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The study of the elastic properties of carbonate rocks on a base of laboratory and field measurement

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The main aim of the paper was to determine the anisotropy coefficient on the basis of the laboratory measurement and also to calculate the dynamic elastic moduli using laboratory measurements and compared them with field measurements. Samples of carbonate rocks like dolomite and limestone for investigation were carried out from different quarries from the south part of Poland. The shape of samples is a cuboid with dimensions 0,1 x 0,05 x 0,05 m. The Pundit Lab+ equipment was used for tests. It measures the transmission time of an ultrasonic wave. P- and S- wave velocity are obtained for each sample. Seismic velocity values allowed for the calculation of the dynamic elastic moduli. The application of laboratory methods allowed obtaining information about the physical properties of rocks. This knowledge makes easier recognition in preliminary stages during engineering study.

Key words: *anisotropy, dynamic elastic moduli, seismic measurement, ultrasonic measurement.*

Introduction

Determination of the physical and mechanical properties of rocks is the most interesting problem which is still not definitely solved despite a lot of scientists who try to do it. It seems to be very important to make an assessment of the elastic properties of rocks which can be studied in laboratory using cylinder samples (Pinińska and Płatek 2002; Gueguen and Schubnel 2003; Mockovčiaková and Pandula 2003; Fener 2011; Hammam and Eliwa 2013; Tripetta et al. 2013; Najibi et al. 2015) and also directly in quarries using geophysical methods (Stan – Kłeczek and Idziak 2008; Vilhelm et al. 2010; Živor et al. 2011). For the simplicity, the elastic properties of rocks are specified on the basis of measurement in the laboratory. We can distinguish two groups of methods. One of them is the static testing where the mechanical properties of rocks are determined on the rock samples. The elastic properties are characterised by the static modulus, which specifies the relation between the applied stress and strain. The second group uses the acoustic waves. The advantage of this method is that it does not destroy the sample. The foundational basis of this method is the theory of the elasticity. By virtue of the velocity values of P- and S- waves, which reflect the properties of the elastic and isotropic media, it is possible to calculate the dynamic moduli such as Young’s modulus, shear modulus, axial modulus and Poisson’s ratio. These parameters characterising rock mechanical properties were estimated and used, for example, to predict the mechanical competency of the formation for hydrocarbon exploration (Eyinla and Oladunjoye 2014). This method which enables the determination of the dynamic modulus is relatively simple and suitable for application also in situ.

This paper presents two methods. First laboratory methods use acoustic waves and second is the seismic refraction method. The results of field measurements are compared with laboratory measurement.

The measurements of the elastic moduli were performed in four Triassic carbonate quarries located in the south part of Poland (Fig. 1). These are two limestone deposits (deposit 1 and deposit 2) and two dolomite deposits (deposit 3 and deposit 4).

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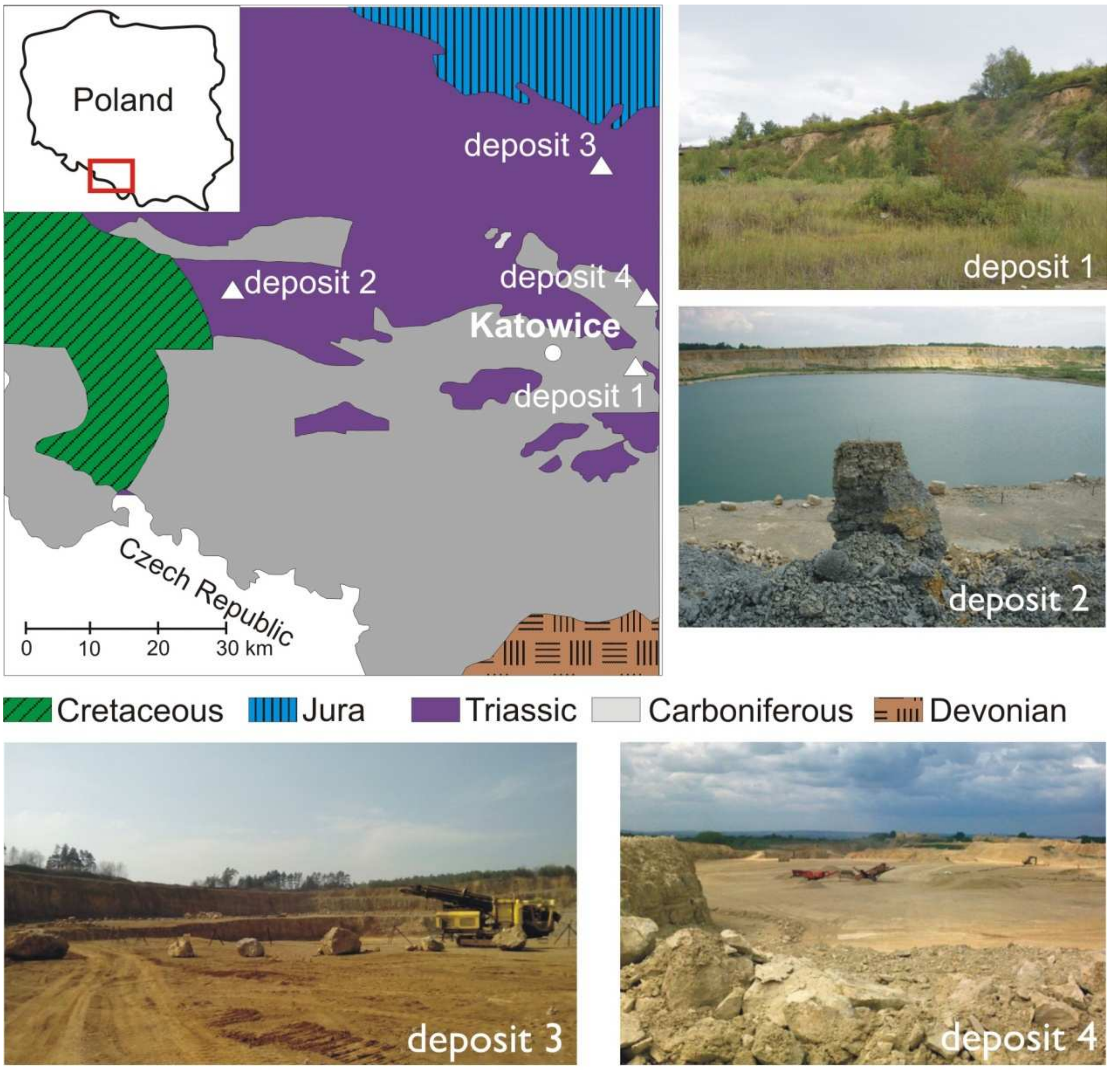

Fig. 1. Localisation of the study area (fot.I.Stan-Kłeczek).

Blocks of rocks for laboratory measurements were collected from each quarry. The seismic measurements were also made in each quarry.

Dynamic elastic moduli

The knowledge of longitudinal and shear waves velocities allow to calculate values of dynamic elastic moduli (Kearey and Brooks 1991; Burger 1992). Young's modulus (E) is the measure of the stiffness of a rock and can be defined in terms of bulk modulus (K) as

$$E = 3K(1-2\nu) \tag{1}$$

where ν is the Poisson's ratio and can be written in terms of P-wave velocity (v_p) and S-wave velocity (v_s) as:

$$\upsilon = \frac{v_P^2 - 2v_S^2}{2\left(v_P^2 - v_S^2\right)}, \tag{2}$$

and bulk modulus as:

$$K = \rho\left(v_P^2 - \tfrac{4}{3}v_S^2\right) \tag{3}$$

where ρ is a density of a rock. We can calculate Young's modulus as:

$$E = \frac{\rho v_S^2\left(3v_P^2 - 4v_S^2\right)}{v_P^2 - v_S^2}. \tag{4}$$

It is possible to calculate shear modulus also from the equation:

$$\mu = \rho v_S^2 \tag{5}$$

Laboratory measurement

The blocks of rocks were brought from each quarry. A cuboid with a length of 0,1 m and section 0,05 m x 0,05 m was cut from the block. Ultrasonic Instrument Pundit Lab+ Swiss company Proceq located in the laboratory of the Institute of Geonics, Academy of Sciences of the Czech Republic in Ostrava is applied to study. The ultrasonic impulses are repeatedly transmitted to the sample and then recorded at the time (t) of the ultrasonic wave passing through the sample. The rock specimens were mounted between the transmitter and receiver transducer holders. The length of the sample is a track (s). It is possible to calculate the velocity (v) using the formula v = s / t. In order to ensure close contact between the surface of the sample and transducers, a special paste was used. The first step of measurement is zeroed using calibration rod on a regular basis and particularly if the transducer frequency is changed. The expected calibration value is marked on the calibration rod. When the instrument is calibrated, the measurement can start by strongly holding transducers and the sample so that the contact is the best. The frequency of elastic signals used for transmission measurement is 250 kHz. The pulse length is 2µs. The accuracy of time of arrival of P- and S-waves is 1 µs. The survey contained about 8,000 measurements of the transit time of an acoustic wave through the sample. The research was performed using each time 3 samples from a 1, 3, 4 deposit and one sample from the deposit 2. The density was determined using the method of weighing samples of regular shapes and furthermore, it was compared with geological documentations of deposits. Both methods gave similar values of density: deposit 1 – 2,66 kg/m^3, deposit 2 – 2,68 kg/m^3, deposit 3 – 2,64 kg/m^3 and deposit 4 – 2,83 kg/m^3.

The anisotropy coefficient k was calculated from the equation:

$$k = \frac{v_{p\max} - v_{p\min}}{v_{p\,mean}} \cdot 100\ \% \tag{6}$$

where v_p mean is the mean velocity, in this particular case determined as the arithmetic average of the velocities corresponding to all measured directions. The values of the maximum and minimum velocities, coefficients of anisotropy and dynamic elastic modulus are summarised in Table 1. Figure 2 presents the configuration of axes which is the same in all samples.

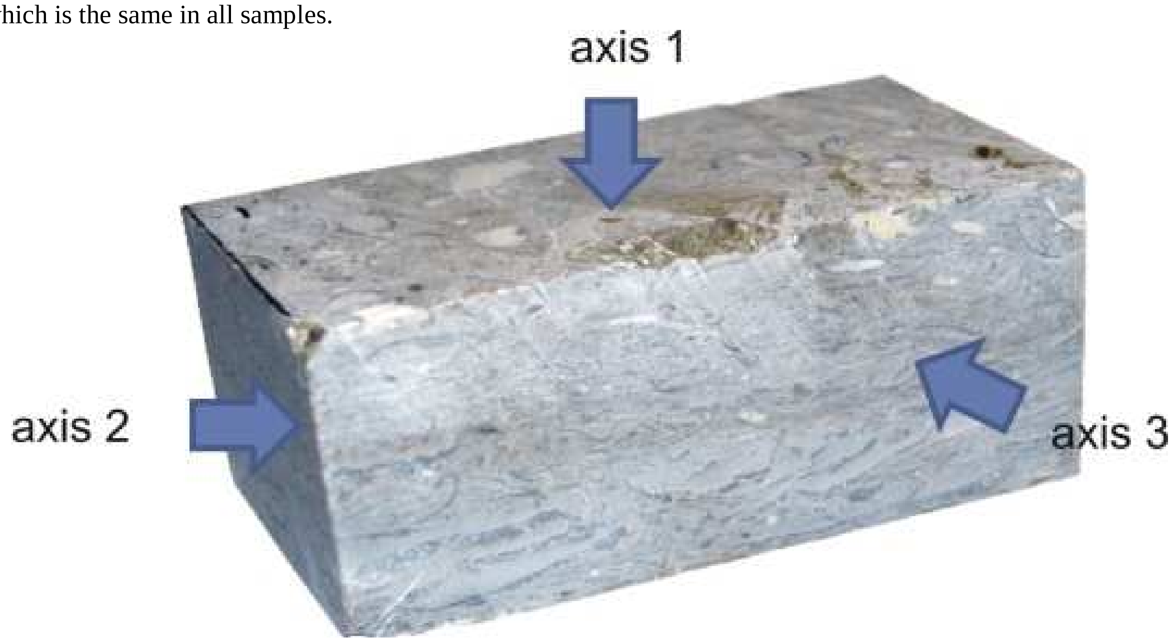

Fig. 2. The orientation of axes (sample from deposit 2).

Tab. 1. Experimental and calculated results from laboratory measurements.

Deposit	**Sample**	**Axis**	$\mathbf{v_P}$ [m/s]	$\mathbf{v_S}$ [m/s]	$\mathbf{v_P/v_S}$	**k** [%]	**Poisson's ratio**	**Dynamic elastic modulus**		
								E [GPa]	**μ** [GPa]	**K** [GPa]
deposit 1	1	1	6840	3495	1,96	11	0,32	84	31,75	79,2
		2	6380	3220	1,98		0,33	71,7	26,97	70
		3	6120	3130	1,96		0,32	67,5	25,5	63,3
	2	1	6060	3310	1,83	3,5	0,29	73,45	28,53	57,4
		2	6280	3200	1,96		0,32	70,56	26,6	67,02
		3	6280	3170	1,98		0,33	69,5	26,2	67,7
	3	1	6010	3040	1,98	12,5	0,33	63,89	24,05	70
		2	6810	3470	1,96		0,32	83,07	31,36	78,9
		3	6300	3180	1,98		0,33	70	26,2	68
deposit 2	1	1	5920	3900	1,52	10	0,12	88,2	39,5	38,3
		2	5400	2960	1,82		0,29	58,6	22,8	45,5
		3	5980	3140	1,90		0,31	67,2	25,6	58,8
deposit 3	1	1	4370	2770	1,58	9	0,16	47,2	20,29	23,3
		2	4740	2680	1,77		0,27	47,9	18,95	34
		3	4780	2690	1,78		0,27	48,5	19,1	35
	2	1	4790	2710	1,77	4	0,27	48,96	19,3	34,8
		2	4990	2890	1,73		0,25	55	22	36,3
		3	4850	2890	1,68		0,22	54	22	32,7
	3	1	4920	2845	1,73	5	0,25	53,3	21,4	35,3
		2	4710	2950	1,60		0,18	54	22,9	28
		3	4970	2900	1,71		0,24	55,2	22,2	35,7
deposit 4	1	1	5160	3040	1,70	3,5	0,24	64,4	26,1	40,6
		2	5260	2920	1,80		0,28	61,6	24,1	46,3
		3	5345	2960	1,81		0,28	63,6	24,9	47,7
	2	1	4770	3100	1,54	15	0,13	61,7	27,2	28,05
		2	5550	3160	1,76		0,26	71,3	28,3	50
		3	5120	3085	1,66		0,21	65,4	26,9	38,2
	3	1	5400	3100	1,74	10	0,25	68	27,1	46,2
		2	4930	2950	1,67		0,22	60,1	24,6	36,1
		3	5480	2970	1,85		0,29	64,4	24,9	51,7

Discussion

The samples of beige limestone from the deposit 1 are characterised by cracks filled with calcite. In the sample, 1 crack is not visible on the surface, but there are numerous small cracks in length in the range of from 1-2 mm to 20 mm. The velocity values and anisotropy coefficient k=11 % indicate on the anisotropy. In sample 2, two cracks perpendicular to axis 1 and 2 are observed. These cracks cause that velocities in this sample have similar values for all axes and the coefficient k = 3,5 % does not indicate the presence of the anisotropy. In sample 3, we can also see cracks with a diameter of 2 mm and filled with calcite, air or clay material. The values of the velocity and the coefficient *k* show a distinct anisotropy. The highest value of velocity 6800 m/s occurs in a plane parallel to the cracks.

The grey limestone sample coming from the deposit 2 contains the inclusions of calcite with the spherical or hemispherical shape. The packing of inclusions parallel to axis 1 and 3 causes the occurrence of the anisotropy.

The obtained values of velocities for deposit 1 are larger than of the deposit 2 what may be due to the effect of unstressing or the difference between the physical properties of the rocks.

The structure of dolomite samples from the deposit 3 has more homogeneous character. Hence, the coefficient *k* has a smaller value. The samples are dark yellow to grey colour. In the samples 1,2 and 3, there are open cracks of 0,1 mm filled with clay material. These cracks are perpendicular to the axis 1.

The samples from the deposit 4 have a light yellow colour. In sample 1, there are not visible cracks, which is reflected in the measured values of velocity, which have a very similar value about 5200 m/s and the anisotropy coefficient *k* is equal 3,5 %. In sample 2, there is a calcite-filled crack with an opening 0,2 mm. This crack is parallel to axis 2, and it is visible in the velocity value equal 5550 m/s. The anisotropy coefficient is equal 15 %. In sample 3, the situation is similar, and the crack is parallel to axis 1. In this sample, there are very small cavities with diameters in the range from 0,8 mm to 6 mm filled with calcite or air.

The average ratio of v_P/v_S for limestone (deposit 1 and 2) is 1,9 (standard deviation is equal 0,13) and for dolomite (deposit 3 and 4) is 1,7 (s.d. = 0,08). Pickett (1963) established v_P/v_S values for core measurement of 1,9 for limestone and 1,8 for dolomite.

Seismic refraction measurement was done directly in the quarries to eliminate the influence of overlayer. The first profile was oriented in N-S direction, and the orientation of next profiles was changed to 10°. Such arrangement of profiles allows investigating the anisotropy of seismic wave velocity. The 24-channel Italian seismographs P.A.S.I. and 12-channel ABEM's Terraloc Mk6 were used for seismic waves recording. The number of geophones (12 or 24) depends on the space available in a quarry. On the basis of the measurements, it was found that 12 geophones are enough for the measurement of seismic wave velocity anisotropy. The space between geophones was 3-meter for deposit 1, 2 and 4 so the profiles were 33-meter long (12 geophones x 3 m) and 2-meter for deposit 3 so the profiles were 46-meter long (24 geophones x 2 m). Seismic waves were generated by an eight-kilogram hammer and a metal plate. The seismic sources were localised at the beginning and the end of profiles and with offset 2 or 3 m. The first break times of P-waves and S-waves were read from recorded seismograms. Wave velocities were calculated from a slope of linear refraction travel time-offset relation obtained by least-squares fitting to experimental data. On the base of P- and S-wave velocities, the dynamic elastic modulus was calculated (Tab. 2).

Tab. 2. Dynamic elastic modulus from seismic measurements.

	$v_{p\,max}$ [m/s]	$v_{s\,max}$ [m/s]	**k** [%]	v_{pmax}/v_{smax}	**Poisson's ratio**	**Dynamic elastic modulus**		
						E [GPa]	**μ** [GPa]	**K** [GPa]
deposit 1	3850	2325	62	1,65	0,21	34,08	14,06	19,71
deposit 2	3570	1450	43	2,46	0,40	15,31	5,46	25,88
deposit 3	3800	1100	29	3,45	0,45	9,29	3,19	33,86
deposit 4	4000	1725	67	2,32	0,39	8,17	2,83	34,06

The dynamic elastic moduli presented in Table 2 was calculated for maximum P-wave velocity and corresponding S- wave velocity. The dynamic elastic moduli calculated for each azimuth can be found in Stan-Kłeczek and Idziak (2008, 2013).

The Poisson's ratio values shown in Figure 3 indicate that the limestone have higher mean values of Poisson's ratio equal to 0,3 (s.d.=0,06) than dolomite 0,24 (s.d.=0,04).

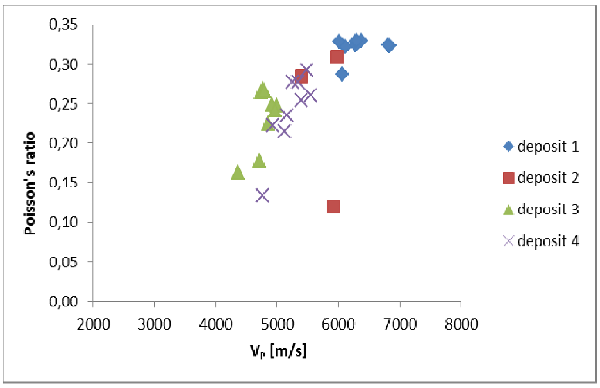

Fig. 3. Values of Poisson's ratio calculated from laboratory measurements.

The obtained data allowed to create diagrams of relations between the dynamic elastic moduli and P-wave velocity from laboratory measurements. The relation between Poison's ratio and P-wave velocity of carbonate rocks can be expressed as a logarithmic function of the equation y=0,32ln(x) – 2,5 (Fig. 4). The comparison of field and laboratory measurements distinguishes a significant difference. Values of Poisson's ratio from field measurement have much higher values than the values obtained in the laboratory conditions.

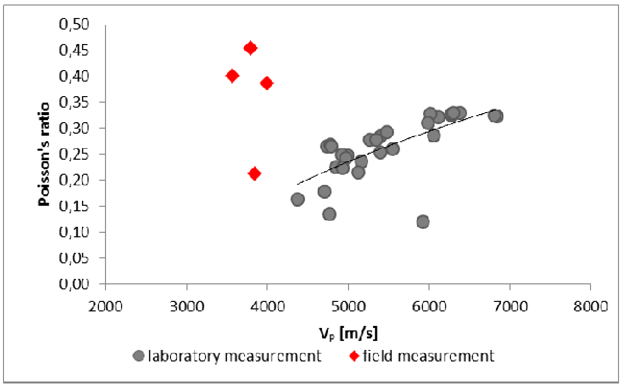

Fig. 4. The relation between Poisson's ratio and P-wave velocity (R^2 = 0,7).

A similar situation can be observed for Young's modulus E (Fig. 5). The values of Young's modulus from laboratory measurements were correlated using a logarithmic function of the equation y=74ln(x)-572. The values of modulus from field measurements have smaller values than from laboratory measurements.

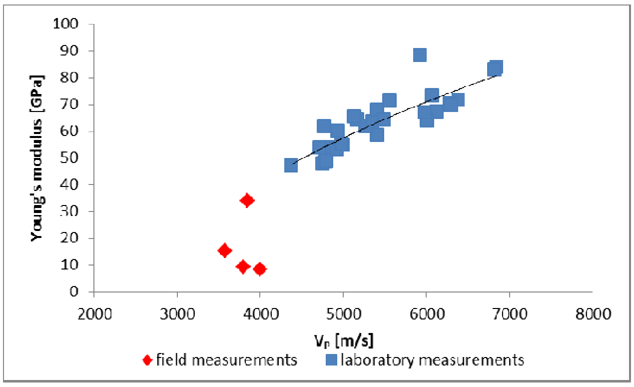

Fig. 5. The relation between Young's modulus and P-wave velocity (R^2 = 0,74).

In the case of shear modulus μ presented in Figure 6, the correlation between values of carbonate rocks is not good (R^2=0,4). Values of modulus are more dispersed, although the modulus reaches values from 18 to 30 GPa and the average value reaches 25 GPa. The range of value is relatively narrow in comparison with other modules. The values from field measurement are different from laboratory measurements.

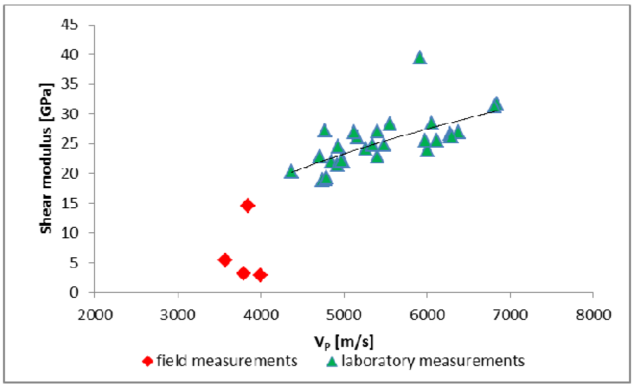

Fig. 6. The relation between shear modulus and P-wave velocity (R^2 = 0,4).

The regression curve of logarithmic function with equation y = 126ln (x) -1032 is the best fit to the data of bulk modulus K from laboratory measurements (Fig. 7). The highest values of the module are measured for limestone from the deposit 1 (from 60 to 80 GPa), the lowest values of modulus observed for dolomite from the deposit 3 (from 23 to 36 GPa). The values from field measurements have similar values like the deposit 3 (from 19 to 34 GPa).

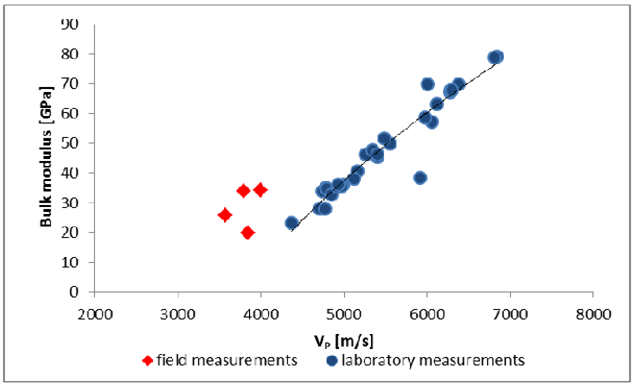

Fig. 7. The relation between bulk modulus and P-wave velocity (R^2 = 0,92).

Conclusion

The number of cracks and their direction has a visible influence on the value of anisotropy coefficient. The highest value of velocity occurs in a plane parallel to the cracks. The cracks perpendicular to wave direction cause that wave velocity has a smaller value.

The values of the anisotropy coefficient are small, of the order of several percent if cracks occur in all directions of propagation of the wave, indicating that there is no anisotropy. The small value of anisotropy coefficient is also present when there are no cracks in a sample.

The values of elastic properties of a sample differ than properties of the same rock in the quarry. This is the effect of compaction, curing and material variability. Velocities measured in the laboratory are higher than velocities measured in quarries. Velocities obtained in the field reflect main crack systems existing in the rock mass. Therefore, the anisotropy of seismic wave velocity is much higher than the velocity in samples. The samples contain only a single crack. It is difficult to compare both velocities. Each of methods provides various information about the rock mass. The study of the elastic properties measured in the laboratory conditions provides additional information about the physical properties of rocks.

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