Determination of static moduli in fractured rocks by T-matrix model

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This article compares static moduli determined using uniaxial tests and dynamic moduli calculated from elastic wave propagation velocities. On the example of limestones, we present the fact that for isotropic and intact rock, static and dynamic moduli correspond to each other over a broad range of frequencies. In damaged rock (cracks, weathering), a difference occurs and static moduli decrease. To include the effect of damage, T-matrix model has been selected. Dynamic moduli, porosity, density changes and information about properties of macroscopic cracks are necessary data for model calculation. These data are provided by other well logging methods such as an acoustic log with full waveform registration, density log, neutron log and acoustic scanner. Using all these data, a T-matrix model can predict static moduli from dynamic moduli. As we are dealing with well log data, we calculated vertical component of the elastic tensor only. This component is represented by Young's modulus E and Poisson's ratio ν. The functionality of the model has been verified on real well logging data and corresponding limestone specimens acquired from borehole core. The differences between values of E modulus, originally up to 20 %, decreased to values similar to differences in intact rock, i.e. the order of the first units of percent. In the case of ν values, demonstrable reduction of dynamic values, approaching static values was achieved, the resulting difference being less than 15 %.

Key words: *fractured rocks, dynamic and static moduli, T-matrix model, elastic wave velocity, well logging*

Introduction

Calculations in building and mining industry commonly demand determination of deformation moduli of the rocks. These moduli can be determined by static tests, most usually uniaxial compressive tests, either in laboratory or in-situ. The results of these tests are static moduli of the rock (Zhang, Bentley, 2005; Karam, 2004; Holt et al., 2013). It is frequently easier to determine dynamic moduli of the rock instead of static ones (Fei et al., 2016). Dynamic moduli can be determined from values of elastic wave propagation velocities. Measurements of these velocities are much easier to perform in the laboratory and in-situ as well (e.g. Stan-Kłeczek, 2016; Konečný et al., 2015). A major problem is a principal difference between static and dynamic moduli because their values can differ for same rock type, often due to different porosity, damage, and weathering. That is why it would be very useful to find a way how to determine static moduli from measured dynamic moduli. This problem has drawn attention in the past as well as in the present, e.g. Fjær, Holt (1994), Fjær (2009), Karam (2004), Zhang, Bentley (2005), Martínez-Martínez et al. (2012).

While comparing different methods of determination of static and dynamic moduli in laboratory and in-situ, we can take in account more viewpoints. If the study is based preferably on tests in a laboratory, one has to keep in mind that the validity of such results is determined by how the specimens represent the rock massif and how did their properties change during sampling and transport to a laboratory. The determination of correct static moduli in-situ using laboratory measurements is according to Sone, Zoback (2013) very difficult.

The validity problem is related to the heterogeneous arrangement of rocks in nature; a sample can be easily non-representative if it is taken from a locally anomalous section of the studied rock body. These well-known phenomena can be eliminated with sufficient certainty by increasing number of specimens and enhancing the set of measured data which the interpretation is based on.

Validity and reliability of in-situ tests can be increased by using a larger volume of specimens. Nevertheless, testing of locally anomalous part of rock massif is possible here resulting in the necessity of increasing the number of specimens as well. Performing of in-situ tests inside the rock massif is rather difficult. The increase in a number of specimens or tests to obtain sufficient data for statistical evaluation can cause serious feasibility problems. For determination of dynamic moduli in-situ, acoustic well logging with full waveform registration (FWS – Full Wave Sonic) can be utilized. This method can provide a large set of measured data suitable for dynamic moduli interpretation, with a sufficient rate of sampling for the record over the borehole profile to be considered continuous (Bała, Jarzyna, 1996). This method is, with respect to the amount of data it can provide, much more time efficient than static methods and many effects connected to the quality of specimens cannot have an impact here.

The purpose of this research is to provide a tool which would, for the purposes of building industry and mining in shallow depths (depths in order of first hundreds of meters under the surface, where we assume that cracks are not completely closed due to overburden pressure) make possible the transition from dynamic moduli determined by using the FWS to static moduli. The main idea of the proposed methodology is porosity data and

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rock damage information implementation in T-matrix elastic medium model resulting in the calculation of equivalent - effective moduli of rock. The effective moduli are equal to static moduli, not determined by measurement but by calculation. On examples of data from two sites, the correspondence of dynamic and static moduli in intact rock is presented and feasibility of determination of static moduli by calculation from dynamic moduli in damaged rock is demonstrated. Good agreement between calculated effective moduli and static moduli determined in laboratory proves the method's validity.

Static and dynamic rock moduli

By term "dynamic moduli", we mean moduli which have been determined using the principle of elastic wave propagation, where very small strain (strains in order of 10^{-6}, Stewart et al., 1980 *in* Cheng; Johnston, 1981) is caused by a force which acts for a very short time period determined by the frequency of the propagating wave. For laboratory or in-situ measurements, usually ultrasound or sonic frequencies are used more often than the lower ones, depending on the purpose of measurement and specimen dimensions. These measurements are quite easy to perform, and theory of elastic continuum is used for data evaluation, mostly in the approximation of linear homogeneous isotropic medium behaving accordingly to linear Hooke's law. This is the approach used in seismic exploration since the 1930s (e.g. Zisman, 1933a, b; Ide, 1936). In the case above of dynamic measurements, results of this approach often show very small differences from the actual material behaviour.

On the other hand, the term "static moduli" refers to moduli determined by measurements where the magnitude of strains is several orders higher (strains in order up to 10^{-2}, Karam, 2004; Fjær, 2009) and force applied to specimen acts for the time period in the order of several minutes. The measurement is typically carried out using a hydraulic press either in laboratory or in-situ. Very frequent way to determine static moduli is the uniaxial compressive test. Thanks to extensive widespread in practice, these tests have a very elaborated methodology (Hawkes, Mellor, 1970; Gercek, 2007). The condition requiring elastic behaviour of tested material not to deviate too much from the linear behaviour described by Hooke's law is frequently not satisfied over any broader stress interval.

Examining the comparison of these two approaches of elastic moduli determination, we can say that in the majority of cases, a nonlinear stress-strain behaviour during static tests is observed, as well as the difference between values of static and dynamic moduli, despite the fact that tested material is identical. This problem is largely caused by the fact that the strain magnitudes during static tests are several orders higher than in the case of dynamic tests. The difference emerges when elastic behaviour of tested material in applied range of stress is no longer linear. In this case, conditions necessary to ensure the validity of linear Hooke's law are no longer met (Tutuncu et al., 1998). This frequently happens, for example, in the case of damaged rocks, containing cracks that react to stress changes by opening or closing. It is possible to state that in a significant majority of cases, the values of dynamic moduli are higher than values determined by static methods (Karam, 2004; Tutuncu et al., 1998; Martínez-Martínez et al., 2012). There are studies showing that the opposite situation can occur as well, but it is quite rare. Lam dos Santos et al. (2013) compared results of elastic moduli determination; higher values of moduli from uniaxial tests were repeatedly received than from dynamic measurements of ultrasound wave traveltimes. The difference exceeded 10 % of static value.

In the case of intact rocks, it is experimentally proven that the correspondence between static and dynamic moduli can be significant (Zisman, 1933a; Ide, 1936; Cheng, Johnston, 1981; Al-Shayea, 2004; Ciccotti, Mulargia, 2004). In this case, there is no problem with using elastic continuum theory and linear Hooke's law for moduli determination (Zisman, 1933b). The more compact rock, the better the correspondence between moduli. If the rock is only slightly damaged, the correspondence can still occur, but for low-stress parts of static tests only, because the closing of cracks does not have a significant effect on the stress-strain curve yet. This fact is confirmed by analogy with engineering practice, where very compact materials are tested using both static and dynamic approach with completely identical results (Ledbetter, 1993). On the other hand, in the case of less compact rocks with the nonlinear stress-strain relationship, the difference between static and dynamic moduli enlarges with increasing amount of damage to the rock (Pola et al., 2014). For example, if at the beginning of the uniaxial compressive test the cracks are open, the stress-strain behaviour is largely controlled by the stiffness of cracks during their gradual closing. Only after the closing of all cracks, the strain reaction to applied stress is controlled by elastic moduli of intact rock. Contrarily the propagation of an elastic wave of sufficiently high frequency is not affected by cracks (Pyrak-Nolte, Nolte, 1992; Worthington, 2008; Vilhelm et al., 2013). The result is the appropriate difference between static and dynamic moduli (Gercek, 2007; Kujundzíc, Grujíc, 1966 in Barton, 2007; Olsen et al., 2004; Heap et al., 2014). Attempts to find a direct relationship between static and dynamic moduli led to results in the form of empirical equations (van Heerden, 1987 in Karam, 2004; Eissa, Kazi,1988 in Karam, 2004; Wang, 2000 in Karam, 2004) or to a certain kind of calibration based on cross-correlation between static and dynamic moduli for tested rock with very local validity (Grujíc, 1974 in Barton

2007; Jizba, 1991; Al-Shayea, 2004; Fjær, 1999 in Fjær, 2009; Ameen et al., 2009; Martínez-Martínez et al., 2012; Fei et al., 2016).

It is necessary to note that if these relationships are available, using them carefully, it is possible to obtain good results as Rasouli, Sutherland (2013) had published.

Damage of rock massif and its effect on elastic moduli

Starting point to calculate static moduli from dynamic moduli is based on analysis of various effects of rock damage on stress-strain behaviour during the uniaxial compressive test and elastic wave propagation. It is possible to start from the fact that for the purpose of interpretation of FWS in intervals of borehole profile where the rock is intact, the difference between static and dynamic moduli can be neglected and dynamic moduli represent identical elastic properties of rock as static moduli do. In other parts of borehole profile, where the rock is damaged, the static and dynamic moduli differ. Damaged parts of borehole profile can be detected by various geophysical methods. In practice, if a well logging measurement is carried out, it usually is not just a single method, but the whole set of well logging methods, which measure various properties of the rock massif and work using different physical principles. By suitable well logging methods, properties of rock massif which change due to damage (cracking) and weathering (alteration) can be measured. Changes in these properties identify and delimit intervals of damaged rock in the profile, where a significant difference between static and dynamic moduli is expected (Han, 1986; Sams, Andrea, 2001; Vanorio et al., 2003; Agersborg et al., 2009 and others).

From the viewpoint of rock damage identification using geophysical methods, it is helpful to distinguish between two types of damage. The first one is mechanical damage; mineral composition remains unchanged. This type of damage manifests itself by the presence of cracks of various sizes. Cracking causes changes in (bulk) density, porosity, electrical conductivity/resistivity (in the case where pore-filling fluid has a different conductivity than the surrounding rock), attenuation and propagation velocity of elastic waves (depends on the size of damaged zone relative to wavelength. The second type of damage is physicochemical damage – result of weathering. In this case, changes in the mineral composition of rock take place. This results in volume changes, which usually cahe use a drop in density and possible increase of porosity (Gupta, Seshagiri, 2000; Bozkurtoğlu, 2013), clay minerals content may rise, which can cause an increase in gamma ray activity of rock and increase of conductivity. The spatial extent of this type of damage is usually larger than extent of local discontinuities and cracked zones caused by mechanical damage, so it has visible effects on elastic waves as propagation velocities reduction and an increase of attenuation (Pola et al., 2014).

Both types of damage rarely occur separately, usually, they combine.

Damage by cracks is detected by density-log as changes in density. In the case of weathering, mineral composition changes have an effect on density changes as well. Independently of the density-log, neutron log detects rock damage as changes of porosity. If the damage is present, FWS tool detects lower propagation velocities of elastic waves and their increased attenuation. Acoustic borehole imager (ABI) determines the shape and position of inhomogeneities, most frequently bedding planes, cracks, and dykes. Resistivity or induction log detects weathered intervals, which have higher conductivity.

T-matrix model

As described earlier, damaged intervals of borehole profile are zones where the difference between static and dynamic moduli is observed. To be able to include the effect of damage on the behaviour of the initially intact elastic medium, we used T-matrix model (Jakobsen et al., 2003a, 2003b; Jakobsen, 2012). Several other models are suitable for this purpose, e.g. DEM, Voigt/Reuss/Hill averages (Mavko et al., 2009). We chose the T-matrix model because its physical background corresponds closely to the problem we are trying to solve and computer implementation of this model is relatively easy. The T-matrix model is based on quantum scattering theory, and it makes possible to model the elastic medium with a finite number of inclusions inside. These inclusions have a certain shape and elastic properties; both can be arbitrary for various inclusions.

The main purpose of the calculation is to calculate fluctuations of the arbitrarily chosen tensor of reference elastic medium $C^{(0)}$. If we assume, that local elastic tensor $C(x)$ varies arbitrarily in space on a scale sufficiently small with respect to specimen size or the wavelength of used acoustic wave, we can write

$$C(x) = C^{(0)} + \delta C(x) \tag{1}$$

Eq.°1 is in such case valid for macroscopic scale. Using the same assumption for basic stress ($\sigma(x)$) – strain ($\varepsilon(x)$) relation

$$\sigma(x) = C(x)\,\varepsilon(x) \tag{2}$$

it is possible to perform averaging of $\sigma(x)$ and $\varepsilon(x)$ in macroscopically homogeneous medium, and we get

$$\langle \sigma(x) \rangle = C^{*} \langle \varepsilon(x) \rangle \tag{3}$$

where C^{*} represents the tensor of effective elastic moduli (it corresponds to $C(x)$ in Eq.°1), further on

$$\langle \sigma \rangle = C^{(0)} \langle \varepsilon \rangle + \langle \delta C \varepsilon \rangle \tag{4}$$

(chevrons denote volume average). Here we can introduce new tensor T as

$$\delta C(x)\,\varepsilon(x) = T(x)\varepsilon^{(0)} \tag{5}$$

By substituting Eq.°5 into Eq.°4 we obtain

$$\langle \sigma \rangle = C^{(0)} \langle \varepsilon \rangle + \langle T \rangle \varepsilon^{(0)} \tag{6}$$

(tensor ε is defined analogically to tensor $C^{(0)}$ defined above). The $T(x)$ tensor formulated as

$$T(x) = \delta C(x) + \delta C(x) \int_{\Omega} G^{(0)}(x - x')T(x')dx' \tag{7}$$

is called T-matrix. $G^{(0)}(x\text{-}x')$ is Green's function over domain Ω. If we now restrict the assumption that *C(x)* varies arbitrarily in space to *C(x)* being piecewise constant, we get to the concept of inclusions embedded in surrounding elastic medium. Detailed derivation can be found in Jakobsen (2003a).

Input data for the model described above are dynamic moduli from FWS, porosity log, information about inhomogeneities from ABI and static moduli determined on specimens taken from certain places of borehole profile. Dynamic moduli are directly used in model calculations, while porosity log and ABI data are evaluated into the so-called porosity of effective medium, which represents damage by cracking. The porosity of effective medium is implemented into the model in the form of families of inclusions embedded in surrounding effective medium and have elastic parameters of pore filling fluid, in our case air. Damage by weathering is represented by static moduli which determine the elastic behaviour of effective medium outside inclusions. Dynamic moduli represent the elastic behaviour of the medium when the elastic wave propagates through it, whereas effective medium in the T-matrix model represents behaviour during the uniaxial compressive test. Elastic moduli calculated by the T-matrix model are called effective moduli and further on are denoted as E_{ef} and ν_{ef} (Young's modulus and Poisson's ratio respectively). These effective moduli have been subsequently compared with static moduli of particular specimens determined by uniaxial tests in the laboratory.

Test sites and executed measurements

In the framework of this research, two experimental measurements have been carried out on two testing sites. First one in 2011 at the site Velkolom Čertovy Schody (VLČS) quarry, the second one in 2014 at the site Kosov quarry. Geologically, both sites belong to the regional area of Barrandien, subunit Prague Basin. Studied rocks are sediments of lower Paleozoic age. In the case of VLČS testing site, there are Devonian bioclastic limestones present, in the case of Kosov testing site, there are bioclastic limestones, limestones with volcanogenic admixture, both interbedded with clay shales and tuffitic shales of Silurian age.

The limestones have in both cases high content of calcium carbonate (calcite and a small fraction of dolomite), and in small amounts quartz and pyrite are present. Other minerals like ilmenite, apatite, muscovite and orthoclase are present in fractions of few percent only. In the case of VLČS, calcite content is over 95 % (with almost no dolomite), locally higher pyrite contend can occur. In the case of Kosov, the calcium carbonate content varies from 89 to 95 %, with calcite component being dominant as dolomite content is under 8 %. Other minerals are present in fractions not exceeding few percent. Locally, there are contents of quartz or pyrite up to 9 %. If tuffitic admixture is present, clay minerals with mixed structures containing members of chlorite and smectite group occur.

VLČS test site

At the VLČS test site, two boreholes (V-1 and V-2) have been drilled, each 15 m deep. The drilling took place on one of the lower etages of the active quarry. Thanks to the thickness of the removed overburden, very intact rock massif was expected, except the top part near the etage surface, where some mechanical damage from blasting should take place. Drilling had been done using coreless hammer drilling machine with drilling diameter 112 mm. Rock specimens for laboratory test have been taken from an outcrop of layers present in boreholes. This was possible thanks to relatively simple deposition of limestone layer, which are not folded and are a few meters thick, and thanks to the mining activity which resulted in exposing part of the profile present in boreholes in the nearby etage wall.

Two visually different types of limestone were encountered in the boreholes. First one has a white colour, coarse biosparitic packstone. From this rock type, four specimens labelled FS (FS1-FS4) have been taken. The second type is light grey fine biomicritic packstone. From this rock type, four specimens labelled FT (FT1-FT4) have been taken as well. Composition analysis has shown that both types are limestone of high purity with minimal content of admixtures.

Kosov test site

In this case, the quarry is no longer active. Borehole K-1 has been situated on the top etage at a place chosen after detailed geological mapping with the intention to be able to encounter intact rocks as well as rocks with various degrees of damage in the borehole. This borehole was drilled using diamond core drilling technology, drilling diameter 112 mm, to ensure that we get as intact core as possible. The core would then be used for laboratory experiments. The borehole depth is 16 m, of which 15 m are in rock massif. The limestone in the upper part of borehole profile (above a depth of approximately 10 m) is quite intact with local discrete cracked zones only. In the lower part (from 10 m down), considerably damaged limestones were discovered as is proven by a geological description of the core and ABI record. Not sooner than in this depth, losses of drilling fluid (water) started to occur. The dominant portion of damage is represented here by numerous cracks. The weak weathering took place along the cracks. The most significant display of weathering was the partial oxidation of present pyrite. Specimens have been chosen from acquired core to cover whole profile evenly.

Specifications of executed laboratory and in-situ tests

Laboratory tests have been carried out on cylindrical specimens with a diameter of 5 cm and height of 10 cm. These cylinders were drilled out of larger blocks (VLČS site) or from core samples (Kosov site). The bulk density of all specimens was determined by weighing. Dynamic moduli determination was carried out using the measurement of the travel time of elastic waves (P- and S-waves) on the trajectory of known length. Elastic waves were transmitted and detected by piezoceramic transducers with a nominal frequency of 1 MHz. Dynamic moduli will be denoted E_D and ν_D.

For the tests carried out to determine static moduli, hydraulic press MTS-815 was used. The axial and radial strain was measured by LVDTs. The test ran in constant loading rate mode. Static moduli will be denoted E_S and ν_S.

In the case of VLČS site, first two samples (FS1, FT1) have been monotonously loaded until specimen failure to determine compressive strength σ_C. Loading rate was 1.31 kN.min^{-1} (10.7 kPa.s^{-1}). Standardized testing procedure for the rest of specimens was designed after. The procedure comprised 5 loading-unloading cycles with increasing value of maximal stress σ_{max} = 5, 15, 30, 50 and 60 MPa.

All the Kosov site specimens were tested following unified procedure which comprised two loading-unloading cycles. The loading rate was set to 8 kN.min^{-1} (62.8 kPa.s^{-1}). In the first cycle, the loading stopped at 50 % of estimated σ_C, in the second one at 70 % of estimated σ_C. Thanks to previous tests and available archival data, it was possible to estimate σ_C with sufficient accuracy. During all the measurements, acoustic emission was being registered to ensure that elastic part of deformation alone will be used for test evaluation and static moduli determination, i.e. stress range in which no fracturing of the specimen occurred. For moduli determination only quasilinear parts of the stress-strain curve have been used.

Well logging methods we used were acoustic log FWS, density-log, neutron log and acoustic borehole imager ABI. The FWS uses piezoceramic transducers with nominal frequency 20 kHz. The ABI tool uses focused seismic ray with wave frequency 1.2 MHz.

The output from FWS data interpretation were velocities of P- and S-wave, density-log determined bulk density of rock surrounding the borehole, the rock porosity determined by neutron log and all macroscopic discontinuities (bedding planes, cracks) present in the borehole wall provided by the ABI tool. FWS tool measures in a direction parallel with borehole direction only, and as signal transmission between tool and borehole wall is provided by drilling fluid (water), it does not distinguish potential polarization of S-waves. In this parallel direction, with the use of isotropic medium approximation, dynamic moduli E_D and ν_D have been determined from acquired data. The values of velocities determined from well logging data from the depth corresponding to the depth where specimens have been taken are later compared with velocities measured in the laboratory on corresponding specimens. The results of measurements are given in the next chapter in Table 1 for VLČS site and Table 3 for Kosov site.

The values of velocities v_P and v_S, moduli E_D, ν_D, E_S, ν_S in Table 1 determined on specimens (FS1-FS4 respective FT1-FT4) represent averages of four values, *s* are their standard deviations.

In Table 3, values of v_P and v_S, determined by laboratory measurements on specimens and measured by FWS in corresponding depth, are presented.

Acquired data do not contain any information about rock properties in a plane perpendicular to the borehole direction. Anisotropy of studied rock has been tested by ultrasonic laboratory measurements in three mutually perpendicular directions and also by (micro) texture analysis of mutually perpendicular thin sections performed on optical and electron microscope. The result is that the anisotropy is very weak, so rocks from both testing sites can be, with satisfactory accuracy, considered isotropic.

The results of measurements and modelling, results discussion

Measurements at the VLČS site verified that in practice, values of static and dynamic moduli E from FWS could come very close to each other, provided we measure in intact rock. The determined average difference value was circa 7 GPa, which corresponds to a relative error of 10 % with respect to E_D values (Tab. 2). Analogous comparison of ν_S and ν_D values from FWS shows an average difference of 0.087, which corresponds to the relative error around 27 % (relative to ν_D). Very slight or no damage of rock was confirmed by ABI measurement. Detected inhomogeneities were either very thin cracks, almost under the detection limit of the tool or bedding planes consisting of a thin layer of material different from the rest.

Tab. 1. Results of measurements at the VLČS site (boreholes V-1, V-2), in each pair of adjacent rows there is the result of laboratory measurement first and then corresponding FWS log result. Average values and their standard deviations are presented.

specimen	v_P [m.s^{-1}]	s(v_P) [m.s^{-1}]	v_s [m.s^{-1}]	s(v_S) [m.s^{-1}]	v_P/v_S	ρ [g.cm^{-3}]	ν_D	s(ν_D)	ν_S	s(ν_S)	E_D [GPa]	s(E_D) [GPa]	E_S [GPa]	s(E_S) [GPa]
FS	6324	22.6	3244	31.3	1.95	2.66	0.32	0.005	0.20	0.012	74.0	1.0	63.9	4.6
FWS (FS)	6275	34.6	3150	17.7	1.99	2.66	0.33	0.005	-	-	70.2	0.6	-	-
FT	6335	75.3	3361	26.0	1.88	2.69	0.30	1.216	0.27	0.078	79.2	1.2	71.5	4.2
FWS (FT)	6350	71.4	3347	19.8	1.90	2.69	0.31	0.003	-	-	79.2	1.5	-	-

In the case of dynamic tests, the moduli from FWS and laboratory ultrasound measurements are practically identical (average difference between E_D from laboratory and FWS in Table 1 is 2 GPa, with respect to measured values of E_D the relative error is between 2 – 3 % (relative to the higher of E_D values). Comparison of ν_D values gives average difference 0.007, adequate to a relative error of 2 %. It is obvious that in the frequency range from 20 kHz to circa 200-300 kHz, acquired moduli values are independent of frequency. When damage consisting from cracking and weathering occurs, moduli values can differ.

Tab. 2. Comparison of differences between moduli values acquired by ultrasonic tests in the laboratory (dynamic; subscript "D"), by FWS log (dynamic; subscript "FWS") and by uniaxial compressive tests in the laboratory (static; subscript "S"). Relative errors δ are calculated with respect to higher of compared values.

compared moduli values	average difference Δν	standard deviation s(Δν)	relative error δ(ν) [%]	average difference ΔE [GPa]	standard deviation s(ΔE) [GPa]	relative error δ(E) [%]
FS_D-FS_{FWS}	0.010	0.0101	3	3.8	1.61	5
FS_D-FS_S	0.125	0.0166	39	10.1	5.56	14
FS_{FWS}-FS_S	0.135	0.0166	41	6.3	5.18	9
FT_D-FT_{FWS}	0.003	0.0052	1	0.0	2.71	0
FT_D-FT_S	0.034	0.0805	11	7.8	5.39	10
FT_{FWS}-FT_S	0.038	0.0809	12	7.7	5.66	10

At the Kosov site, the K-1 borehole perforated limestones with various degrees of damage. In the upper part of the borehole profile, the damage is isolated in discrete cracked zones with intact blocks in between, more consistently damaged limestones are in depths below 10 m. Thanks to drilling through the same rock type in intact and in damaged state, it was possible to test the effect of damage on moduli values. Values presented in Table 3 relate to that. Specimens 1A, 2A, and 4C come from a less damaged part of the borehole. The differences in values of E_D from FWS and E_S are from 3 to 9 GPa, in average 6 GPa. The corresponding relative error with respect to higher of compared values (E_D) is circa 8 %. Comparison between ν_D from FWS and ν_S gives a difference of 0.047, which with respect to the higher value of ν corresponds to a relative error of 17 %. On the other hand, specimens 5B and 7A come from consistently damaged part of borehole profile and have the difference between E_D from FWS and E_S from 12 to 15 GPa, in average 13.5 GPa (relative error

18 – 21 %). In the case of average difference between ν_D from FWS and ν_S a value of 0.025 (relative error 11 %) was calculated.

The comparison between values of E_D determined by laboratory tests and FWS log has shown that in the case of specimen 1A, 2A and 4C, this difference is smaller than 3 GPa, on average 2 GPa, which with respect to compared values of E_D gives a relative error of 2 – 3 %. For determined values of ν_D, this difference is 0.013 (relative error up to 5 %). This finding is in correspondence with VLČS results described earlier. In the case of specimens 5B and 7A, the difference for E_D is 6 respective 4 GPa, the relative error is here in the range of 6 – 8 %. For values of ν_D the difference is 0.040 (relative error 18 %).

Tab. 3. Results of measurements at the Kosov site (borehole K-1); in each pair of adjacent rows there is the result of laboratory measurement first and then corresponding FWS log result.

depth of sampling / measurement [m]	specimen	v_P [m.s^{-1}]	v_S [m.s^{-1}]	v_P/v_S	ρ [g.cm^{-3}]	ν_D	E_D [GPa]	ν_S	E_S [GPa]
2.95	1A	6145	3367	1.82	2.69	0.29	78.3	0.28	74.8
	FWS(1A)	5932	3470	1.71	2.65	0.24	79.2	-	-
4.50	2A	6056	3354	1.81	2.69	0.28	77.3	0.19	68.2
	FWS(2A)	6086	3388	1.80	2.55	0.28	74.7	-	-
9.15	4C	5934	3260	1.82	2.71	0.28	73.8	0.20	67.0
	FWS(4C)	5809	3166	1.83	2.77	0.29	71.7	-	-
11.10	5B	6005	3301	1.82	2.69	0.28	75.3	0.20	62.9
	FWS(5B)	5427	3265	1.66	2.67	0.22	69.3	-	-
14.30	7A	5084	3054	1.66	2.82	0.22	64.0	0.17	49.5
	FWS(7A)	5325	3266	1.63	2.72	0.20	69.6	-	-

Comparison of dynamic, static and effective moduli

Moduli determined from laboratory measurements, well logging and T-matrix modelling (effective; subscript "ef") are presented in Table 4. Determinations and modelling have been done on data from specimens and data from corresponding depths of K-1 borehole (Kosov site).

Tab. 4. Comparison of dynamic (subscript "D") and static (subscript "S") moduli determined in borehole K-1 at the Kosov site with effective (subscript "ef") moduli calculated using T-matrix model. Superscript "lab" denotes laboratory measurement, superscript "FWS" denotes FWS well log and superscript "well" denotes density-log.

depth [m]	specimen	ν_D^{lab}	ν_D^{FWS}	ν_S	ν_{ef}	E_D^{lab} [GPa]	E_D^{FWS} [GPa]	E_S [GPa]	E_{ef} [GPa]	ρ_V^{lab} [g.cm^{-3}]	ρ_V^{well} [g.cm^{-3}]
2.95	1A	0.29	0.24	0.28	0.26	78.3	79.2	74.8	72.6	2.69	2.65
4.50	2A	0.28	0.28	0.19	0.26	77.3	74.7	68.2	68.8	2.69	2.55
9.15	4C	0.28	0.29	0.20	0.27	73.8	71.7	67.0	65.8	2.71	2.77
11.10	5B	0.28	0.22	0.20	0.21	75.3	69.3	62.9	59.8	2.69	2.67
14.30	7A	0.22	0.20	0.17	0.19	64.0	69.6	49.5	55.6	2.82	2.72

Comparisons analogous to ones carried out in the previous chapter have been done to compare moduli determined by measurements and modelled ones (Tab. 5).

Tab. 5. Comparison of differences between average values of dynamic moduli from FWS, static moduli from laboratory and effective moduli calculated using T-matrix model. Static and dynamic values of moduli have been averaged for less damaged upper part of borehole profile from values determined on specimens 1A, 2A, and 4C. For consistently damaged lower part of borehole profile, values determined on specimens 5B and 7A have been averaged. Relative errors δ are calculated with respect to higher of compared values.

	$\nu_D^{FWS}-\nu_S$	$\delta(\nu_D^{FWS}-\nu_S)$ [%]	$\nu_{ef}-\nu_S$	$\delta(\nu_{ef}-\nu_S)$ [%]	$E_D^{FWS}-E_S$ [GPa]	$\delta(E_D^{FWS}-E_S)$ [%]	$E_{ef}-E_S$ [GPa]	$\delta(E_{ef}-E_S)$ [%]
less damaged part of borehole	0.047	17	0.040	15	5.2	7	-0.9	1
consistently damaged part of borehole	0.025	11	0.015	7	13.3	19	-1.2	2

Table 5 shows that the difference between dynamic moduli from FWS and static moduli from the laboratory is always bigger than the difference between static and effective moduli. The difference reduction is clearly visible in the drop of relative error values. This corresponds with the initial intention to be able to move via modelling from dynamic moduli from FWS to static moduli determined by standard methods in the laboratory.

In the less damaged upper part of borehole profile, the difference between values of modulus E decreased from the original 5 GPa (corresponding to a relative error of 7 %) to 1 GPa (corresponding to a relative error of 1 %). In the case of ν, the original difference was 0.047 (corresponding to a relative error of 17 %), and it decreased to 0.040 (relative error 15 %).

In consistently damaged lower part of borehole profile, the difference between values of modulus E decreased from the original 13.3 GPa (corresponding to a relative error of 19 %) to 1 GPa (corresponding to a relative error of 2 %). In the case of ν, the original difference was 0.025 (corresponding to a relative error of 11 %), and it decreased to 0.015 (relative error 7 %).

The comparison of effective values of Young modulus E_{ef} with values E_S determined by measurements clearly shows correspondence in order of first GPa.

In the case of values of Poisson's ratio ν, the correspondence of ν_{ef} and ν_S is good; nevertheless, relative errors are considerably higher than in the case of E. The results of ν could be improved by incorporating fluid movements in pore space as is defined in Jakobsen et al. (2003b). In the framework of this research, this modification was not used.

Conclusions

This research aimed at differences between static and dynamic moduli of rocks with relation to rock damage. In the case of intact rock massif, the good correspondence between dynamic moduli from FWS well log and ultrasonic tests have been verified. The differences between compared moduli values were in terms of relative error in the order of first units of percent. This finding has proven independence of moduli values of frequency in the range from 20 kHz to 200 – 300 kHz. The difference between dynamic moduli from FWS and static moduli from the laboratory was in the case of E up to 10 %, in the case of ν, it was up to 27 %. In consistently damaged rock massif, these differences were for E up to 21 %, for ν it was up to 11 %.

Thanks to the knowledge of geological profile and rock properties acquired by well logging, the geological description of the core and the laboratory measurements, it was possible to implement a model, capable of producing so-called effective moduli values, equivalent to static moduli values, from values of dynamic moduli determined by well logging. Apart from dynamic moduli, well logging results are needed, such as values of porosity, density, a record of the position and character of discontinuities present in the borehole wall. Furthermore, static moduli values determined in the laboratory on specimens from core samples are necessary. The functionality of the model has been verified on real well logging data and specimens acquired in K-1 borehole, which has been drilled in the framework of this research at the Kosov test site. After the modelling was carried out, significant correspondence between effective and static moduli values was observed in depth intervals respective to specimens used for uniaxial compressive laboratory tests. The differences between values of E modulus, originally up to 20 %, decreased to values similar to previously described differences in intact rock, i.e. order of the first units of percent. In the case of ν values, demonstrable reduction of dynamic values, approaching static values was achieved, the resulting difference being less than 15 %.

In the future, the functionality of this model will be tested on a whole borehole profile; effective values will be calculated between the depths of core sampling. The result will be the curve of effective moduli covering the borehole profile completely.

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