A new heuristic non-linear approach for modeling the variable slope angles in open pit mine planning algorithms

Seyed-Omid Gilani[1] *and* ***Javad Sattarvand***[2]

In this paper, a new non-linear heuristic approach has been developed to model variable slope angles in the open pit optimization algorithms that are based on a simple inverse distance formulation. In the primary approaches, a fixed value of slope angle is considered, and it is usually based on special block configurations such as 1:5, 1:9 and 1:5:9 patterns that suffer from creating the higher or lower angles than desired. The cone template based methods that were employed later, considered variable slope angles and improved the solution quality a little but also suffer from modeling just in non-cardinal directions. Implementation of the proposed approach on a hypothetical block model showed its capability in generating accurate and smooth pit shapes using any number of regions with different mechanical behaviors.

Keywords: *open pit mine; production planning; variable slope angles; inverse angle formulation*

Introduction

The extensive application of computers in the field of planning and designing of the large open pit mines has engaged researchers to develop superior and comprehensible algorithms to solve the problems as fast as possible, requiring less computer and human resources. Finding optimal pit limits, designing push backs, determination of yearly production schedules and cut off-grade strategies are examples of these problems that directly affect the subsequent issues. The main objective of these algorithms is to maximize the total net present value (NPV) of the mining projects while satisfying the constraints related to the pit wall slopes as well as the technical and marketing limitations. However, most of the developed algorithms have so far encountered to the severe restrictions during the implementation of the large realistic block models due to the high amount of required computational resources. This makes developers apply a high level of simplification in order to be able at least to create a sub-optimal answer.

The most recognized algorithms in this field are the graph theory-based algorithms such as Lerch-Grossmann algorithm [1], network flow techniques [2, 3], dynamic programming [4, 5], parameterization techniques [6] and various versions of heuristic and metaheuristic algorithms such as moving cone [7], Korobov [8], modified Korobov [9], Genetic algorithm [10], simulated annealing [11], particle swarm optimization [12] and ant colony algorithms [13].

In almost all of the proposed procedures, the limitation of the pit slope angle is considered as one of the main constraints in the optimization model. It can be defined as the determination of the precedence of the mining blocks in order to build a stable pit wall that affects the size and the shape of the pit, layout of the access roads, waste, and low-grade ore dumps, stockpiles, processing plant, and other surface facilities. Therefore, the minable reserves, stripping ratio and the amount of ore and waste to be removed during the life of mine are highly dependent on the slope angles. The selection of an appropriate slope angle is a function of geological conditions of the site, rock mass failure mode, and slope governing boundary conditions [14]. Considering the different behavior of each part of the mine because of its different geotechnical properties such as rock strength, presence of faults, joints, water, and other geologic structures, it should be noted that using multiple and variable slope angles for each region of the mine and in different directions is undeniable (

Fig. 1). Nevertheless, a fixed value is used for slope angle in the most of the early developed open pit optimization approaches that normally governed by the block dimensions.

[1] *Seyed-Omid Gilani,* PhD student, Department of Mining Engineering, Sahand University of Technology, Tabriz, Iran, omidgilani@gmail.com

[2] *Javad Sattarvand,* Associate professor, Department of Mining Engineering, Sahand University of Technology, Tabriz, Iran, sattarvand@sut.ac.ir

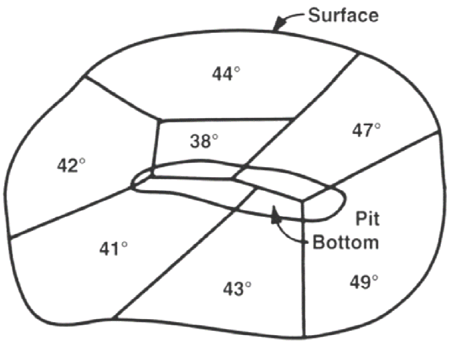
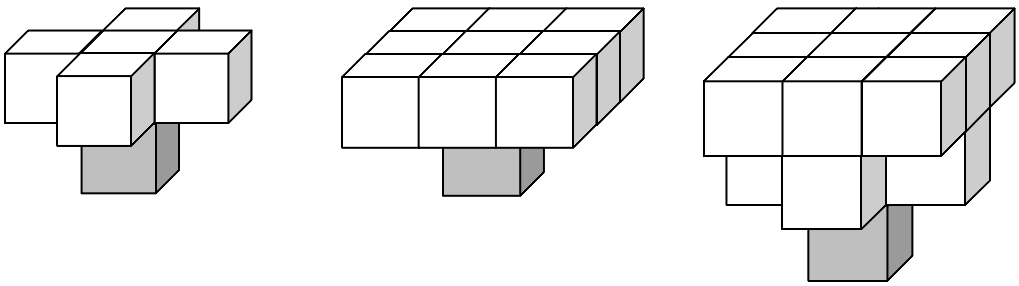


Fig. 1. Illustration of the pit slopes variation in the deposit

Primarily slope angle modeling routines were usually based on the special block configurations such as 1:5 or 1:9 patterns, i.e., any given block is considered accessible if the 5 or 9 blocks located above it have been removed before (Fig. 2a, b). These approaches were suffering from the creation of the higher or, the lower slope angles than desired. Later, a knight move pattern was proposed to estimate the conical extensions on the surface in order by using a 1:5:9 pattern that was able to create enhanced slopes (Fig. 2c) [15]. However, the major drawback of these routines was the dependency of created slope angles to the block dimensions. For example, in a cubic block model, the average slope angle would approximate 45° to 55° using the 1:5 pattern and the 1:9 pattern produces pits with angles between 35° and 45°, while a close approximation to 45° could be obtained by implementation 1:5:9 configuration.



a) 1:5 pattern *b) 1:9 pattern* *c) 1:5:9 pattern*

Fig. 2. Non-cone-based patterns.

The problem has been partly overcome by introducing the idea of cone template that consists of constructing a cone template (Fig. 3), putting its apex on a block and finally considering all of the blocks located inside the cone as the predecessors of the selected block. Chen attempted to apply variable slope angles concept on Lerchs and Grossmann algorithm using the concept of cone template [16]. Dowed utilized this concept in their proposed algorithm for production scheduling [9]. Whittle have frequently reported the incorporation of the variable slope angles by a linear interpolation [17]. The basic concept of incorporation of the variable slope angles into Lerchs - Grossmann algorithm has been addressed basically by Khalokakaie [18, 19]. They generated a smooth cone template assuming four different slope angles along the four principal geographical directions. They used an elliptic equation to interpolate the slope angle of cone side between any two consecutive directions (Fig. 4). However, in the real cases, we could not consider different slope angles just along four principal directions only. It also is unpractical method when the orientation of the block model does not match with the principal directions. Sattarvand et al. presented a spline interpolation based technique to define the cone template by outlining its horizontal sections in each level of block model as a set of closed spline curves, (Fig. 5) [20]. Although it was capable of considering an unlimited number of variable slope angles in different directions, however, difficulties in the modeling of cone templates with few directions engaged the authors to develop a new routine based on a simple inverse distance formulation.

The paper describes the basic theory of the newly non-linear proposed interpolation method and discloses the results of its implementation on a hypothetical block model.

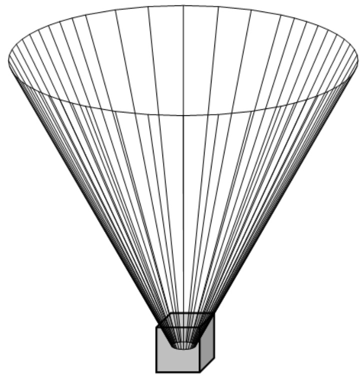


Fig. 3. Construction of cone from base block.

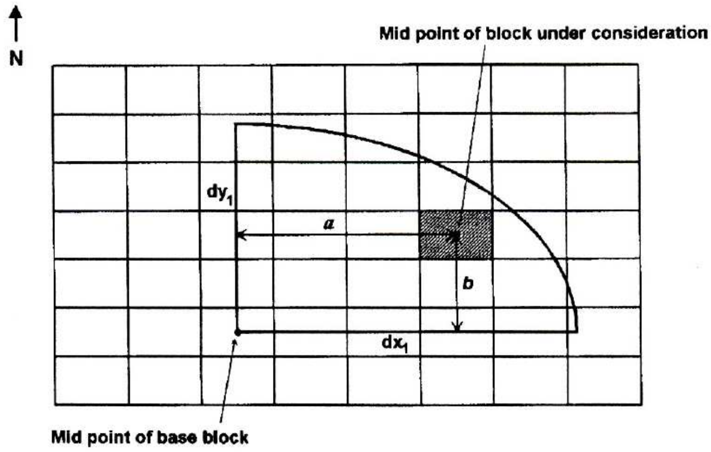


Fig. 4. Slope modeling using elliptical equation and slope angles along four principal directions.

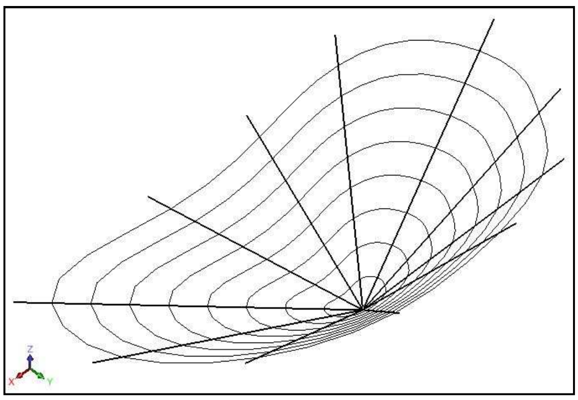


Fig. 5. Cone template generation by spline interpolation [20].

New inverse distance based method

The variable slope angle modeling in the open pit optimization algorithms consist of fitting some 2D closed curves over the intersection points of the slope lines and horizontal levels in different azimuths. As described, in prior procedures these curves were constructed in a linear manner that cannot meet the real open pit mining operations. The new proposed methodology utilizes the inverse distance law, and its capability to produce better interpolation has been proven in many engineering issues. According to the inverse distance law, any specified physical quantity or intensity is inversely proportional to the distance from the source of that physical quantity that can be written as follows:

$$Intensity \propto \frac{1}{distance^{m}}, \quad m \geq 1 \tag{1}$$

The modified equation used in our study was obtained easily by the substitution of distance quantity with an angle in the current. For a better understanding of the methodology, consider a simple block model in which its blocks are distinguished by the indices (i, j, k) representing their sequential number in x, y and z directions and the lowest southwestern block have been chosen as the origin of this block model (Fig. 6). The pseudo-code of cone template construction procedure with variable slope angles for a given set of azimuths and slope angle is sketched in Figure 7. Parameters involved in calculations are presented in Table 1.

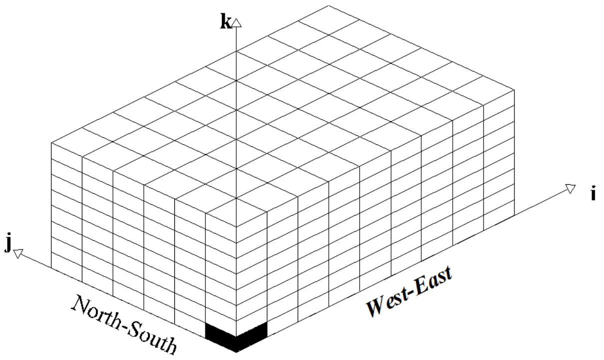


Fig. 6. Block model of deposit and coordinate system.

Procedure of Cone Template Construction
 input *azimuths and slope angles*;
 base block ← $block_{i,j,k}$;
 for $m \leftarrow 1$ **to** M **do**
  for $l \leftarrow 1$ **to** L **do**
   calculate $radius_{m,l}$ *using equ.(2)*;
  end for
 end for
 for $n \leftarrow 1$ **to** N **do**
  $\overrightarrow{EM}_n$ ← *horizental line between base block and* $block_n$;
  $azimuth_s \leftarrow azimuth_m$ *right after* EM_n;
  $azimuth_p \leftarrow azimuth_m$ *right before* EM_n;
  calculate $radius_n$ *using equ.(4)*;
  if $\left|\overrightarrow{EM}_n\right| \leq radius_n$
   $block_n$ *is inside block*;
  else
   $block_n$ *is outside block*;
  end if
 end for
end procedure

Fig. 7. The pseudo code of cone template construction procedure.

The consequence of the methodology that will be explained in detail later is a series of 2D closed curves at some levels above the cone apex. Sometimes it is easier to use a lookup table of passing points of these curves (with a certain accuracy) instead of defining its analytical formulation. For example, as illustrated in Figure 8, for a block model with block dimensions and considering four different slope angles in four azimuths, a series of

360 points could be generated instead of defining the curve formulation. The figure also shows the cone section and its corresponding internal blocks on the seventh level of the model.

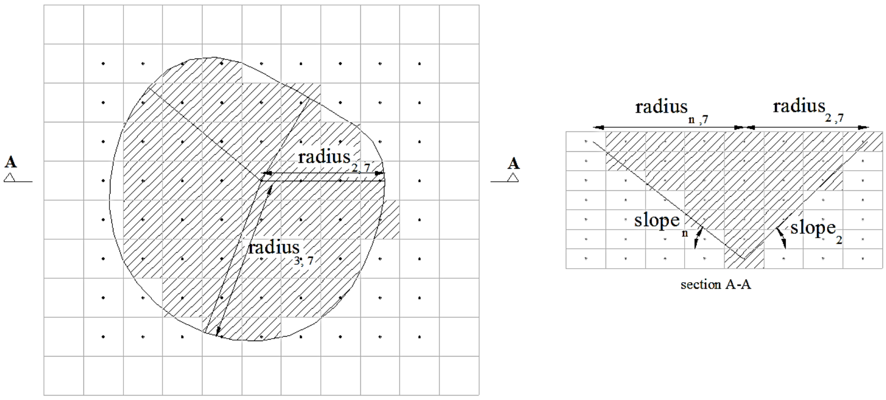


Fig. 8. Extraction cone of base blocks and all blocks within it.

Cone radiuses along M major azimuths in each level will be calculated using the simple trigonometric function according to their related slope angles and the vertical distance between the considering level and the apex of the cone. Considering a base block $X_{i,j,k}$ as the cone apex and assuming some input data, the needed parameters for construction of the cone that is illustrated in Figure 8 can be calculated as following.

$$radius_{m,l} = \frac{l \times dim_k}{\tan(slope_m)}, \quad for\ l = 1\ to\ L\ \ and\ \ m = 1\ to\ M \tag{2}$$

Tab. 1. Input data and the needed parameters.

	Parameters	description
Block definition:		
	dim_i , dim_j, dim_k	Block dimension in the *i*, *j* and *k* direction
	N	Number of blocks
Slope angles:		
	$azimuth_m$	Different azimuths based on number of regions
	$slope_m$	Slope angle at m^{th} azimuth $(1 \le m \le M)$
	$radius_{m,l}$	The radius of cone at l^{th} level and m^{th} azimuth $(1 \le l \le L)$
	M	Number of different azimuths
	L	Number of levels

The calculated $radius_{m,l}$ for L level above the base block and in M azimuths are assumed as the base radiuses for interpolating the radius of the cone in other azimuths based on inverse distance law. The interpolation process starts by considering M different slope angles on M azimuths that can be done in two ways. In a first way, we suppose 36 azimuth angles between 0 and 360 degrees that each of which has been located between two sequential predefined azimuths. Then, as illustrated in Figure 9, the slope angle of the cone in these azimuths, or in other words the radius of the cone along these azimuths in all levels, is interpolated between two precedent and subsequent values of $radius_{m,l}$ using the equation 3.

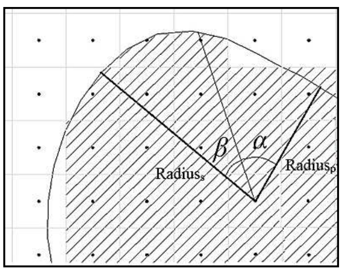


Fig. 9. Needed parameters to define a radius at any azimuth.

$$radius_n = \frac{\beta^d}{\beta^d + \alpha^d} \times radius_p + \frac{\alpha^d}{\beta^d + \alpha^d} \times radius_s, \quad n = 0^\circ \text{ to } 360^\circ \tag{3}$$

By replacing the α and β with the appropriate parameters, the interpolation equation becomes as following:

$$radius_n = \frac{(azimuth_s - azimuth_n)^d}{(azimuth_s - azimuth_n)^d + (azimuth_n - azimuth_p)^d} \times radius_p + \frac{(azimuth_n - azimuth_p)^d}{(azimuth_s - azimuth_n)^d + (azimuth_n - azimuth_p)^d} \times radius_s, \quad n = 0^\circ \text{ to } 360^\circ \tag{4}$$

Where:

$azimuth_n$, $azimuth_p$ and $azimuth_s$ represent the current, precedent, and subsequent azimuths, respectively;

$radius_n$, $radius_p$ and $radius_s$ are the cone radiuses along the current, precedent and subsequent azimuths, respectively;

d is the power of intensity. According to several implementation of the algorithm, it was found that assigning the value of d equal to 1.5 to 2 leads to more realistic results.

In a second way, the azimuth between the midpoints of any block and the base block at each level should be calculated using equations (7) to (10) and the radius of it easily calculated using the equation (11).

$$X_{i',j',k'} = dim_i \times (i' - i) \tag{5}$$

$$Y_{i',j',k'} = dim_j \times (j' - j) \tag{6}$$

$$azimuth_{i',j',k'} = 0, \quad \text{if } X_{i',j',k'} = 0 \text{ and } Y_{i',j',k'} \geq 0 \tag{7}$$

$$azimuth_{i',j',k'} = 180, \quad \text{if } X_{i',j',k'} = 0 \text{ and } Y_{i',j',k'} \leq 0 \tag{8}$$

$$azimuth_{i',j',k'} = 90 - \arctan(Y_{i',j',k'} / X_{i',j',k'}), \quad \text{if } X_{i',j',k'} > 0 \tag{9}$$

$$azimuth_{i',j',k'} = 270 - \arctan(Y_{i',j',k'} / X_{i',j',k'}), \quad \text{if } X_{i',j',k'} < 0 \tag{10}$$

$$R_{i',j',k'} = \sqrt{X_{i',j',k'}{}^2 + Y_{i',j',k'}{}^2} \tag{11}$$

Where:

$X_{i',j',k'}$ and $Y_{i',j',k'}$ represent the horizontal distances between the considering block (i', j', k') and the base block (i, j, k) along the *i* and the *j* axes;

$azimuth_{i',j',k'}$ and $R_{i',j',k'}$ are the azimuth and the length of horizontal line between the midpoints of two blocks (i', j', k') and (i, j, k).

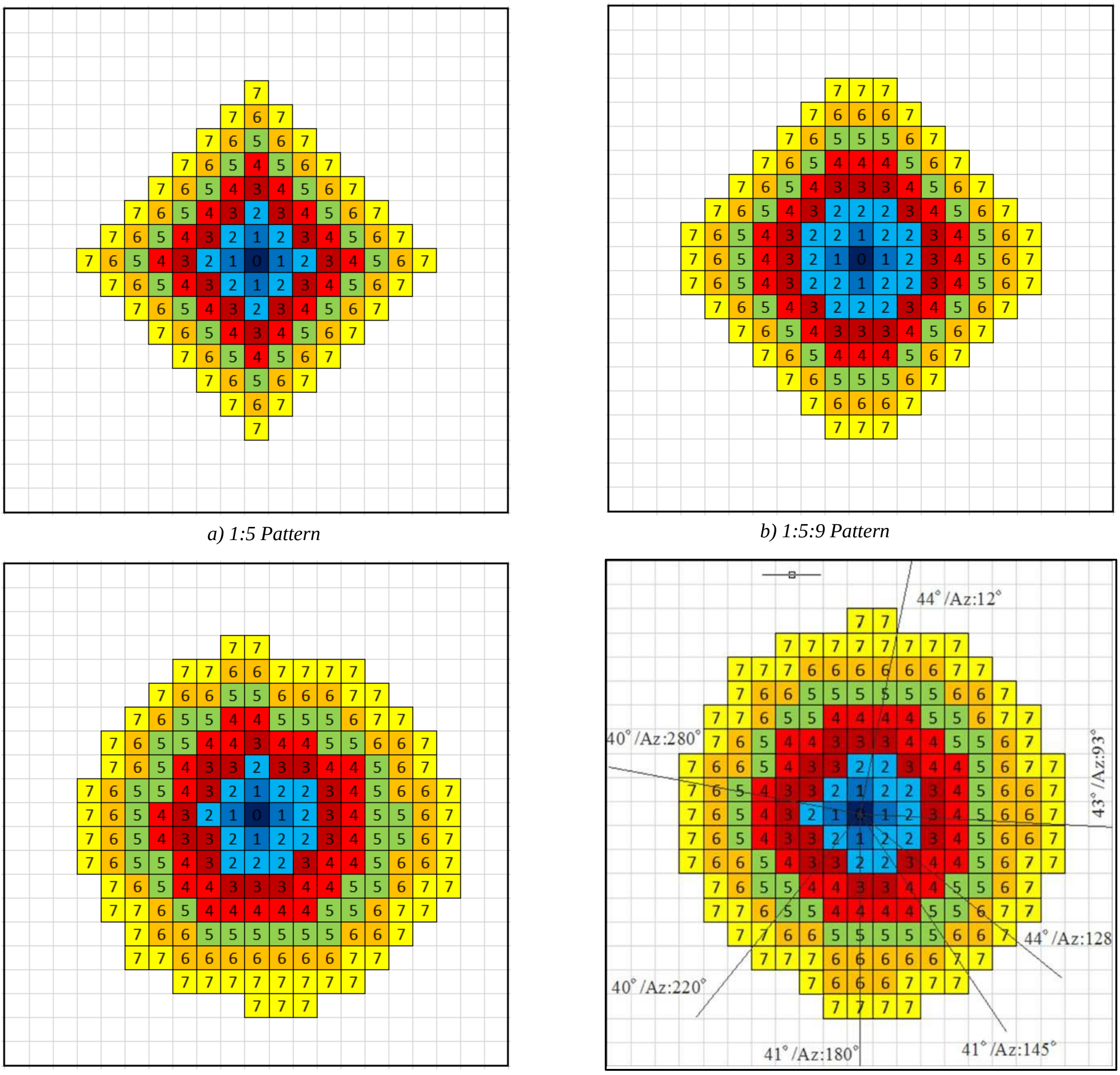
The next and the final step after determination of the boundary of the extraction cone template at each level is specifying the blocks located inside it, which is done by the comparison of the radius of the intended block ($R_{i',j',k'}$) and the cone radius ($radius_{azimut_{i',j',k'}}$). If $R_{i',j',k'}$ is less than or equal to $radius_{azimut_{i',j',k'}}$, it is concluded that the block (i', j', k') is within the extraction cone and must be removed from the base block (i, j, k). Otherwise, the block is considered as an outside block. Blocks that lie within the extraction cone could be submitted in any optimization algorithm.

Results and discussion

The developed algorithm has been applied to a hypothetical block model containing 29×29×9 blocks with a dimension of 10×10×10 m in which its main directions are parallel to the principal geographical directions. As shown in Tab. 2, a slope configuration with 7 variable slope angles has been considered based on the different geotechnical behavior of the ore body in some regions. A computer program is developed in C++ programming environment for implementation of the calculations. The algorithm starts to create the cone by considering the block (15, 15, 1) as the apex of the extraction cone, or in other words, the radiuses of the cone side at each level. Then, a comparison process has been done on all blocks up to 7 levels above the base block (cone apex) to determine whether they are within the extraction cone or not. The results at each level show the reliability of the method as well as the creation of a smooth and realistic pit wall. The constructed cone was compared with traditional 1:5 and 1:5:9 block configurations and also the spline based method, and their plan of view has been illustrated in Figure 10. Regarding being realistic and smoothness of the created pit walls by a different approach, the proposed approach leads to better results especially related to 1:5 and 1:5:9 configurations. As showed in Figure 10c), although the created pit walls by the spline approach have been improved when compared to the conventional approaches, it has some limitation in the modeling of cone templates with few directions (northeast and northwest in Figure 10c). A more detailed comparison between the new proposed approach and the other three approaches mentioned above is illustrated in Table 3 in order to show the number of blocks included in all 7 levels and the whole pit. The proposed approach led to an increase in the blocks included in the whole pit with 5.6%, 26.1%, and 65.7% compared to the spline based method, 1:5 and 1:5:9 configurations, respectively. As illustrated in Figure 11, the highest difference is at the 4th level and above.

Tab. 2. Input data set characteristics (different regions and their appropriate slope angles).

	Different region and directions						
	Reg.1	Reg.2	Reg.3	Reg.4	Reg.5	Reg.6	Reg.7
Azimuths	12	93	128	145	180	220	280
slope angle	44	43	44	41	41	40	40



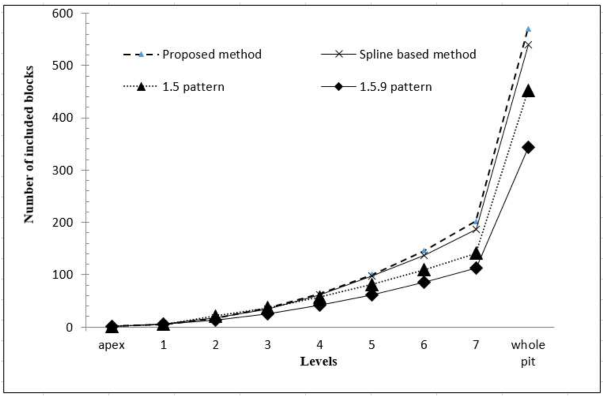
a) 1:5 Pattern *b) 1:5:9 Pattern*

c) Spline method *d) Constructed Cone*

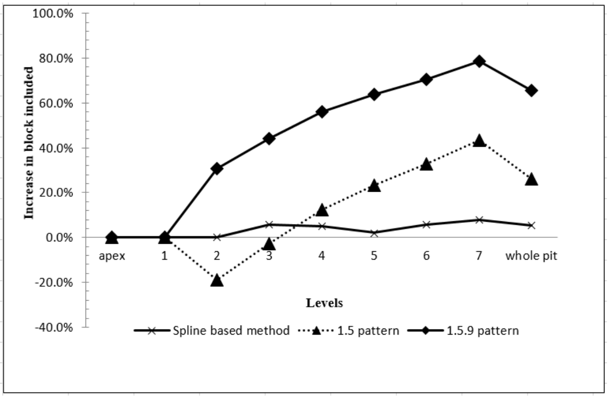
Fig. 10. Comparison between constructed cone and traditional block sequencing configuration.

Tab. 3. Comparison between proposed, spline-based and conventional approaches in term of included block numbers.

	Number of blocks in different methods				Block number difference of proposed methodology compared to that of different methods		
	Proposed method	**Spline method**	**1.5 pattern**	**1.5.9 pattern**	**Spline method**	**1.5 pattern**	**1.5.9 pattern**
Apex	1	1	1	1	0.0%	0.0%	0.0%
Level 1	5	5	5	5	0.0%	0.0%	0.0%
Level 2	17	17	21	13	0.0%	-19.0%	30.8%
Level 3	36	34	37	25	5.9%	-2.7%	44.0%
Level 4	64	61	56	41	4.9%	14.3%	56.1%
Level 5	100	98	80	61	2.0%	25.0%	63.9%
Level 6	145	137	108	85	5.8%	34.3%	70.6%
Level 7	202	187	140	113	8.0%	44.3%	78.8%
total	570	540	448	344	5.6%	27.2%	65.7%



a) Number of block in different approaches



b) Block number difference of proposed methodology compared to that of different methods

Fig. 11. Comparison of block numbers of created pits for proposed, spline-based and conventional approaches.

Conclusion

Difficulty in the incorporation of the variable slope angles is one the most common problems amongst open pit mine design algorithms. Early procedures were operating by the fixed slope angles and were heavily controlled by the dimensions of the blocks. Latterly developed approaches that were mostly based on the slope cone concept were still suffering from specific problems. Some were constrained to use slope angles in four major geographical directions. Some others led to a diamond formed cone shape instead of a smooth configuration. The other methodologies that were succeeded in smoothing of the cone outline would lead to unfamiliar conformation if only a few slope angles are defined or if the defined angles are concentrated in a particular direction. The presented algorithm is a new slope cone based approach that can generate suitable and reliable pits in almost all of open pit designing and scheduling procedures, especially in situations where the variable slope angles should be considered. It generates a list of precedent blocks for a given block that could be applied as a template for all blocks of the model. There is no limitation on the number of regions with different slope angles or their concentration in a particular azimuth. Considering that the generation of the extraction cone and finding the precedence list is running only one time during the whole implementation of the design algorithms. Since the time needed to determine the slope constraints list takes only a portion of a second, the effect of the proposed methodology on the runtime of the design algorithms will be negligible.

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