

## The influence of extraction speed on the value of the coefficient of subsidence rate

Piotr Strzałkowski<sup>1</sup>, Wiesław Piwowarski<sup>2</sup> and Roman Ścigala<sup>1</sup>

The presented paper discusses the issues of prediction of land surface transient deformation state caused by underground mining, using Budryk-Knothe theory, which is still the most popular solution in this field in Poland. The research results known from the literature and authors' own observations indicate the relationship between the value of the coefficient of subsidence rate "c" and the speed of extraction progress as well as the depth of underground mining works. Firstly in this paper, a short description of present knowledge state in this field has been presented. As the next part of this work, an empirical formula (9) has been proposed, that links the value of parameter "c" with the depth of mining works and the extraction speed. The formula (9) has been worked out on the basis of 9 cases, where extraction was led with the different speed at different depths. Then, the obtained formula was compared with some research results known from the literature. Presented calculations results are consistent with them, concerning analyses of surveys conducted above underground mining extraction carried out with high speed. This allows considering the proposed empirical relationship (9) as valid and useful for the practice of mining in the field of prediction of land surface transient deformation state. In the final part of the paper, some suggestions have been pointed, regarding the further possible development of the research toward increasing the accuracy of predictions of transient deformation state caused by underground mining.

**Key words:** prediction of post-mining transient deformations, geometric-integral theories, speed of extraction front.

### Introduction

As it is well known, underground mining generates many adverse changes to the natural environment as well as constructions in urbanized areas (Peng, 2015; Stojilkovic et al., 2014; Strzałkowski, 2010a; Mikulenska, 2007; Bell, Genske, 2001). Economic indicators of Polish hard coal mining industry inspire to seek new strategies for increasing the efficiency of underground mining with taking into account issues of minimization of its influence on the environment by using different methods, for example, described in (Wang et al., 2016). One of the possible solutions for improving efficiency is the concentration of extraction by, among others, increasing the speed of extraction advance, as the simplest solution – especially that the existing mining machinery has some power reserves. The impact of high-speed progress of mining extraction on the land surface deformation state was a subject of many publications (Dżegniuk, Sroka, 2002; Chudek, 2010; Hejmanowski, 1997; Kowalski, 1993, 2007; Smolnik, 2009; Sroka, 1974, 1999; Strzałkowski, 2010a). On the other hand, it should be noted that high extraction speed causes a significant increase in deformation rates over time, which negatively affects the construction of buildings and is an important factor causing mining damages.

A. Sroka (1999) concluded, basing on the experience of the Ruhr Coal Basin, that the cost of mining damages increases with the square of the extraction advance speed. The need for continuous extraction advance, without weekend breaks, was also pointed in his work. As an expression of this finding, there was a proposal of defining the categories of mining areas basing on the subsidence rate and its acceleration.

Extraction speed should, of course, be adopted to specific conditions, with particular emphasis on the resistance of surface building objects to mining damages. It is worthy to mention that the research results published in (Kratzsch, 1983) indicate (based on the experiences of the Ruhr area) that the extraction speed close to 5 m/day did not result in typical building objects damages, while in case of objects with low resistance, safe was the speed of 3 m/day.

In Poland, extraction is usually led with a moderate speed at about 3–5 m/day. Only in the case of "Staszic" coal mine, the speed of extraction was significantly greater – up to 20m/day (Kowalski, 1993; Kowalski, 2007). Presently, in the Lublin Coal Basin, "Bogdanka" mine operates with extraction speed at about 10 m/day, while the total longwalls advance reaches several kilometres.

In the USA ("Twentymile" mine, Colorado) average extraction speed amounts about 40 m/day, but maximum reaches up to 80 m/day (Smolnik, 2009). German experiences come from the extraction speed at the maximum level of 20m/day.

<sup>1</sup> Piotr Strzałkowski, Roman Ścigala, Silesian University of Technology, Faculty of Mining, Safety Engineering and Industrial Automation, Akademicka 2, 44-100 Gliwice, Poland. piotr.strzalkowski@polsl.pl, roman.scigala@polsl.pl

<sup>2</sup> Wiesław Piwowarski, AGH University of Science and Technology, ul. Mickiewicza 30, 30-059 Krakow, Poland

All analyses concerning deformation state changes over time with consideration of high-speed extraction must be founded on the mathematical models verified by surveys. Most of such models based on the proposed by S.Knothe solution of a differential equation, expressing the law of limited growth (Knothe, 1953):

$$\frac{dw(t)}{dt} = c \cdot (w_k(t) - w(t)) \quad (1)$$

where :

- $w_k(t)$  – final (asymptotic) value of subsidence,
- $w(t)$  – transient value of subsidence,
- $c$  – the coefficient of subsidence rate (often called: „time factor”).

S.Knothe gave the solution of equation (1) for a theoretical case of instantaneous extraction of elementary field (with the assumption:  $w_k(t,x)=w_k=const.$ ), with the initial condition  $t=0 \Rightarrow w(t)=0$ , as follows:

$$w(t) = w_k \cdot (1 - e^{-ct}) \quad (2)$$

In work (Piwowarski et al., 1995) there was pointed, that for given mining–geological conditions:  $w_k = const$  and  $c = const$ ,  $w(t)$  – trajectory in the  $z$  direction, the subsidence rate is estimated according to:

$$\frac{dw(t)}{dt} = c \cdot w_k \cdot e^{-ct} \quad (3)$$

for  $t \rightarrow 0$  one has the speed of subsidence expressed as:

$$\frac{dw(t)}{dt} = c \cdot w_k \Rightarrow max \quad (4)$$

In practice, subsidence process  $w(t)$  cannot start with its maximum rate, as it goes from the equation (4), so it is necessary to take into account the condition:  $w_k=w_k(t) \neq const.$  Such assumption means that final subsidence  $w_k$  varies along with extraction front development. With this assumption, the solution of the inhomogeneous differential equation (1), with the condition:  $w(t=0)=0$  has the form:

$$\left. \begin{aligned} w(t) &= A \cdot e^{-ct} + c \int_0^t w_k(\tau) d\tau \\ &= e^{-ct} \left[ A + c \int_0^t w_k(\tau) \cdot e^{c\tau} d\tau \right] \\ w(t) &= c \cdot e^{-c \cdot t} \cdot \int_0^t w_k(\tau) \cdot e^{c \cdot \tau} d\tau \end{aligned} \right\} \rightarrow A = 0 \quad (5)$$

where:

- $\tau$  – independent time variable.

Assuming that extraction field is of rectangular shape and active extraction edge moves with constant speed  $v$ , one obtains, according to S.Knothe theory, final subsidence expressed by the formula:

$$w_k(t) = \frac{w_{max}}{r^2} \int_{x_0}^{x_1} \int_{y_1}^{y_2} \exp \left[ -\pi \left( \frac{\xi^2(\tau) + \eta^2}{r^2} \right) \right] d\xi(\tau) d\eta \quad (6)$$

where:

- $w_{max}$  – maximum possible value of subsidence,
- $r$  – parameter (so-called the radius of major influence range),
- $y_1, y_2$  – space coordinates describing the size of the extraction field in the  $Y$  direction,
- $x_0$  –  $x$  coordinate of longwall’s starting roadway,

$x_t$  –  $x$  coordinate describing the location of active (moving) longwall edge at time  $t$ :  $x_t = x_0 \pm v \cdot t$ ,  
 $v$  – the speed of extraction,  
 $t$  – the extraction lasting time,  
 $\zeta, \eta$  – independent space variables,  
 $\tau$  – independent time variable.

It is necessary to point here, that parameter  $c$  cannot be measured directly. It is important, therefore, to plan the optimal location of displacement field sensors for its identification purposes. Such analysis should concern space kinematics of the process. The process model in the  $R^{3+1}$  space is a formal structure involving four independent variables, and the description is usually given by employing differential equations. Classical techniques of identification are not effective here, which goes from the necessity of recalculation of the information matrix.

### Characteristics of the problem and research state

Aiming at determining the deformation state of rock mass and land surface, several models have been worked out for predicting the displacement of transformed subspace and procedures of displacement field observation have been developed. Solutions should be distinguished here: based on the S. Knothe model and other concepts described, for example, in works (Piwowski et al., 1995; Sroka, 1999). Other works which come from developing equation (1) with the assumption that  $c \neq \text{const.}$  were presented, among others, by P. Strzałkowski (Strzałkowski, 1998) and J. Białek (Białek, 1991), which additionally modified function describing asymptotic deformation state by introducing two radii of influence range. Interesting discussion on this idea is given in work (Orwat, Mielimąka 2016). There is also a solution that bases on the phenomenon of diffusion (Piwowski, 1989). In the 70s, the theory of dynamical systems was considered, which expanded the possibilities of describing the phenomena encountered in nature (Piwowski et al., 1995). In a transient process, the value of a given variable at the time  $t$  depends on the value of this variable at the time  $(t-1)$ , and also determines its value at the time  $(t+1)$ . The same variable can be treated as both – a cause and consequence. Such approach stays in contradiction with the traditional way of unidirectional thinking about the relationship between cause and effect and creates a kind of causal loop.

The results of analyses performed by different authors (Hejmanowski 1997; Kowalski 1993, 2007; Sroka, 1974) point, that in case of extraction led with high speed, not always significant decrease in values of deformation indices is observed, as it was judged earlier. Discussions of survey results from Niederberg coal mine presented in (Sroka, 1999) show the ratio of transient tilt decrease in relation to its maximum values at the level of 0.95, which refers to so-called "dynamic variable"  $u$  ( $u=c \cdot r/v$ ) value of about 7.5. The coefficient of deformation reduction –  $f_B$ , one should understand as :

$$B_{\max} = f_B \cdot B_{\max}^k \quad (7)$$

where:

$B_{\max}$  – maximum value of given deformation index in a transient state,  
 $B_{\max}^k$  – maximum value of the given deformation index in the asymptotic (final) state.

The same author in work (Sroka, 1999) gives the following reduction coefficients, characteristic for central part of Upper Silesian Basin:

- for tilt:  $f_t = 0.60$ ,
- for horizontal extensive strain:  $f_{e+} = 0.50$ ,
- for horizontal compressive strain:  $f_{e-} = 0.28$ .

One can find similar values in (Kowalski, 2007):

- for tilt:  $f_t = 0.51 \pm 0.12$ ,
- for horizontal extensive strain:  $f_{e+} = 0.50 \pm 0.20$ ,
- for horizontal compressive strain:  $f_{e-} = 0.84 \pm 0.15$ .

In work (Sroka, 1999) Author states, that on the basis of mentioned above findings, it may be presumed, that relation between values:  $\{c, r, v\}$  exists. It was further found, that the impact of the extraction speed on the parameter characterizing influence range leads to different profiles of asymptotic troughs for the same extraction field mined out with different speed. It was also noted that the value of parameter  $c$  depends on the speed of extraction, recalling the results of (Sroka, 1974).

In the framework of this work, the empirical function which binds values of parameter  $c$  with the extraction speed –  $v$  has been worked out. Next, the results of transient deformation state calculations obtained with using the proposed formula along with the analysis of their agreement with values known from the literature have been presented.

### An attempt to establish the relation between parameter $c$ values and the speed of extraction

First, the identification of S.Knothe parameters, on the basis of asymptotic subsidence trough profile taken from surveys led in one of Upper Silesia Basin coal mine had been performed. The extraction was led at the depth  $H=500\text{ m}$  with caving. The thickness of the extracted deposit was  $g=1.7\text{ m}$ . The sketch of the mining field against observing line location is shown in Fig.1. The following values of parameters were identified: coefficient of roof control:  $a\approx 0.6$ , parameter describing influence dispersion:  $tg\beta\approx 2.0$ . Next, using these parameters, values of  $c$  were determined on the basis of trajectories of chosen observing points with using dedicated software (Ścigała, 2008). An exemplary course of subsidence over time for observing point No 17 is presented in Fig.1, along with the obtained value of parameter  $c$  and percentage fit error.

In Table 1 the values of  $c$  parameters are given, obtained on the basis of survey for different extraction depths –  $H$  and the speed of extraction front –  $v$ . Values of  $c$  for extraction speeds of  $v=8\text{ m/day}$  and  $v=14.2\text{ m/day}$  have been taken from works (Kowalski, 1993; Sroka, 1999). In Table 1, the values of time factor  $c_{calc}$  have been added, that were obtained by calculating them from the empirical formula (9) presented below.

The empirical formula has been worked out with the assumption, that the dependence between extraction speed  $v$ , its depth  $H$  and the value of parameter  $c$  exists.

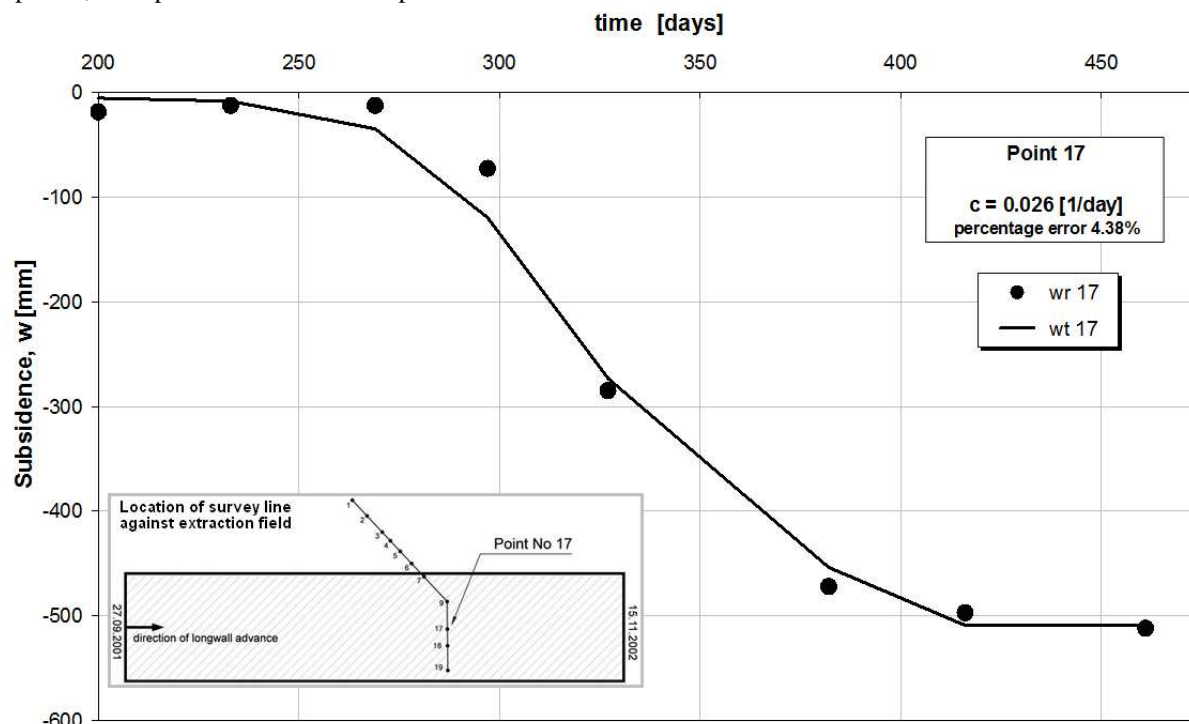


Fig.1. The exemplary course of subsidence over time – measurement  $w_r$  and calculation results with using S.Knothe model –  $w_t$

Tab. 1. Obtained values of parameter  $c$  for different considered cases of extraction

No	H	v	c	$c_{calc}$
	[m]	[m/day]	[1/day]	[1/day]
1	840	14.20	0.040	0.045
2	500	5.25	0.026	0.030
3	500	4.60	0.033	0.027
4	495	8.00	0.052	0.044
5	440	1.50	0.011	0.012
6	440	2.40	0.014	0.018
7	360	1.30	0.015	0.012
8	330	2.60	0.022	0.024
9	310	2.00	0.023	0.020

The following regression formula has been used for estimation:

$$c = f(\{\Xi\}, H, v) \quad (8)$$

where:  $\{\Xi\}$  – vector of model parameters.

As an effect of regression analysis performed using Mathematica software, the following formula has been obtained:

$$c = 1.463 \cdot \left(\frac{v}{H}\right)^{0.847} \quad (9)$$

For the formula (9), the value of determination coefficient amounts to  $R^2=0.976$ . In Fig.2 the course of parameter  $c$  value according to formula (9) is presented with a thick black line, as well as source data for estimation (Table 1) – dots. Additionally, the confidence intervals at the level of 0.95 are marked on the graph as greyed bands.

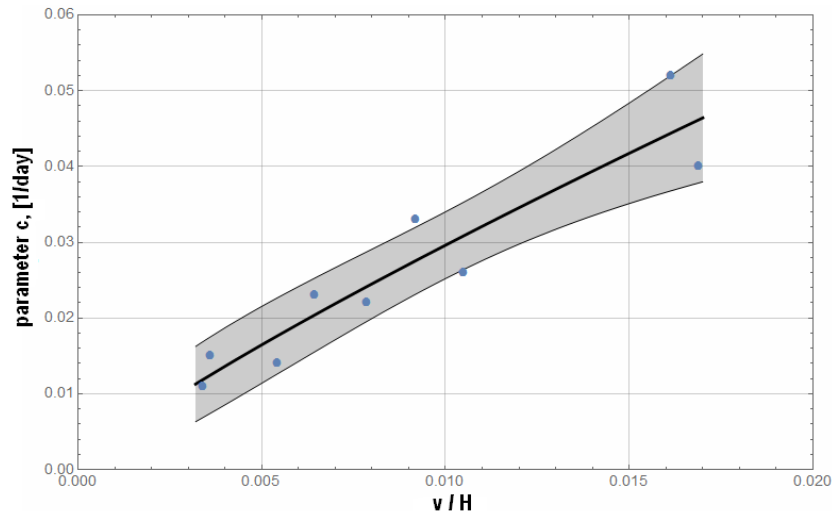


Fig. 2. The graph of empirical formula (9) with confidence intervals at the level of 0.95

### The analysis of post-mining deformations changes over time with using an obtained regression formula

For analysis purposes of formula (9) practical use, some testing calculations were performed. Theoretical case of single longwall extraction was assumed, with the length of 250m and the total advance of 1000m. It was also assumed that maximum subsidence equals to  $w_{max} = -a \cdot g = 1 \text{ m}$ . Extraction was located at a depth of  $H=600 \text{ m}$ ; coal seam is horizontal and planar. Different speeds of extraction were considered  $v$ : 3, 5, 7, 10 i 14 [m/day]. The following values of S.Knothe model parameters were used:

- Parameter  $tg\beta = 2.0$  (typical for Upper Silesia Basin),
- Coefficient of proportionality in the Awiershin's relationship  $B = 0,32r$ .

Values of parameter  $c$  were calculated by using formula (9), proposed in this paper. So, according to this formula, they were different for diverse extraction speeds – not fixed as it was assumed in work (Strzałkowski, 2010b). Used values of parameter  $c$  are presented in Table 3.

For calculation point located at the surface level exactly above the centre of the longwall, the values of deformation indices were calculated, with using DEFK–Win software (Ścigała, 2008) – Fig.3. Calculations were performed in a special way – it was a simulation of extraction advance in the eastern direction, assuming the start of extraction on January 1<sup>st</sup>, 2017 and 5–day step of extraction advance. As an effect, the distributions of deformation indices over time (along with extraction development) were obtained.

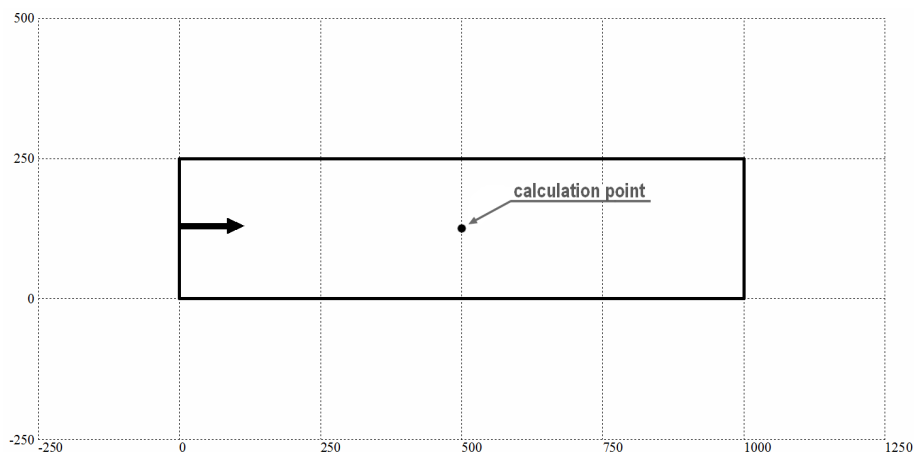


Fig. 3. The sketch of extraction location against calculation point

The following deformation indices were calculated : subsidence  $w$ , maximum tilt  $T_{max}$ , extreme horizontal strain  $E_{max}$ . The course over time for these deformation indices are shown in figures 4–6. In Table 3, their extreme values are presented with subsidence rate value  $dw/dt$  added. Every row in the table presents deformation indices for different value of parameter  $c$  and “dynamic variable”  $c \cdot r/v$ .

Tab. 3. Extreme values of deformation indices obtained for considered extraction speeds

v [m/day]	c [1/day]	$\frac{c \cdot r}{v}$	$T_{max}$	$E_{max} +$	dw/dt
			[mm/m]	[mm/m]	[mm/day]
3	0.016	1.600	1.57	0,66	4,68
5	0.025	1.500	1.52	0.63	7.58
7	0.034	1.457	1.50	0.62	10.48
10	0.046	1.380	1.47	0.61	14.50
14	0.069	1.307	1.42	0.56	19.68

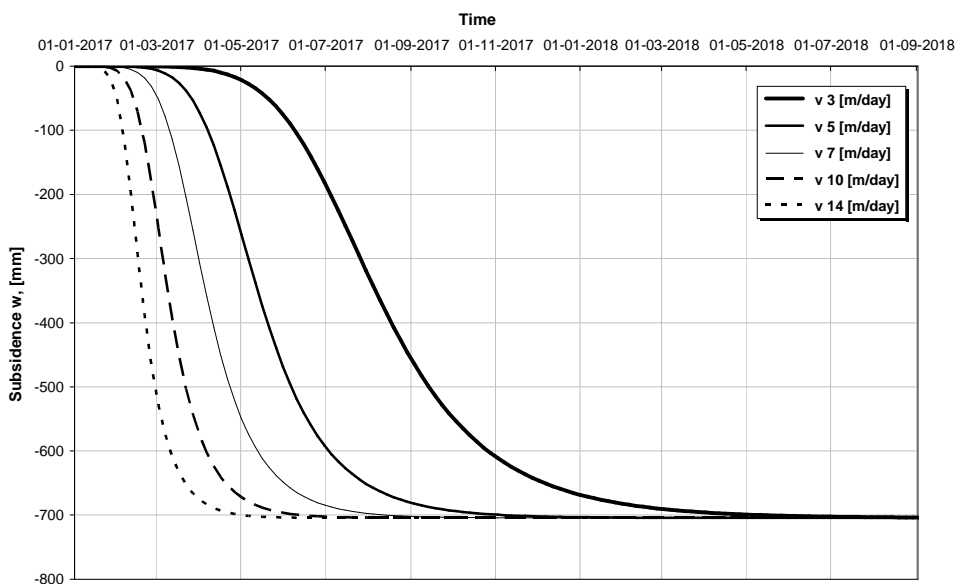


Fig. 4. The course of subsidence over time for different extraction speeds

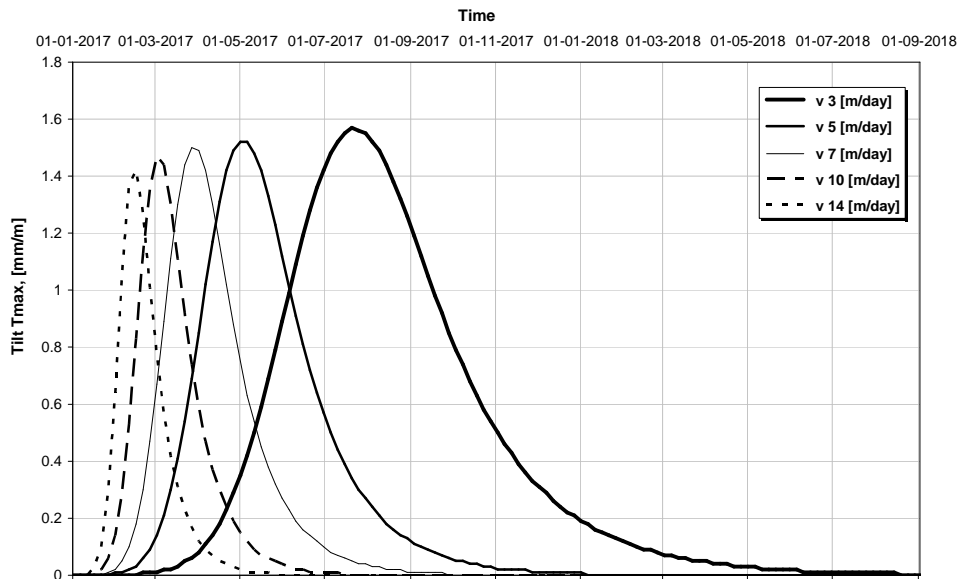


Fig. 5. The course of maximum tilt over time for different extraction speeds

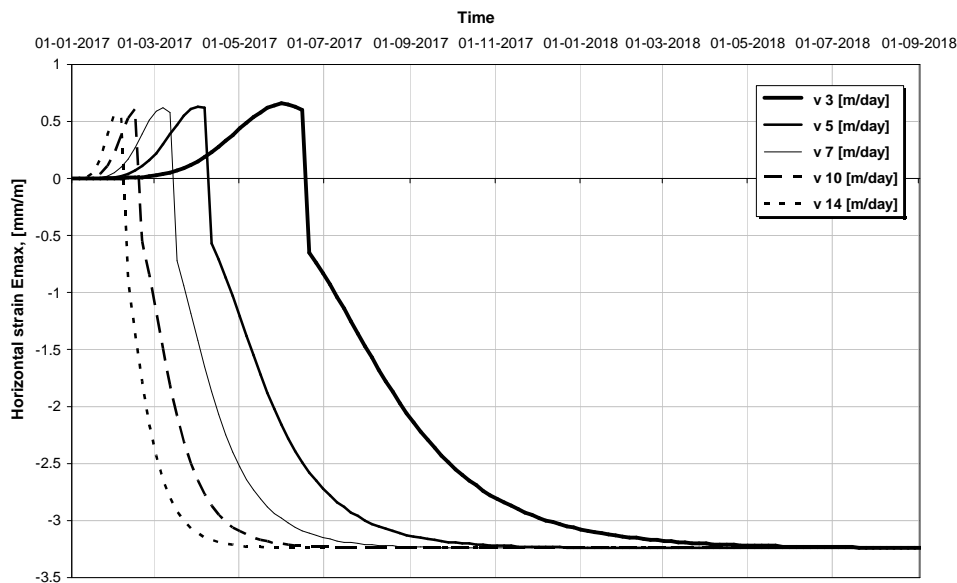


Fig. 6. The course of extreme horizontal strain over time for different extraction speed

As it is apparent from the foregoing drawings and Table 3, an increase in extraction speed does not reduce significantly – in the light of the results of calculations – the values of the deformation indices relative to the predetermined minimum speed of extraction – 3 m/day. It can be concluded, that increasing the extraction speed did not result – in the light of presented calculations results – in a significant reduction of the deformation indices. On the other hand, the subsidence rates  $dw/dt$  were significantly greater, from approximately 5 mm/day (for  $v = 3$  m/day) to nearly 20 mm/day (for  $v = 14$  m/day).

Further calculations were carried out for 33 points located along a straight line on the surface above the same extraction field – Fig. 7. Calculation points were positioned at mutual distances of 25 m. The calculations were performed for the last day of the given extraction period with the same set of different speeds as used above. The profiles of transient subsidence troughs and asymptotic one are shown in Fig. 8. In Table 4, the maximum values of instantaneous and final: tilt –  $T_{max}$  and horizontal strain –  $E_{max}$  (positive and negative).

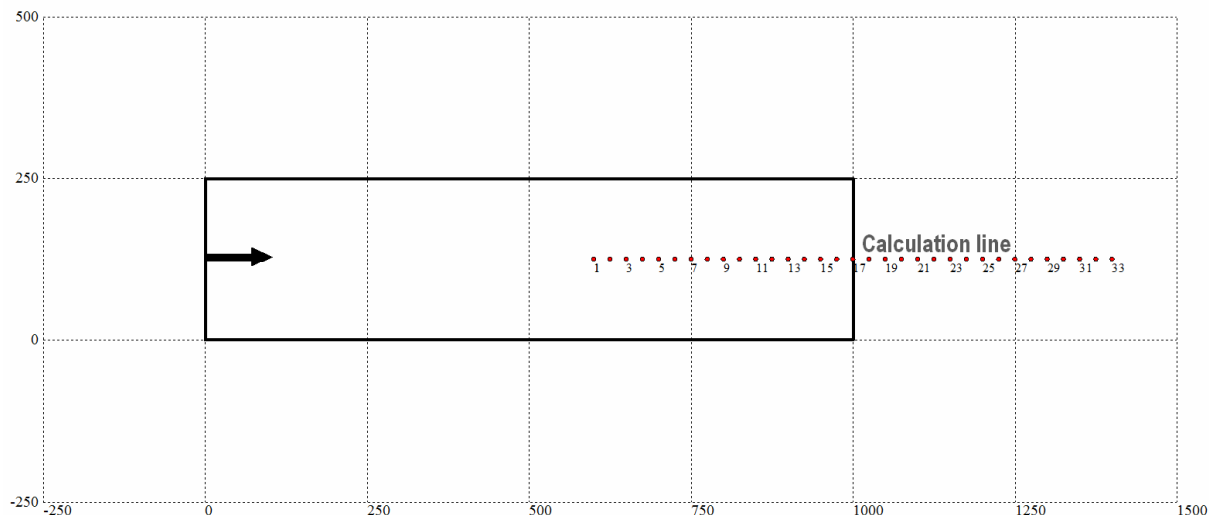


Fig. 7. The sketch of extraction location against calculation line

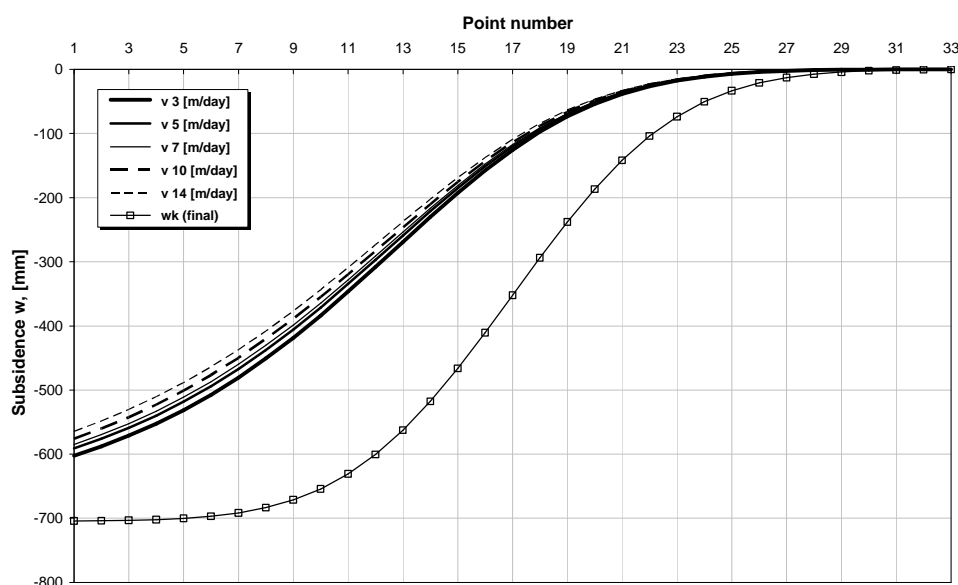


Fig. 8. The profiles of transient subsidence troughs obtained with different extraction speed and the profile of asymptotic trough –  $w_k$

Tab. 4. Maximum values of transient and asymptotic values of considered deformation indices and reduction coefficients

v [m/day]	$T_{\max}$ [mm/m]	$E_{\max+}$ [mm/m]	$E_{\max-}$ [mm/m]	Reduction coefficients		
				$f_T$	$f_{E+}$	$f_{E-}$
3	1.56	0.65	-2.77	0.664	0.570	0.855
5	1.52	0.63	-2.72	0.647	0.553	0.840
7	1.50	0.62	-2.69	0.638	0.544	0.830
10	1.47	0.60	-2.65	0.626	0.526	0.818
14	1.43	0.58	-2.60	0.609	0.509	0.802
asymptotic values	2.35	1.14	-3.24	1.000	1.000	1.000

In Table 4, the coefficients of deformation reduction  $f_i$  are given – for tilt and horizontal strain (compressive and tensile). It is worthy to emphasize here, that there is an agreement of their values with those cited from publications in the introductory part of the paper. The agreement is clear when referring to A. Kowalski work (1993), whereas some differences one can find in relation to work (Sroka, 1999), regarding compressive strain.



### Suggested directions for further research

The analyzed problem of mining displacement process approximation by solutions of differential equations in the form (1) needs to assess whether the process has the trajectories of locally finite  $p$ -variation. Equation (1) was examined separately as homogeneous and inhomogeneous, as they require different solution definitions as well as various parameters. It can be seen that empirical trajectories of displacement are equivalent to the so-called problem with stochastic disturbance. It seems that it would be advisable to consider the employment of the deterministic equivalent for the stochastic inclusion for a convex, time-varying set of observations of the process.

It can be proved (Skorochod, 1961), that for any continuous functions:  $y$ ,  $y_0$  and  $x_0$ , functions exist:  $x$ ,  $k$  such that:

$$Ax_t = y_t + k_t \geq 0 \quad \text{for } t \in R^+ \quad (10)$$

$k_0 = 0$ ,  $k$  is non-decreasing

$$\int_0^t x_s dk_s = 0 \quad t \in R^+ \quad (11)$$

Function  $k$  is given as follows:

$$k_t = \sup_{s \leq t} (y_s)^- \quad (12)$$

It can be proved, that above-mentioned lemma stays true if instead of continuous functions, one employs the C adl ag functions (right-continuous with left limits – RCLL) of type:

$$x_t = y_t + k_t \geq l_t; \quad t \in R^+ \quad (13)$$

So, for every  $t \in R^+$  we have:

$$\left. \begin{aligned} x_t &= \max \left\{ \min \left| x_t^- + \Delta y_t \right|, l_t \right\} \\ k_t &= \max \left\{ \min \left| k_t^-, u_t - y_t \right|, l_t - y_t \right\} \end{aligned} \right\} \quad (14)$$

Hence the problem arises of existence and uniqueness of solutions to deterministic equations of the form:

$$x_t = x_0 + \int_0^t f(s, x_s^-) db(s) + \int_0^t g(s, x_s^-) da(s) + k_t; \quad t \in R^+ \quad (15)$$

where integral with respect to  $a$  is a generalized Riemann–Stieltjes integral.

A solution to the problem (15) has a finite  $p$ -variation for any function  $y$  of locally finite  $p$ -variation, which goes from the equation (16):

$$\left| \nabla_x g(t, x) \right| \leq C_{g,N} \quad (16)$$

where:  $C$  – is a constant

The solutions of the equation (15) one can approximate by series of iterations in the form:

$$(x_0, k_0) = ESP(x_0, l, u)$$

This issue seems to be important, prompting the authors to undertake further research in this field.

## Conclusions

Presented in this paper recognition of literature, carried out calculations and analysis of their results entitle to formulate the following statements and conclusions :

1. It should be considered, both in the view of findings known from the literature as well as authors' research, that value of the coefficient of subsidence rate  $c$  is a function of, among others, the speed of extraction. One possible way of taking this variability into account is the formula (9) presented in this work.
2. A linear relationship between the extraction speed and coefficient of subsidence rate  $c$  entails coherence in the form of obtaining in predictions the transient subsidence troughs with the same maximum tilt for different extraction speeds. Therefore, it was decided to seek empirical formula (9), basing on a power function.
3. The results of calculations contained in this work confirm the validity of the proposed formula, as obtained transient values of deformation indices stay in similar relations to their final maximum values, as those known from the literature.
4. As a part of the work, the need has been noted for research related to the estimation of whether the mining displacement process described by solutions of the differential equation (1) is the process having the trajectories with locally finite  $p$ -variation. This issue authors intend to devote more attention in a separate publication.

## References

- Bell, F.G., Genske, D.D. (2001) The influence of subsidence attributable to coal mining on the environment, development and restoration: Some examples from western Europe and South Africa. *Environmental and Engineering Geoscience* 7(1), pp. 81-99.
- Białek J. (1991) Opis nieustalanej fazy obniżenia terenu górniczego z uwzględnieniem asymetrii wpływów końcowych. *Zeszyty Naukowe Pol. Śl., s. Górniczo, nr 194, Gliwice.*
- Chudek M. (2010) Mechanika górotworu z podstawami zarządzania ochroną środowiska w obszarach górniczych i pogórnich. *Wydawnictwo Politechniki Śląskiej, Gliwice.*
- Dzeganuk B., Sroka A. (2002) Prędkość i przyspieszenie procesu osiadania w aspekcie przerw w prowadzeniu eksploatacji. *Prace Naukowe GIG. Seria Konferencje nr 41. Problemy Ochrony Terenów Górniczych, Katowice.*
- Hejmanowski R. (1997) Optymalizacja prędkości eksploatacji górniczej i przerw eksploatacyjnych – oprogramowanie komputerowe. *Conference Proceedings: IV Dni Miernictwa Górniczego i Ochrony Terenów Górniczych, Ryto.*
- Knothe S. (1953) Wpływ czasu na kształtowanie się niecki osiadania. *Archiwum Górniczo i Hutniczo, Vol 1, Issue 1, Kraków.*
- Kowalski A. (1993) Deformacje terenu powstałe w wyniku szybkiej eksploatacji górniczej. Nowe doświadczenia. *Conference Proceedings: II Dni Miernictwa Górniczego i Ochrony Terenów Górniczych, Ustroń.*
- Kowalski A. (2007) Nieustalone górnicze deformacje powierzchni w aspekcie dokładności prognoz. *Prace naukowe Głównego Instytutu Górniczo, nr 871, Katowice.*
- Kratzsch H. (1983) Mining Subsidence Engineering. *Springer-Verlag, Berlin, Heidelberg, New York.*
- Mikulénka V. (2007) Influence of the mining works on the roads in Ostrava-Karviná's district. *Acta Montanistica Slovaca, Vol.12, issue 3, p.465-470.*
- Orwat, J., Mielimaka, R. (2016) Approximation of average course of measured curvatures of mining area with reference to their forecast values by Bialek's formulas, *Proceedings of 14th International Conference of Numerical Analysis and Applied Mathematics, pp. 130003-1 - 130003-4, ISBN 978-0-7354-1538-6; AIP Conference Proceedings, Volume: 186, Rhodes (Greece).*
- Peng S. (2015) Topical areas of research needs in ground control - A state of the art review on coal mine ground control. *International Journal of Mining Science and Technology* 25(1), pp. 1-6.
- Piwoarski W. (1989) Opis przemieszczeń pionowych aktywnego procesu deformacji górotworu w warunkach eksploatacji górniczej. *Zeszyty Naukowe Akademii Górniczo – Hutniczej, Vol. Geodezja, Issue No 106, Kraków.*
- Piwoarski W., Dzeganuk B., Niedojadło Z. (1995) *Współczesne teorie ruchów górotworu i ich zastosowania. Wydawnictwo AGH, Kraków.*
- Skorochod A. V. (1961) Stochastic equations for diffusion processes in a bounded region. *Theory of Probability and its Applications, Vol. 6 Issue 3, 264-274. Society for Industrial and Applied Mathematics, Philadelphia.*
- Smolnik G. (2009) Najnowsze trendy w badaniu interakcji górotworu i ścianowej obudowy zmechanizowanej. *Maszyny Górnicze 4/2009, 31-37. Quarterly by KOMAG Mining Mechanization Centre, Gliwice.*

- Sroka A. (1999) Dynamika eksploatacji górnictwa z punktu widzenia szkód górniczych. *Polska Akademia Nauk, seria Studia, rozprawy, monografie, nr 58, Kraków.*
- Sroka A. (1974) Wpływ postępu frontu eksploatacji górnictwa na kształtowanie się wskaźników deformacji górotworu. *Phd thesis, AGH Univ. of Science and Technology, Kraków.*
- Stojiljkovic E., Grozdanovic M., Marjanovic D. (2014) Impact of the underground coal mining on the environment. *Acta Montanistica Slovaca, Vol.19, issue 1, p.6-14.*
- Strzałkowski P. (1998) Model nieustalonych przemieszczeń pionowych górotworu w obszarze objętym oddziaływaniem eksploatacji górnictwa. *Zeszyty Naukowe Pol. Śl., s. Górnictwo, nr 237, Gliwice.*
- Strzałkowski P. (2010a) Zarys ochrony terenów górniczych. *Wydawnictwo Politechniki Śląskiej, Gliwice.*
- Strzałkowski P. (2010b) Forecasts of mine-induced land deformations in consideration of the variability of the parameter describing the process kinematics. *Archives of Mining Science. Volume 55. Issue 4, Kraków.*
- Ścigała R. (2008) Komputerowe wspomaganie prognozowania deformacji górotworu i powierzchni wywołanych podziemną eksploatacją górnictwa. *Wydawnictwo Politechniki Śląskiej, Gliwice.*
- Wang X., Zhang D., Sun C., Wang Y. (2016) Surface subsidence control during bag filling mining of super high-water content material in the Handan mining area. *International journal of oil gas and coal technology Vol. 13, issue 1, p. 87-102 DOI: 10.1504/IJOGCT.2016.078049*