Maximal precision increment method utilization for underground geodetic height network optimization

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Nowadays, optimization of the measurements is an important theme of the geodesy and especially engineering surveying. The optimization can be categorized into orders, from the zero to the third order. The zero order design is the coordinate system selection; the first order is the configuration optimization; the second order is the optimization of the number of the repetitions; the third order is the improvement of the existing network by adding point and/or observations. In the engineering surveying, the second order design needs to be addressed. Thus, there are known possible locations of network points, the desired accuracy, possible measurements within the existing situation, and it is necessary to choose such measurements and their numbers of repetition so that their number was minimal (resp. used energy on their implementation) and yet the required accuracy was met. There were many attempts to meet this objective, but usually by the deployment of general mathematical optimization methods that do not respect the character of geodetic measurements and rational optimization results demands. There was designed a new method which, in our opinion, meets these requirements and is based on a gradual selection of the most beneficial measurement to meet the accuracy requirements. The method was successfully tested on the leveling network optimization. The method is very suitable for use in mining and tunnel surveying, wherein it allows economic planning of the accuracy of the measurement, which is important both in economic terms and in terms of safety. The economic aspect is important not only in terms of money spent on the measurement, but also in terms of minimizing downtime of a mine operation and equipment.

Key words*:* *Geodetic measurements, geodetic networks, optimization, underground surveying, tunnel surveying*

Introduction

Optimization of geodetic measurements is currently a major topic of geodesy and especially engineering surveying. This optimization can be divided into the design of zero to third order. The zero order is the prime choice of the coordinate system, the first order determination (optimization) of geodetic network configuration (the point location), second order optimal selection of the number of individual measurements (from zero to *n*) and the third in order to define the optimum modification or extension of the network.

In the field of engineering surveying, it is necessary to solve the design of the second order in particular, because the coordinate system is usually given, and the configuration is determined by the possibilities of practical solution within the existing spatial arrangement (e.g. location of points in areas of interest, visibility on the building site).

There are thus known locations of the network points, the desired accuracy, and possible measurements within the existing situation. It is necessary to choose such measurements and their numbers, on the condition, that the number of the measurements was minimal (resp. energy expended on their implementation was minimal, in the case of the geometric leveling it is the length of the trace) and the required accuracy was met. The optimized criteria may also be the reliability or some other one. It should also be noted that it is possible to optimize one criterion (e.g. the maximum coordinate deviation) or solve the problem as the multi-criteria one, but this case is much more complicated.

Solving these problems is not unambiguous and is not easy to implement without additional conditions, except the simplest cases. There can be found a lot of the attempts of reaching of the optimization goal, using the general optimization methods usually, without the respect to the character of the geodetic measurements and without complying with the rational demands on the optimization results.

Publications (Baarda, 1968) and (Grafarend, 1974), or later (Grafarend and Sanso, 1985) were discussing the topic of the optimization and laid the fundamentals of optimization in geodesy. An interesting overview of the fundamental principles presents (Amiri-Simkooei et al., 2012).

Among the countless techniques used for optimization can be classified the use of global optimization methods, e.g. the simulated annealing is used in (Baselga, 2011) and (Berné and Baselga, 2004), particle swarm optimization (PSO) is used in (Yetki et al., 2008). On the optimization of the reliability is focused

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method described in (Amiri-Simkooei, 2004), optimization of the configuration together with the number of measurements is performed in (Kuang, 1991). There are many other methods and procedures in the literature, including attempts of analytical solution, which is very difficult to apply (except the special cases) and unreliable respectively (e.g. (Pecár, 1985)). On the other hand, there are some optimization methods easily understandable as is the one described in (Koch, 1985). To document the importance of the optimization of the geodetic measurements and especially the geodetic networks, one can mention the work (Bartoš et al., 2011) documenting the historical monuments by various geodetic 3D methods including photogrammetry and laser scanning, or paper (Fraštia et al., 2014) describing the complex monitoring of the rock slide, or a deformation measurement of the gabion wall by the photogrammetric methods described in (Marčiš et al., 2014). The measurement with use of UAV is a separate chapter on the current development of geodetic and mapping science, as described and used e.g. in (Blišťanová et al., 2015; Kršák et al., 2016) for the purposes of the crisis management or in (Blistan et al,. 2016) for the documentation of the rock outcrops. None of these geodetic projects can be executed without a quality geodetic frame – a geodetic network, well designed and determined with necessary and sufficient precision. Naturally, there are geodetic networks necessary as such, e.g. in geodetic high precise deformation measurement of the buildings and areas (e.g. described in (Stroner et al., 2014)), or in tunnel building process, where the precision is the key as described in (Urban and Jiřikovský, 2015). It is also necessary to mention the simple addition or renewal of points of the geodetic network with the required precision (Sabová and Pukanská, 2012), where of course it is appropriate to use the optimization.

A new method of the optimization of the second order was designed by the authors. It is computationally simple and easy to implement, it allows measurements to be grouped into logical groups with the same number of repetitions (e.g. when measuring by the total station), the numbers of the repetition are integers and is based on a gradual selection of the most beneficial measurements for the meeting of the accuracy requirements. The designed method has been successfully tested on optimization of the various configurations of the leveling network. As the optimization criterion in tested cases, selected an achievement of a state, when all of the standard deviation of the heights of points are smaller than the selected maximum value, was selected. (The criterion can be easily chosen differently, however).

Maintaining accuracy and its economic planning are very important, especially in mining and tunneling surveying where there is usually practically impossible independent control because, there is only one entry to the underground, with some few exceptions. Failure to comply with the required accuracy could also have incalculable consequences in terms of safety and also in terms of economic losses. Test examples given in the second half of the article represent the possibility of using the proposed method in this field for height measurements.

Last mentioned example is a really optimized measurement of the geodetic network in the underground corridors of the Czech town of Jihlava, which demonstrates the practical applicability of the method.

A priori precision analysis

The basic element determining the accuracy of the adjustment (or a simple calculation with zero redundant measurements) result is the covariance matrix $\boldsymbol{M}$ describing the accuracy of the resulting coordinates, which can be obtained as (according to (Hampacher et al., 2015), for adjustment by the least squares method (for the case of the adjustment with additional conditions (free network) are the equations a bit more complicated, but the principle is the same)):

$$\boldsymbol{M} = \sigma_0^2 (\boldsymbol{A}^T \boldsymbol{P} \boldsymbol{A})^{-1}, \tag{1}$$

where $\boldsymbol{A}$ is the matrix of the experiment plan (Jacobi's matrix, the matrix of derivatives) describing the configuration of the geodetic network, $\boldsymbol{P}$ is the weight matrix of the individual measurements (1 to m), in the case of the independent measurements, only the diagonal of the matrix is nonzero:

$$\boldsymbol{P} = diag(p_1 \quad p_2 \quad \cdots \quad p_i \quad \cdots \quad p_m) \, . \tag{2}$$

The weight of the matrix is determined by use of the chosen constant σ_0 (to simplify the calculation is often chosen to be equal to 1) and standard deviation of the one-time realized i-th measurement σ_i and number of the repetitions of the individual measurement n_i:

$$p_i = n_i \frac{\sigma_0^2}{\sigma_i^2} \, . \tag{3}$$

For the next derivation, it is advantageous (as will be seen) to separate the number of the repetitions from the other variables, the matrix $\boldsymbol{P}$ can be then written as:

$$\boldsymbol{P} = \boldsymbol{n} \cdot \boldsymbol{P}_0 = (n_1 \quad \cdots \quad n_m) \cdot \ diag\left(\frac{\sigma_0^2}{\sigma_1^2} \quad \cdots \quad \frac{\sigma_0^2}{\sigma_m^2}\right). \tag{4}$$

Matrix $\boldsymbol{A}$ for m measurements and k unknowns (usually determined coordinates):

$$\boldsymbol{A} = \begin{pmatrix} \frac{\partial l_1}{\partial x_1} & \frac{\partial l_1}{\partial x_2} & \cdots & \frac{\partial l_1}{\partial x_k} \\ \frac{\partial l_2}{\partial x_1} & \frac{\partial l_2}{\partial x_2} & \cdots & \frac{\partial l_2}{\partial x_k} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial l_{m-1}}{\partial x_1} & \frac{\partial l_{m-1}}{\partial x_2} & \cdots & \frac{\partial l_{m-1}}{\partial x_k} \\ \frac{\partial l_m}{\partial x_1} & \frac{\partial l_m}{\partial x_2} & \cdots & \frac{\partial l_m}{\partial x_k} \end{pmatrix}. \tag{5}$$

Variance-covariance matrix $\boldsymbol{M}$ is then calculated:

$$\boldsymbol{M} = \sigma_0^2(\boldsymbol{A}^T\boldsymbol{n}\,\boldsymbol{P}_0\boldsymbol{A})^{-1}. \tag{6}$$

For the purposes of optimizing, all elements of the right side of the prior equation (6) are constant except the vector $\boldsymbol{n}$. The precision of the individual measurements, the configuration of the points and measurements is given by the matrix $\boldsymbol{A}$ and matrix $\boldsymbol{P_0}$.

The criterion of optimization K evaluating the required accuracy reaching is a function of the covariance matrix:

$$K = f(\boldsymbol{M}). \tag{7}$$

As an example, it is possible to write the criterion of the maximal standard deviation in one coordinate:

$$K = f(\boldsymbol{M}) = \sqrt{\max\,(M_{i,i})}\text{ , for } = 1 \ldots k\ . \tag{8}$$

For the optimization, it has to be defined, which criterion K_T should be achieved, so the target is to fulfill the inequality:

$$K \leq K_T\ . \tag{9}$$

Goals of the optimization and possibilities of the verification

The ideal goal of optimization is, in this case, the fulfillment of the selected criteria of accuracy with an additional condition, that the sum of the number of individual measurements is minimal. Accuracy criterion may be, for example, the maximum size of a standard deviation of coordinate, major axis size of the error ellipse or error ellipsoid, coordinate standard deviation or position standard deviation, etc. The criterion is a function of the covariance matrix M. (As was mentioned in the introduction, there are other options, for example, reliability, but it is not the objective of this article. Moreover, the method can be modified for these purposes easily.)

It should be noted that optimization solution may not be only one, there may be more of them, or even very much. The quality of the optimization in case of integer solutions can be verified very simply, in principle, by brute force. The criterion is determined for all combinations of elements of $\boldsymbol{n}$, where all elements are smaller than the selected maximum $n_i < n_{max}$. Then, optimal solutions are those that meet the criterion, and the sum of the measurements repetitions is the smallest one. This begs the question; why do you optimize by brute force. The answer is simple; it is very demanding in terms of computing power, e. g. for a network with 14 measurements with a maximum of 2 repetitions, it is necessary to count $3^{14} = 4\,782\,969$ times the criterion, including covariance matrix inversion. For the most complex case described at the end of the article, the calculation in the Matlab software lasted about 2 days, and determining $3^{18} = 387\ 420\ 489$ combinations possible, a number of the measurements is only 18, variants of the repetitions 3; it is 0, 1 or two repetitions. In the case of a maximum of 3 repetitions, the number of variants would be already 68 719 476 736. In this manner, verification or direct calculation are unquestionable, but it can be done only with some reasonably small networks.

In general, for the optimization method is true that in more complex problems it cannot be expected that the obtained solution is not the best existing one, the calculation is often stopped after a finite number of operations or after a certain time of calculation. Solving these problems is typically very computationally intensive. As an example, the solution by the global optimization methods can be presented ((Baselga 2011; Berné and Baselga 2004; Yetki et al. 2008), where in the real (for example, according to (Hampacher and Štroner 2015)) individual local minimums without certainty, that the minimum is the global one, are searched.

The number of measurements must be considered in terms of expended energy. When measuring by the total station, it is actually a number of measurements, but for instance, in the case of geometric leveling from the center between the rods with unequally long distances between points, it is a length of overall leveling line.

The designed optimization method principle

The principle of the proposed optimization method is based on the initial calculation with a vector $\boldsymbol{n}$ (the number of repetitions of each measurement) as (practically) zero, i.e.:

$$\boldsymbol{n} = (n_1 \quad \cdots \quad n_m) = (\varepsilon \quad \cdots \quad \varepsilon)\ , \tag{10}$$

where ε is a very small number close to the zero, however, so large to calculate the inversion during the covariance matrix $\boldsymbol{M}$ calculation, and simultaneously so small to have a negligible impact in terms of accuracy.

$\varepsilon = 10^{-12}$ was used successfully. Furthermore, individual measurements are added to the system sequentially each time, so that the most improved the worst characteristic. Selection of the worst characteristics is performed so, that according to equation (6), the $\boldsymbol{M}$ covariance matrix is calculated and all the characteristics from it for all points of the optimized network points. From these, the worst one is selected with the characteristic K_w. Then, the number of the repetitions of individual measurement in vector $\boldsymbol{n}$ is increased by one (for each one sequentially) and calculated the characteristics $\mathrm{K}_{\mathrm{w,n}_i+1}$ again. Measurement having the biggest impact on the decrease of the characteristic of the precision $(\mathrm{K_w} - \mathrm{K}_{\mathrm{w,n}_i+1})$ is then added to the system, so it increased the number of repetition of the *i*-th measurement, for which holds true:

$$\max_i\left(\mathrm{K_w} - \mathrm{K}_{\mathrm{w,n}_i+1}\right)\ . \tag{11}$$

The procedure is then repeated until the relation (9) is true, until all of the precision characteristics of all points selected as the criterion are better than requirement K_T. In terms of the computational cost for each added measurement, the inversion is calculated (when calculating covariance matrix) m times, and once to find the worst point.

Example of the application on the simple leveling network

An extensive testing of the method was performed. For the initial testing with easily interpretable results, the method of geometric leveling from the center between the rods in the height network was used.

The first testing was made with a simplifying assumption, that all the measured height differences have the same length and thus the same accuracy. Fig. 1 shows the simple testing network. The number of the possible measurements is 10, unknown coordinates 5. The matrix $\boldsymbol{A}$ describing the configuration and the matrix $\boldsymbol{P}$ with weights are:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix},\ \boldsymbol{P} = \frac{1}{\sigma_h^2} diag\,(1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1)\ .$$

Optimizing criterion can be chosen easily, as a standard deviation of the height of the points A to E, which is squared on the diagonal of the matrix $\boldsymbol{M}$. The standard deviation of the one measured height difference was chosen σ_h = 1 mm, required standard deviation of the height 0.95 mm. Optimization then runs according to described algorithm, results are shown in Fig. 1. Measurements 2, 4, 6 were not used, measurement 3 should be done 3 times; measurements 1, 5, 7, 8, 9, 10 one times. The sum of the measurements repetitions is 9.

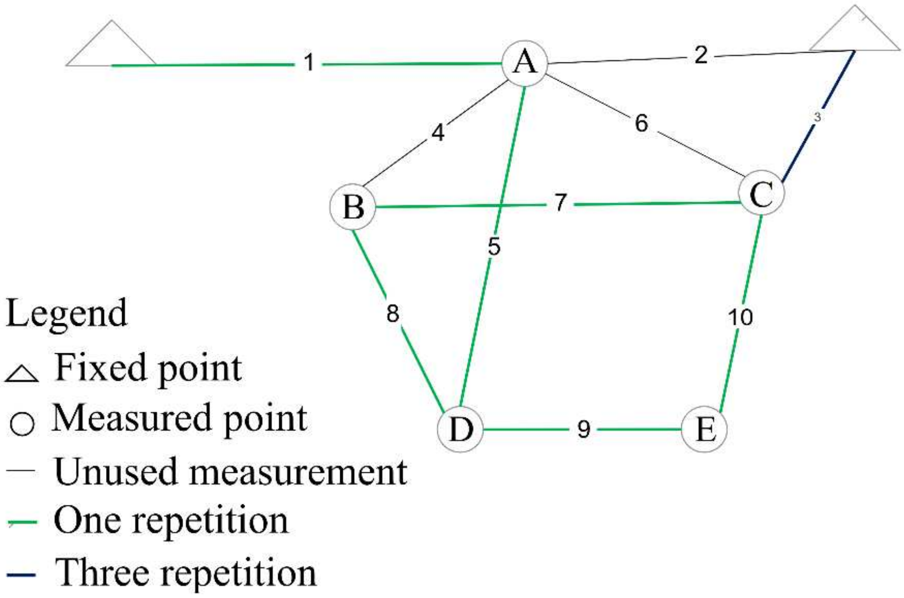

Fig. 1. Testing leveling network 1.

Brute force verification was made with a maximum number of tree repetitions. In the sum, there was calculated 1 048 576 variants (while calculating the optimizing algorithm only 91). Totally, there were 434 091 variations satisfying the target precision, of which 2 solutions with 9 measurements, 116 solutions with 10 measurements, 1086 solutions with 11 measurements, etc. It is obvious that the proposed optimization method is working correctly.

Another example of a simple network (Fig. 2) was tested with similar results. The standard deviation of the one measured height difference was chosen σ_h = 1 mm, required standard deviation of the height 1.1 mm. Optimization then runs according to described algorithm, results are shown in Fig. 2.

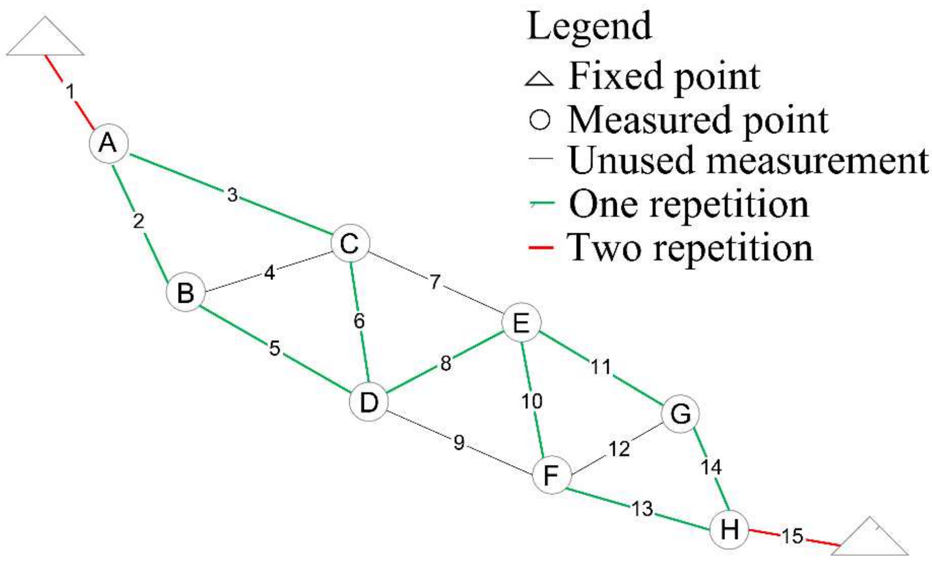

Fig. 2. Testing leveling network 2.

A result of the optimization consists of 13 measurements in total, according to the verification by the brute force with maximal 2 repetitions it is the optimal solution. Besides this one, three other combinations with 13 measurements exist.

Example of the application on the leveling network with different leveling lines lengths

The successful application of the proposed optimization method has been shown in the previous paragraph, but it was a simplified case. As the next example shown the case of different lengths of leveling lines between points is shown. The optimizing algorithm was applied in the same network as shown in Fig. 1, but leveling lines between the points were defined having a different length (in km, from 0.1 km to 0.6 km):

$$\boldsymbol{d} = (0.4 \quad 0.4 \quad 0.1 \quad 0.3 \quad 0.6 \quad 0.4 \quad 0.6 \quad 0.4 \quad 0.4 \quad 0.4)\,.$$

Thus, matrix $\boldsymbol{P_0}$ (matrix $\boldsymbol{A}$ is identical):

$$\boldsymbol{P_0} = \left(diag(\sigma_0^2 \cdot d_1 \quad \sigma_0^2 \cdot d_2 \quad \cdots \quad \sigma_0^2 \cdot d_m)\right)^{-1}.$$

The standard deviation of the one measured height difference was chosen $\sigma_0 = 1$ mm, required standard deviation of the height 0.42 mm. The result of the optimization (numbers of the repetitions of measurements):

$$\boldsymbol{n} = (2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 0 \quad 1 \quad 2 \quad 1 \quad 2)\,.$$

This represents an overall distance of measurement of 6.2 km. By brute force verification, it was found that there exist two better solutions, namely the one with the overall distance 6.0 km and five with the overall distance 6.1 km. The result obtained by the proposed optimization method is worse by 3.3%, it can certainly be considered as a very good result.

Optimizing method was also tested on the example of the more complicated network with 18 possible measurements (shown at Fig. 3). Levelling lines lengths (in km, from 0.1 km to 0.6 km):

$$\boldsymbol{d} = (0.1 \;\; 0.2 \;\; 0.1 \;\; 0.3 \;\; 0.3 \;\; 0.2 \;\; 0.5 \;\; 0.6 \;\; 0.2 \;\; 0.1 \;\; 0.6 \;\; 0.3 \;\; 0.6 \;\; 0.2 \;\; 0.3 \;\; 0.2 \;\; 0.2 \;\; 0.4)$$

The standard deviation of the one measured height difference was chosen $\sigma_0 = 1$ mm, required standard deviation of the height 0.5 mm.

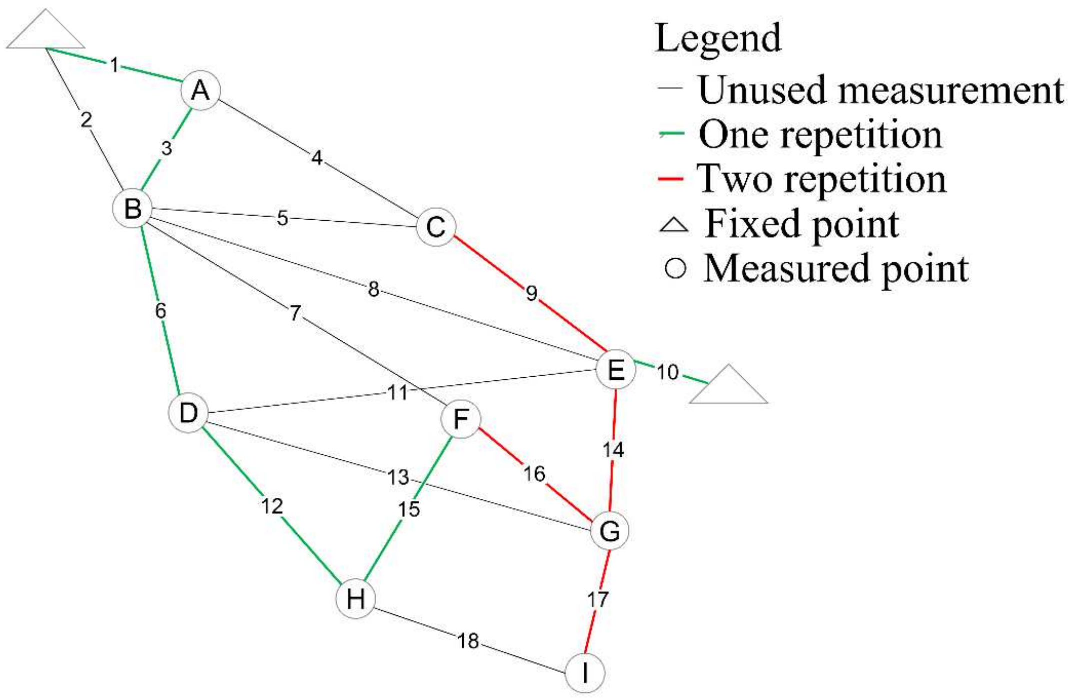

Fig. 3. Testing leveling network 3.

Result of the optimization (numbers of the repetitions of measurements):

$$\boldsymbol{n} = (1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 2\ 1\ 0\ 2\ 0\ 2\ 1\ 2\ 2\ 0).$$

This represents the overall distance of the leveling lines 3.0 km. The same solution by brute force verification was found as the best one (seven combinations in total).

Optimization method application example - the height geodetic network in the Jihlava underground catacombs

Catacombs in Jihlava originated from the beginning of the construction of the city as a system of cellars. In the 14th century, these cellars began to be deepened and linked by the corridor system; this trend lasted until the 16th century. Three floors were excavated in the granite and gneiss bedrock over time, the total length of the corridors was about 25 km.

The underground was briefly opened to the public from 1936 until World War II. It was made available again between the years 1946 and 1969. After this year construction works began in the town and the catacombs were closed for safety reasons. Then, the square began to succumb; therefore, most of the historically significant part of the basement was encased in concrete. A small part of the catacombs was open for public in the year 1990 again.

For the purposes of Jihlava's underground mapping, a quite complicated geodetic network was built. It is necessary to keep the measurement accuracy in the underground because access to the corridors is possible by one entrance only and thus an independent control is not possible. The connection is possible from one height point only (4005), the network is displayed together with the outline of corridors in Fig. 4.

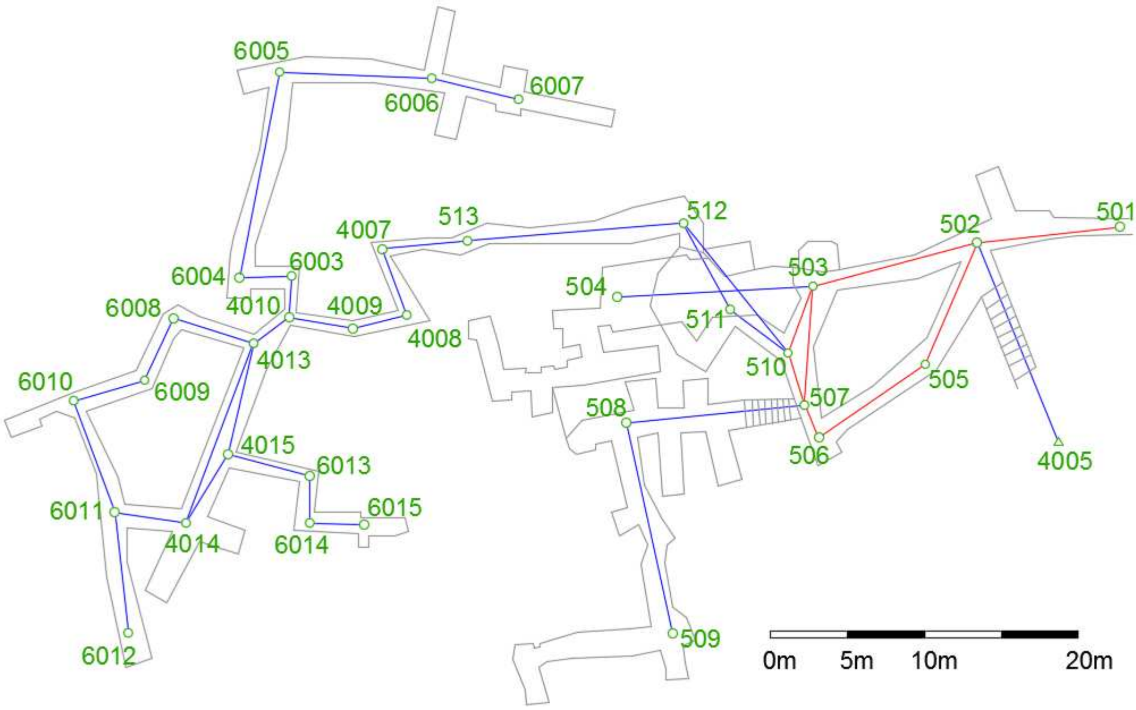

Fig. 4. Jihlava's underground height network (red lines – height difference measured by the geometric leveling, blue ones – by trigonometric method).

The required resulting accuracy of each determined height was set to 0.7 mm for optimization purpose. The optimization results are the numbers of repetitions of each measurement, which together with initial data are given in Table 1 and also in Fig. 5, where the numbers of repetitions are shown graphically (excluded measurements are represented by the dotted line).

The height differences measured by the method of geometric leveling from the center between the rods are marked by the asterisk in Table 1; others were measured by the trigonometric method. Standard deviations of the onetime measured height differences given in Table 1 were calculated based on the total station measurement precision (the standard deviation of the distance measurement is 1.5 mm, the standard deviation of the zenith angle measurement is 0.6 mgon; there was a premise of using the TS06 Leica instrument). The instrument was placed in the center between starting and ending point of the height difference. The same survey mark was used for both sides when measuring the trigonometric method, so the inaccuracy of height determination of the survey mark is irrelevant.

As can be seen from the beginning of this paragraph, two different methods of the height difference measurement (a trigonometric one and the geometric leveling from the center between the rods) with different precision were combined. Due to short sight lines, the total number of the measurements was chosen to be an optimized criterion.

The result shows that in the case of so high demanded precision, it is (logically) very substantial to determine the primary connection with adequate precision (the height difference 4005 - 502), which is necessary to measure seven times according to the optimization results. Due to such high number of the repetitions, a different and more precise measurement method should be considered. For the total of eight height differences, it is necessary to measure by two to four repetitions. These are longer, trigonometrically measured ones, which connect the entrance point with the far ones. Three measurements were entirely excluded; these are not needed to achieve the precision.

The results of the performed optimization are in compliance with basic principles of the geodetic measurements and the real situation too. Note that the high number of repetitions (higher than approximately five or six) signals the appropriateness of using other (more accurate) methods or adjustment of initial conditions (configuration of the network, precision premise).

Tab. 1. Input values and results of the optimization.

Meas. No.	**From point**	**To point**	**Standard deviation of the height difference** [mm]	**Number of repetitions**
1	4005	502	0.96	7
2	502	501	0.37	1
3*	502	503	0.10	1
4*	503	510	0.10	1
5*	510	507	0.10	0
6*	503	507	0.10	1
7*	507	506	0.10	1
8*	506	505	0.10	0
9*	505	502	0.10	1
10	503	504	1.03	4
11	507	508	0.56	2
12	508	509	0.38	1
13	510	511	0.50	0
14	511	512	0.21	1
15	510	512	0.41	3
16	512	513	0.37	3
17	513	4007	0.30	2
18	4007	4008	0.33	3
19	4008	4009	0.12	1
20	4009	4010	0.18	1
21	4010	6003	0.10	1
22	6003	6004	0.05	1
23	6004	6005	0.36	3
24	6005	6006	0.25	2
25	6006	6007	0.21	2
26	4010	4013	0.27	2
27	4013	6008	0.21	1
28	6008	6009	0.16	1
29	6009	6010	0.10	1
30	6010	6011	0.22	1
31	6011	6012	0.19	1
32	6011	4014	0.19	1
33	4014	4015	0.12	1
34	4014	4013	0.33	1
35	4013	4015	0.24	1
36	4015	6013	0.17	1
37	6013	6014	0.11	1
38	6014	6015	0.08	1

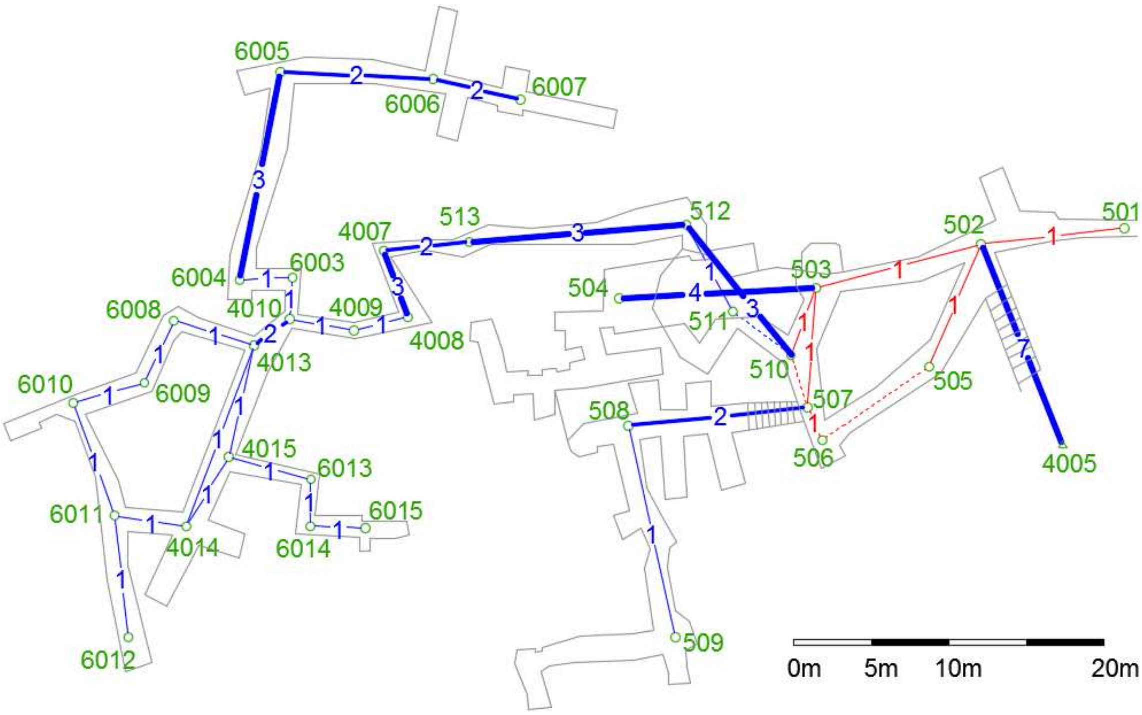

Fig. 5. Jihlava's underground height network after optimization with numbers of the repeated measurements.

Conclusions

A new method of optimization of geodetic measurements was designed and described, namely the number of repetitions of each measurement (second order design). The method is based on the selection of the worst point of the network in terms of optimized characteristics and the subsequent search for such measurement mostly improved this characteristic. This measurement is considered advantageous, and the number of its repetitions is increased by one. This procedure is repeated until the optimization criterion is fulfilled, usually the maximum value of the selected accuracy characteristic optimized for all points. The method was tested on a larger number of leveling networks in two basic variants, with the same standard deviation of individually leveled height differences and with a different standard deviation of individually measured height differences. Four specific cases are listed in the paper. In all cases, the method showed good results, the verification was conducted by brute force, by calculating all possible combinations.

It should be noted that the method is not mathematically defined as the optimum seeking method, it only gradually increased the number of measurements that in the given situation mostly improve the worst characteristic. The results are solutions in an optimal way, usually (especially in the case of the higher number of repetitions of measurement) there can be a better solution, but as is shown in the example, this solution differs in ones of percent. Another feature of the method is that the maximum number of repetitions of individual measurements in comparison to the "superior" combination is always smaller, which is preferable in the real geodesy. It is known that with a larger number of repetitions of measurement, the accuracy is not increasing with the square root of the number of repetitions. This is because of the residual systematic errors, and therefore the external precision is always lower (closer to reality) than the inner precision (calculated from the repetitions only).

Testing of the method and its behavior will continue, both in terms of the general behavior and in terms of application on other types of measurements, especially for terrestrial positioning and spatial geodetic networks measured by the total stations using angles and distances, as well as on other measurement methods such as e.g. global navigation satellite systems.

The main advantages of the method are the simplicity, compared to other methods, low demands on both of algorithmization and computational power, providing of integer and unambiguous results, and of course, according to our previous testing, it gives very good results. The method is also very easily adaptable to optimization of the different characteristics, and it is practically useful.

The last presented example of the height geodetic measurements shows the benefits of using the method in mining and tunneling measurements, where it enables measurement to be planned to meet the requirements on precision and at the same time be economically very close to the optimal one.

Acknowledgement: *The paper was written as a part of solving of the grant project SGS16/060/OHK1/1T/11 "Optimization of acquisition and processing of 3D data for purpose of engineering surveying and laser scanning".*

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