mined material.

Therefore, the optimum cut-off grade that maximizes the objective function is the any value between g_m and g_c . This can be presented as [25]:

$$g_m \le \gamma \le g_c \tag{14}$$

In Figure 3 the left-hand curve shows the total cash flow from mining and processing if the mine is entirely mining limited. It has its maximum at a cut-off of g_m . The curve which shows the cash flow per year if the mine is entirely processor limited (the right-hand curve) peaks at g_c . Since both the mining and milling throughput limits must be honoured, it can be only operated at or below both curves, so that it is limited by the "maximum feasible" curve. In this case the cut-off which gives the highest point on this curve G_{mc} is also the point at which milling rate is at its limit.



Fig. 3. Total cash flow against cut-off for processor and mining limited operations.

The point of intersection corresponds to the balancing cut-off grade g_{mc} .

If G_{mc} is the optimum cut-off grade that meets the capacity constraints of mining and processing, then:

$$G_{mc} = g_m, \text{ if } g_{mc} \ge g_m$$

$$G_{mc} = g_c, \text{ if } g_{mc} \ge g_c$$

$$G_{mc} = g_{mc}, \text{ otherwise.}$$
(15)

Case study

The copper deposit Kraku Bugaresku - Cementacija is one of the copper deposits in Serbia, which is exploited within the company Mining and Smelting Complex Bor – Group (RTB Bor Group), one of the largest regional producers of copper and precious metals in Eastern Europe. The company conducts business as a corporation comprising several companies operating under central management (Figure 5).

This deposit belongs to a type of porphyritic deposits, which are characterized by the secondary sulphide enrichment.

Copper mineralization is located in the hydrothermally altered zone, approximately 2,000 m long with a maximum width of about 800 m (average 600 m), which dips to the east. The deposit is in the horizontal projection elongated in the direction of NW - SE. It is of irregular shape in the vertical projection.

Sulphide copper in deposit is presented with minerals chalcocite and covellite, and oxide copper with minerals as tenorit, cuprite, malachite and azurite.

Interpretation of deposit and surrounding area in the form of block model includes partition of the area, which covers the deposit, in the blocks of proper size. Taking into account all influential factors, adopted size blocks for the copper deposit Kraku Bugaresku - Cementacija is $15 \times 15 \times 15$ m. The block model covers an area of approximate length of 1,450 m and width of 650 m. A block model was formed with 60 levels, 160 columns and 256 rows. Each block in the block model is defined by the following values: rock types, density, content of basic component (Cu %) and associated components (Au, g/t; Ag, g/t), and economic value of the block, achieved by excavation of the block [26].

Figure 4 shows a block model of the deposit.

For mining of copper ore and waste rock deposits at the site Kraku Bugaresku - Cementing applied discontinuous technologies, with operations drilling and blasting, loading and transportation. The planned mining capacity is 5,500,000 tons of ore per year.

Management of the company has decided for high-capacity equipment for loading and transport of ore and waste rock as follows: excavators with bucket volume of 15 m³ and 22 m³ will be used for loading, and trucks with a capacity of 172 and 220 t will be used for transport. Excavated ore is transported to the primary crushing, and products are deposited in a closed stockpile. The next technological phase is the secondary and tertiary crushing; such crushed ore is sent to the next stage of processing - two stages of grinding, with rods and balls. The process of flotation processing at the site Kraku Bugaresku – Cementacija ends up getting the pulp, which, after concentration, is hydraulically transported to the Flotation Plant Veliki Krivelj. After the flotation process, the copper concentrate is sent to the Smelter and Refining, where the final product - copper cathode is obtained [27].



Fig. 4. Block model of a copper deposit Kraku Bugaresku – Cementacija.

Figure number 5 shows a schematic representation of the process of mining, processing and refining of ore at the site Kraku Bugaresku - Cementacija as part of the production process of copper in RTB Bor Group.

Problems accompanying the mining and processing of copper ore from the deposit Kraku Bugaresku - Cementacija originate from the increased grade of oxide copper in the deposit.

So far, there was no conducted technological research in terms of the negative impact the oxide copper in the process of flotation processing, which would determine the cut-off grade of oxide in which the optimal flotation recoveries would be achieved. Considering the above mentioned, the process of exploitation is carried out unselectively, without separating the ore with the increased content of copper oxide, so that the all excavated ore is sent to the process of flotation processing. The consequence of this is a very low copper recovery in the flotation processing, and thus the increased processing costs, or lower profit. Figure number 6 shows the recoveries in the flotation depending on the grade of oxide copper in total copper in the ore for the period covering May 2012 to December 2013 [28].



Fig. 5. Process of mining, processing and refining at the site Kraku Bugaresku - Cementacija within RTB Bor Group.



Fig. 6. The relationship between processing recovery and grade of copper oxide.

The relationship between the average copper grade of ore, copper grade in concentrate (head grade) and processing recovery is shown in Figure 7 [28].



Fig. 7. The relationship between copper head grade, copper grade in concentrate and mill recovery.

Since the problem is detected, the first step in its solution represents determining the maximum grade of oxide copper, which can be present in total copper in the ore, which is sent to the flotation processing. Due to this reason, the technological tests were performed that have shown that maximum permitted percentage participation of oxide copper in total copper, in which the flotation process takes place without negative consequences, with optimal recovery of metals from ore, is up to 10 % [28].

Figure 8 shows Kraku Bugaresku – Cementacija grade tonnage graph.

The next step, based on technological tests, is that two types of ore are separated in the block model of deposit: ore with the copper oxide grade ≤ 10 % in total copper and ore with the copper oxide grade ≥ 10 % in total copper. Tests have shown that the processing recovery of ore with a copper oxide grade ≤ 10 % in total copper is at a level of 80 %, which is considered as optimum recovery [28].

Finally, the changes in the technology of mining are predicted in terms of using a selective method for mining two types of ore.

Taking into account the above mentioned, in making a long-term plan for ore mining and processing, the following is planned: ore with the copper oxide grade $\leq 10\%$ in total copper will go directly to the processing or will be disposed of in a stockpile, from which it will later supply the processing during the life of the mine; ore with the copper oxide grade > 10% in total copper will be disposed in a stockpile and wont be returned in the processing. Its further treatment is not considered in this case study.



Fig. 8. Kraku Bugaresku - Cementacija grade tonnage graph.

In addition, the following analysis was performed for:

- 1. the existing process of ore mining and processing, without separation of ore containing copper oxide grade in total copper higher than 10 % (Scenario A) and
- the process of ore mining and processing containing copper oxide grade in total copper up to 10 % (Scenario B)

Analysis was performed using the software for economic analysis and strategic planning Whittle.

Table 1 shows the input techno-economic parameters used for the open pit optimization for Scenario A and Scenario B.

Parameter	Unit	Scenario A	Scenario B
Mining capacity	t/god	15,000,000	15,000,000
Processing or milling capacity	t/god	5,500,000	5,500,000
Copper selling price	\$/t	5,000	5,000
Gold selling price	\$/kg	35,000	35,000
Silver selling price	\$/kg	500	500
Mining cost	\$/t	1,5	1,5
Processing or milling cost	\$/t	6,0	6,0
Copper refining cost	\$/t	450	450
Gold refining cost	\$/kg	150	150
Silver refining cost	\$/kg	15	15
Copper recovery (processing and refining)	%	0,529	0,784
Gold recovery (processing and refining)	%	0,504	0,504
Silver recovery (processing and refining)	%	0,396	0,396
Discount rate	%	10	10

Tab. 1. Techno-economic parameters for the open pit optimization.

After open pit optimization, the next step is, according to the applied methodology, the cut-off grade optimization for the scenario that has a higher NPV. For this purpose, in this Case Study three stockpiles were designed: two for ore with the copper oxide grade ≤ 10 % in total copper and one for ore with the copper oxide grade ≥ 10 % in total copper and one for ore with the copper oxide grade ≤ 10 % in total copper oxide grade ≤ 10 % in total copper oxide grade ≤ 10 % in total copper with the content of 0.2 % to 0.25 % Cu, and from 0.15 % to 0.20 % Cu is being stockpiled on the stockpiles SP1 and SP2, respectively. The ore, which containing more than 10 % copper oxide in total copper, is being stockpiled on the stockpile SP3.

The capacity of these stockpiles is not limited, and the process of material returning in the production process is independent of the mining capacity. The rehandle costs amount to \$0.27 per tonne of material. Cut-off grade optimization was conducted using the software Whittle.

Results and discussion

The performed analysis shows that due to an increase in the processing recovery has been a significant increase in the amount of ore that can be exploited in an economically viable manner, resulting in the increase of NPV in Scenario B. This increase is 24.77 %.

Further steps are analyzing the possibilities for improving the NPV for Scenario B by cut-off grade optimization. The goal is to achieve higher profits increasing the cut-off grade in the early years above the value

of the marginal cut-off grade. This means that some parties of ores with lower copper content are discarded as waste or they can be disposed on stockpiles to return it later into the process, which is much better in terms of increase the NPV.

Table 2 shows the results of open pit optimization and NPV analysis for Scenario A and Scenario B and the results of cut-off grade optimization of the copper in ore with copper oxide grade ≤ 10 % in total copper for Scenario C.

Tab. 2. Scenario A, B and C – Results									
	SCEN	SCENARIO A SCENARIO B		SCENARIO C					
Period	Copper to process	Open pit cashflow	Copper to process	Open pit cashflow	Copper to process	Open pit cashflow			
[year]	[%]	[\$ disc]	[%]	[\$ disc]	[%]	[\$ disc]			
1	0.278	-6,857,551	0.243	-13,302,615	0.243	-13,302,615			
2	0.372	20,912,870	0.321	17,844,472	0.346	19,553,816			
3	0.300	11,640,120	0.332	22,861,295	0.350	24,663,926			
4	0.314	13,708,915	0.274	14,421,012	0.279	13,712,179			
5	0.311	15,488,022	0.265	11,499,553	0.304	15,650,136			
6	0.279	11,387,060	0.286	14,386,357	0.314	18,406,110			
7	0.244	6,880,224	0.303	15,752,074	0.296	16,335,742			
8	0.237	5,047,815	0.273	9,727,949	0.256	11,683,298			
9	0.253	6,828,434	0.245	6,860,688	0.237	8,154,479			
10	0.262	6,560,273	0.225	4,877,313	0.229	5,770,656			
11	0.241	6,339,081	0.219	4,866,233	0.241	6,017,591			
12	0.261	4,764,820	0.234	5,038,979	0.250	7,390,766			
13			0.248	6,328,040	0.237	6,592,143			
14			0.231	5,335,653	0.226	416,720			
15			0.274	1,647,656					
Total	0.280	102,700,083	0.265	128,144,659	0.307	141,044,947			

Figure 9 shows the effects of cut-off grade optimization.



Fig. 9. Effects of cut-off grade optimization

Figure 10 shows the flows of the average grade of copper, and Figure 11 shows the discounted cash flows for the mine life for Scenarios A, B and C.



Fig. 10. Flows of the average grade of copper for the mine life.



Fig. 11. Discounted cash flows for the mine life..

Cut-off grade optimization has achieved an increase in NPV of 10:07 %, so that Scenario C provides the best economic results, and the highest NPV and it is adopted as the basis of long-term planning of copper ore exploitation of in the deposit Kraku Bugaresku - Cementacija.

Conclusion

This case study presents a procedure that through cut-off grade optimization achieves the set goal of maximizing the NPV for long-term planning of the exploitation at the open pits of copper ore.

In the example of the copper deposit Kraku Bugaresku - Cementacija, which is characterized by the presence of sulphide and oxide copper in the ore, it is shown that in long-term planning the requirements and constraints must be taken into account that occur in the technological process of copper production as the final product.

This follows from the fact that the operations of mining, processing and smelting treatment constitute an integrated system. Thus, in order to achieve the planned refining recovery, the process requires that the concentrate is of appropriate quality, which in turn affects the level of processing recovery.

In the case of oxide copper presence, it is necessary to determine its allowed percentage participation in total copper in the ore (total copper consists of oxide copper and sulphide copper) which will not have a negative impact on the processing. The technological tests were performed that have shown that maximum permitted percentage participation of oxide copper in total copper is up to 10 %. On that basis, there are two types of ore are separated in the block model of deposit: ore with the copper oxide grade ≤ 10 % in total copper and ore with the copper oxide grade ≥ 10 % in total copper. The obtained Ultimate Pit Limit for Scenarios A and B shows an increase in NPV as much as 24.77 % in the Scenario B.

Maximizing the NPV was carried out through the process of optimizing the cut-off grade, which results an additional increase in NPV of 10:07 %, Scenario C.

The subject of future research will be the development of a model for cut-off grade optimization for longterm planning of open pits with two types of copper ore (oxide and sulphide ore), with consideration the effects of the application of the hydrometallurgical methods of preparation and concentration of ore with copper oxide content > 10 % in total copper on the economy of the mine.

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