Copper and Aluminium as Economically Imperfect Substitutes, Production and Price Development

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Abstract

Copper and aluminium prices have long been influenced mainly by non-renewable resources and the industry's widespread use of copper and aluminium for their desired properties. Metal commodities are irreplaceable for the industry of developed countries, and their shortage in the covid times also increases the price and consequently the price of products made from them. As copper ore stocks continue to decline, suitable substitutes should be sought. The paper discusses the potential of copper substitution by aluminium and subsequently the development of prices and production of copper and aluminium, including a prediction about the future development. Research data were obtained from Market. business insider (2021) and Investing.com (2022) converted to time series. The price is shown in US dollars per tonne and the production value in millions of tonnes. Development data were processed using artificial intelligence and recurrent neural networks, including the Long Short Term Memory layer. Neural networks, as such, have great potential to predict these types of time series. The annual copper and aluminium production data were processed using a regression function. Neural networks could not be used due to the smaller data range. The results show that the 1NN30L neural network with an LSTM layer and considered a 30-day delay is the most suitable network for forecasting future copper prices, and the 3NN30L neural network with an LSTM layer and considered a 30-day delay is the most suitable network for forecasting future aluminium prices. The forecast has confirmed that the price of copper will fall at the end of 2021, and the trend will be constant in the next planned period. Aluminium will also fall sharply at the end of 2021; at the beginning of 2022, the price level is predicted to rise to that of 30 October 2021, and thereinafter the trend will be almost constant. Research has confirmed that copper and aluminium may be imperfect substitutes in some respect, but they can generally be considered complementary. Copper mining has stabilised in recent years, but aluminium production has increased significantly in the last decade, and it can be expected to grow in the near future.

Keywords

Copper price, aluminium price, substitutes, time series, forecast, future development, neural networks, production

Introduction

The economical use of rare production resources depends on proper decision-making, use and distribution. Demand for minerals has grown in recent years due to the developing global economy and higher product intensity (Harmsen et al., 2013; Bilan et al. 2017). However, the unfavourable consequence of this growing demand for metals is the depletion of resources. Even the slightest saving with resources can prolong their depletion. However, it will happen one day that the continuing need for the demanded and rare raw materials will make them unavailable to humankind (Ahmad et al. 2022; Neagu et al. 2022). It is nothing new that the fear of depletion is a persistent problem in the management of natural resources (Castillo & Eggert, 2020; Simionescu et al. 2022). So how long would these resources last? Estimates and speculations of concern may refer to decades (Villena & Greve, 2018). On the contrary, Singer (2017) does not look for a specific date of depletion; the demand for copper is not governed by time but by the population and its income. However, higher demand can be expected due to the increase in population, which comes closer to the exhaustibility of resources. Henckens (2021) provides indicative estimates of the thirteen mineral resources that will be available, provided that the world's population size stabilises at ten billion people. Tilton et al. (2018) identified a number of mineral commodities that are most at risk of resource depletion at the maximum production over the next 50 years, which raises further concerns about depletion. According to Ponomarenko et al. (2021), the depletion of mineral resources will have a special impact on economies that are based on these resources.

Mining is limited by the number of resources present in the bark. Supplying global demand for metallic raw materials requires known mineral deposits in the earth's crust that contain the required quantity of metals in excess of current ore reserves (Douce, 2016). Gradual ore depletion is characterised for all minerals (Kuipers et al., 2018). It is unlikely that undiscovered mineral deposits would exist in sufficient numbers to meet the expected global demand for the rest of this century (Douce, 2016).

Currently, the primary source of information on reserves and resources is the United States Geological Survey (USGS), which collects extensive information from mines and deposits worldwide and for all mineral commodities.

Some mine operations, especially in the United States (the U.S.), have even been suspended due to COVID-19 positive workers, as well as discontinued due to restrictions imposed by the government. Thus, U.S. production has fallen by an estimated 13% at a minimum (USGS, 2021).

Unfortunately, there is no systematic and comprehensive overview of substitutability for any precious metal, or in our particular case, for copper (Reijnders, 2021).

In prehistoric times, copper was probably the first metal used by man, and it is the third most used metal after iron and aluminium nowadays. In the past, copper used to be the second largest use, but it is now often replaced by aluminium due to its high price (Campbell, 2008). The future availability of copper and the basic satisfaction of ever-increasing demand is becoming a bigger challenge (Hunt et al., 2021). This concern leads to the development of forecasts and estimates for future copper demand. Subsequently, Elskaki et al. (2016) developed a copper demand and supply model related to energy consumption by 2050. Using regression and stock dynamics, Schipper et al. (2018) developed copper models with a demand estimate until 2100. They are based on the fact that copper reserves are stable and will gradually be consumed. Both studies outlined how many years would remain until they were completely exhausted.

Substitutes in metallic materials only have a similar function. One can never find the perfect substitute. For this reason, metals can be substituted by perfect and imperfect substitutes. If the substitute is perfect, the metals do not differ from each other. In this case, the consumer will be indifferent and impartial to the form of a perfect substitute. Imperfect substitutes cannot be changed one for the other. The availability of substitutes can create a price cap for the products needed. However, there are no perfect substitutes for some commodities, and they probably will never exist (Novotný & Sejkora, 2016).

The entire twentieth century can be characterised by an eruption of progressive material innovation. The presence of more than one hundred thousand materials may currently be estimated. How to choose suitable representatives of material substitutes for copper? The most important properties are high electrical and thermal conductivity, corrosion resistance and formability.

Currently, even subject to covid measures, there has been a global increase in material extraction. According to the United States Geological Survey (USGS), the total world copper production has increased significantly over the last 10 years. Together with the population growth, concerns about the availability of raw materials are on the rise. Calvo et al. (2017) explain the possibility of using the Hubert model due to exponential mining to satisfy the given demand. This will help determine which mineral resources can become scarce in the coming decades. After reaching the so-called peaks, the raw materials will not run out, but they can be taken as a warning alarm that we should get another source in due time.

Fluctuations in mineral prices can put developing countries' economies at significant risk as long as their economies are heavily dependent on mineral production (Renner, 2020).

New technologies are constantly being developed to accelerate economic development. Korhonen (2018) says that we are able to overcome the shortage of mineral resources thanks to innovations and technologies. Will technological progress overcome the shortcomings that the future may bring? Can technology perfectly replace a given metal? Necessity is said to be the mother of invention. However, there are still no guarantees of human ingenuity to overcome all obstacles.

Since the 1980s and the recession in the mining industry, businesses have mainly survived by introducing innovations and technological changes, which has also led to higher productivity (Aydin, 2020). Mitra (2019) says that technology development is successfully advancing while the harmful effects of depletion on the productivity of the copper sector are successfully compensated. Given the growing problems and the impending ban on use in some sectors, an effective substitute for copper must be sought. Many just mention aluminium replacements.

Recently, higher copper prices have boosted the replacement in the automotive industry, such as in Zheng (2018) in brake composites research. Also, Mahale et al. (2019) describe a step to replace copper in the automotive industry, in particular brake pads. Xuegian et al. (2018) discuss a substitution in ferroalloys. Henckens & Worrell (2020) note that aluminium is currently used as a copper substitute in high voltage overhead line cables. At the same time, the advantage is that power cables carry the same current at a lower weight at a significantly lower cost. Reijnders (2021) also agrees to replace copper in certain industries while opposing Henckens & Worrell (2020), who disagree with the substitutability of copper in electricity transmission.

Some metals can be replaced by trade-offs such as lower cost or weight or even lower efficiency. The most common type of substitution is element by element. For example, aluminium can be used as a substitute for copper in many electrical applications (Manberger, 2018). The existence of an absolute deficiency depends on the perception of the possibility of substitution. One can talk about absolute scarcity only if there is no substitute for the given rare resource, i.e. metal in the form of copper. Every deficiency that the economy has studied so far has been resolved with the help of some substitution. In order to replace the X metal for an X' alternative or substitute, the latter must first and foremost be available and have certain, at least partially substituting properties. The need for a substitute or alternative does not only result from scarcity; economic theory suggests that price increases will eventually lead to lower-cost opportunities such as substitutes, innovative materials or increased recycling rates. The problem is that a significantly increased demand for substitutes can also result in price increases of given substitutes, precisely because of the higher demand. It should be noted that the main factor is commodities' price, which may be influenced by factors other than mere geological.

The paper aims to clarify the relationship between two economically imperfect substitutes, especially regarding their production volumes and prices, using the example of metal commodities of copper and aluminium. The following research questions were formulated with regard to the aim of the paper:

Given the study of the issue of metal substitution and the lack of a systematic overview of metal-to-metal substitutability, we ask the below questions:

Can copper and aluminium be classified as economically imperfect substitutes?

Due to the recent high demand for both metals, we will study the volume and price development over the last 10 years.

2. How has the volume of copper and aluminium production changed from 1 January 2011 to the present?

3. How has the price of copper and aluminium changed from 1 January 2011 to the present?

Based on the knowledge of the volume and price changes over the last 10 years, we can predict the development in the coming years.

4. What development and changes in copper and aluminium production volumes and prices can be expected in the upcoming years? And how will they be affected by the fact that substitutes are/are not economically imperfect?

Literature Review

Data analysis and the processing of forecast trends in mining are amongst the key questions in the use of metals. This paper describes several methods that can solve problems for the mining system. The prediction of other time series for metals is difficult. Time series forecasting is essential in economics, business, and financing (Šuleř, Rowland & Krulický, 2021). The key methods examined in the paper are based on mathematical and statistical modelling in combination with correlation and regression analysis and further forecasting with trend lines.

Tarasyev et al. (2019) argue that forecasting ability is very important in the production of natural resources. They present two prognostic methods: the first is trend line analysis or regression analysis, which is determined on the basis of trend line analysis data using MS Excel software chart. The result is a series of fluctuating data points as commodity prices rise and fall over time. The second method is correlation analysis for monitoring the relationship between quantities and processes. Momber et al. (2017) refer to it as the "Pearson's" correlation coefficient, defined as the ratio of covariance and the product of standard deviations. The strength of a relationship between variables can range from -1 to +1, where approaching one of the values means a stronger correlation. In

general, there are different levels of correlation strength. For example, Fu et al. (2020) break down the basic levels of correlation into 4 types. It depends on the determination of the threshold and the subsequent distribution of correlation values.

The price of copper not only depends on supply and demand but also on the overall global economy, the current dollar exchange rate and other factors that underline the volatility tendency. Various economic, geopolitical and technological factors affect commodity prices, both positive and negative. These inconsistent price and market trends are difficult to predict. It is necessary to use stochastic approaches, time series, or econometric techniques to present the dynamics of mineral commodity markets and predict prices (Tapia Cortez et al., 2018). However, these techniques will not provide comprehensive market dynamics due to the inability to render developments and the cumulative effects of factors on prices.

The main task of the analysis and design of the mining system is to predict the behaviour reported by prices in the future. Alipour, Khodavar and Jafari (2019) evaluated the application of various forecasting methods in econometrics and financial management. They applied techniques such as ARIMA, TGARCH and stochastic differential equations to the time series of monthly copper prices in the period from early 1987 to late 2014. The study results show that better forecasting results will be obtained for copper price time series using the Stochastic Differential Equations (SDE) method compared to traditional linear or nonlinear stock prices movement modelling techniques such as ARIMA or TGARCH.

Carasco et al. (2018) studied the potential of neural networks in the context of forecasts in chaotic copper price series. Using two algorithms of Feed Forward Neural Network (FFNN) and Cascade Forward Neural Network (CFNN), two models of neural networks were created to predict the price of copper on the London Metal Exchange. The resulting knowledge proves a better use of the artificial neural network (ANN) in financial forecasts of copper prices. The forecasting of copper prices using a different number of inputs can be improved by a different quantity of neurons or by changing the transmission or performance function.

Wang et al. (2019) also encourage and use hybrid techniques intertwining complex networks and traditional ANN techniques of price forecasting. Initially, the technique will transform the original time series into a price volatility network (PVN) to extract characteristics from its topological structure. The results confirm that the PVN-ANN hybrid techniques have a favourable forecasting effect if they are compared to traditional techniques. It is, therefore, welcome to support hybrid techniques for copper price predictions.

The metal price predictions are used for subsidy estimates of future profits and future metal mining, and at the same time, they can instruct on purchasing, sales and other activities in the metal industry. Kriechbaumer et al. (2014) and their wavelet analysis makes it possible to examine the usefulness of the Autoregressive Integrated Moving Average (ARIMA) approach for predicting the monthly prices of aluminium, copper, lead and zinc. The performance results of the ARIMA metal forecast models have shown increased accuracy of the forecast one month in advance as a result of the combination of wavelet transformation and exponential decay amount function used in the MRA multi-resolution analysis to enhance accuracy. For example, the increase in forecast accuracy for one month upfront was $ 126.00 / t for copper and $ 53.00 / t for aluminium.

However, significant fluctuations in mineral prices such as copper have recently led to lower performance of standard forecasting approaches such as ARIMA to estimate price changes accurately (García & Kristjanpoller, 2019). According to Kim & Ron (2018), the GARCH model has also led to a low forecast horizon due to price deviations and the subsequent ability to predict.

According to Hu et al. (2020), it is challenging to predict copper price fluctuations given the time-varying characteristics of many factors that affect copper prices. They propose a new hybrid method for predicting copper price volatility using a synthesis of two techniques. One is a standard model called GARCH able to encode useful statistical information, and the other is a powerful deep neural network with a combination of GARCH predictions, long short-term memory (LSTM), and a traditional artificial neural network (ANN). Using GARCH predictions positively affects informative functions used to increase the neural network model's predictive power and integrate LSTM and ANN networks to create more efficient deeper neural networks and increase prediction performance relative to the forecast horizon.

Vochozka et al. (2021) study daily closing historical copper prices using artificial intelligence and recurrent neural networks (LSTM) with a great potential for copper price time series prediction. Thanks to neural networks, more accurate price forecasting data are obtained for the first three months than for the rest of the reference period. For a longer time series, the question remains whether to use the whole time series or only a part of it to obtain a relevant result. Neural networks employ LSTM layers and a gateway to oblivion, but there is no guarantee that the result will not be distorted by information from the beginning of time series and data development (Vochozka, Vrbka & Šuleř, 2020).

Thus, there are many time-series methods used for forecasting, which is an important factor in economics, business and finance. The newly developed deep learning algorithms such as LSTM outperform the traditional algorithms such as ARIMA (Siami-Namini et al. 2018). The better of the algorithms have a lower error rate compared to ARIMA, thus showing its predominance. Another finding from the better algorithm was the so-called "epochs" of training times that in LSTM had no effect on the performance of the prediction model, thus showing

only random behaviour. Based on this knowledge and given its focus on the commodity market, LSTM is the best method for predicting the required time series of copper and aluminium prices. The data necessary for analysis and prediction will be collected using the content analysis method.

Material and Methods

The analysed data were taken from Markets.business insider (2021) and Investing.com (2022). For data concerning the selling price of copper and aluminium, all tradable days are available from 3 January 2011 to 29 November 2021. The final price of the day applies to both commodities. The copper mining and aluminium production data are only available in annual aggregates. The formulated research questions include a basic statistical and scientific description, comparison, correlation analysis and last but not least, regression analysis using neural networks. The formal logic tools such as deduction, induction and generalisation will help to obtain answers. Before the actual processing of the collected data, it should be considered whether we can achieve a relevant result when using the data of the whole time series or only a part of it. Neural networks using the LSTM layer also include a gateway to oblivion; since we are using quite current data, we expect that the result of the neural network will not be distorted by information from the beginning of the development of time series (Vochozka, Vrbka & Šuleř, 2020). The answer to the RQ1 question will be determined from the available literature and subsequently verified or refuted using the Pearson correlation coefficient. Pearson's correlation coefficient is marked by the letter *r* and calculated from a two-dimensional random vector with a range *n*.

$$\binom{x_1}{y_1}, \binom{x_2}{y_2}, \dots, \binom{x_n}{y_n}, \tag{1}$$

The Pearson correlation coefficient is calculated according to the following formula:

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}\overline{(x_i - \overline{x})^2} \sum_{i=1}^{n}\overline{(y_i - \overline{y})^2}}} = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{xy}}{(n-1)s_x s_y}, \tag{2}$$

where
$\overline{x}\ a\ \overline{y}$ sample averages
$s_x\ a\ s_y$ standard deviations,
$x\ a\ y$ random variables.

The resulting value will be graphically illustrated.

The whole time series for the period 2011-2021 will be used to find answers to **RQ2 and RQ3 research questions**. For the purpose of analysis, the development of both examined commodities in the time series 2011–2021 will be examined using regression analysis and graphically illustrated. Regression analysis is a statistical tool used to examine the relationships between two or more numeric variables. The purpose of regression analysis is to identify the suitable theoretical regression functions to illustrate the nature of the observed dependence, point and interval estimates of regression parameters of the regression function and values of the theoretical regression function, and verify compliance of the regression function with experimental data. In general, the search for a given regression function is associated with residue analysis. F-test will be used to test the statistical significance of the regression model based on the determination coefficient or reliability coefficient R^2.

The answer to the **RQ4 research question** will be obtained based on the prediction of the best generated neural network. The prediction of copper and aluminium production will be processed by regression analysis, taking into account the use of annual totals of copper mining and aluminium production. Mining or production may be affected by seasonal fluctuations or adverse weather during the annual period, so the annual totals are more appropriate for forecasting.

The forecasting of future copper and aluminium prices will be performed accordingly using the methodology of Vochozka et al. (2021). After the selection of NN, the future development is forecasted for each trading day from 3 January 2011 to 29 November 2021 (inclusive).

Results

Relationship between copper and aluminium in terms of use. According to Sazegaran et al. (2021), it was identified from monitoring the effect of aluminium and copper on graphite morphology, microstructure and pressure behaviour of ductile iron samples produced by sand casting that aluminium has a greater effect on the

mechanical properties of ductile iron as well as the modulus of elasticity, and yield strength, or the maximum compressive stress and fracture stress have improved.

There were 55 8-hydroxyquinoline derivatives used to evaluate the fluorescence activity of metal complexes of aluminium, cadmium, copper and zinc. The examination results showed that aluminium complexes, followed by zinc and cadmium complexes, showed the best fluorescence properties, while almost no fluorescence was observed in copper complexes (Zhang et al., 2021).

Medvedev et al. (2021) examined the use of new materials in the power electronics market, mainly due to the constant increase in component prices. Aluminium could be used instead of copper in the motherboard of the power supply module, where an insulated metal base plate is currently one of the most widespread technologies. The subject of the study was an insulated metal substrate based on anodised aluminium. The paper's main aim was to compare the adhesion of copper topology to an anodised aluminium oxide layer formed on various aluminium alloys with an aluminium content of at least 99.3 wt. The result of the research revealed that the obtained values of the adhesion strength of copper metallisation to AAO (anodic alumina) from commonly available aluminium alloys of the A1100 series with reduced copper content meet worldwide requirements for DBC structures (ceramics with direct bonded copper). Based on the summary of the results, we can say that the introduction of composite materials based on anodised aluminium in the packaging of power modules is technologically and economically justified.

The authors discussed intermetallic compounds (IMC) in copper-aluminium brazed joints, where it is very difficult to avoid the formation and growth of IMC, which depends on the mutual diffusion between the copper (Cu) substrate and the aluminium (Al) substrate. In addition, defects (cavities and cracks) in the joint are mainly due to the formation and growth of brittle IMCs, leading to stress concentration as a source of cracks and accelerating excessive consumption of scattered atoms to form cavities and cracks. Effective methods for IMC copper-aluminium joint management include heat input control, optimising the joint design, or adding a third element to the filler metals (Long et al., 2021).

According to Kurganov and Chen (2022), aluminium, together with its alloys, would be a promising material for the new generation, especially for mechanical engineering. There are several technological problems, and their solution allows to use of the potential of aluminium to achieve its new properties. There has been a technique proposed for the efficient introduction of light nanofibers with a diameter of about 10 nm using transport powders. This technique is based on the principle that copper powders of different fractions are rubbed with nanofibers to form conglomerates that are introduced into the aluminium melt. The structure of the produced samples was studied, the hardness was measured, and the material behaviour under impact loading was estimated. Comparative studies of the structure and properties of the reinforced material and the starting material showed higher efficiency of copper powder (particle size 180-200 μm) under these conditions. Nanofibers have been found to create a modifying effect and improve their mechanical properties.

The new proposed winding is made of thin conductive sheets with industrial tools as these tools are commonly used in the manufacturing of electrical machines. The flat winding topology allows easy manufacturing without significant changes to the production line. The authors presented an experimental verification of a 1.4 kW and 26 Nm prototype with copper and aluminium flat windings to compare the performance of both materials. In terms of efficiency and losses, copper or aluminium could result in excellent performance depending on the operation. Aluminium is favourable in the high-speed range due to suppressed eddy current losses, while copper is favourable in the high torque / current range due to suppressed line losses. Regarding its price and weight advantage, the obvious choice would be to use aluminium (Cakal and Keysan, 2021).

In his study, Dehghanpour (2021) generated Al-Cu-containing nanocomposites using pulsed ablation with an Nd: YAG (solid-state laser) laser (1064 nm) in a water-enclosed plasma. The selected electron diffraction (SAED) region of copper-containing nanoparticles shows crystallographic surfaces such as bulk aluminium, while the SAED of Al-Cu-containing nanocomposites did not show a regular crystalline structure. It was identified that the Cu - Al - containing suspension showed a different behaviour compared to the aluminium and copper suspensions.

Two different materials, copper and aluminium doped cobalt ferrite nanoparticles, were produced to investigate the effect of the number of additives on hydrogen synthesis and process stability. Copper-added material increased hydrogen production more than aluminium-added material under the same conditions. (Wenqing et al., 2022)

Gudic et al. (2021) conducted comparative corrosion studies of copper and copper alloys in aqueous chloride solution using open circuit potential, potentiodynamic polarisation, and electrochemical impedance spectroscopy measurements. The results showed that copper alloys had better corrosion resistance. The scanning electron microscopy / energy-dispersive X-ray spectroscopy (SEM / EDS) analysis used to measure polarisation showed uniform dissolution of pure copper as well as the presence of a surface oxide layer. However, aggressive anodic polarisation severely damaged the barrier layers on copper alloy samples.

Samples of aluminium and copper show 1.5-1.7 times higher corrosion in 3% sodium chloride (NaCl) solution than in 0.1% NaCl solution. The authors concluded that corrosion of copper in contact with aluminium in a 3%

NaCl solution was lower than in its samples. Polarisation curves of aluminium in contact with copper placed in 0.1% NaCl are shifted to the area of less negative potentials where the final diffusion and corrosion are an order of magnitude greater than an individual for copper and aluminium (Slobodyan et al., 2017).

Development of time series of copper mining and aluminium production. The use of regression analysis was tested using a matrix F test containing the annual copper mining and aluminium production data. The results are provided in Table 1 and are considered valid because the F test shows a higher value than the F crit.

Table 1. Two-sample F-test for scattering

Pointer	Mining copper	Production *aluminium*
Mean value	18,98	55,62
Dispersion	2,299555556	61,37733333
Observation	10	10
Difference	9	9
F		0,037465876
P(F<=f) (1)		0,000019274
F crit (1)		0,314574906

Development of time series of copper and aluminium prices. Figure 1 illustrates the evolution of the whole time series (2011-2021) of copper and aluminium prices on a daily basis. Copper and aluminium prices have a similar trend.
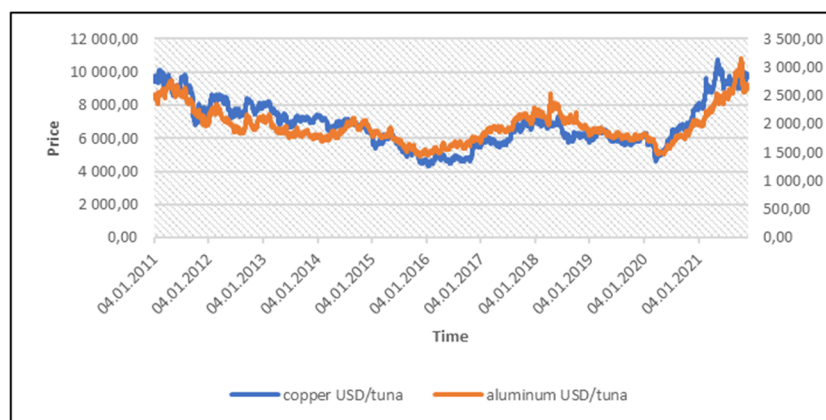

Fig. 2. Development of time series of copper and aluminium prices for the period 2011-2021

Interdependence of copper and aluminium price and production - Pearson's correlation coefficient. The dependence measurement of random observations undoubtedly plays a key role in statistics. We are often interested in compressing the dependence into a single number where such a number that is usually defined in the interval $[-1,1]$ or $[0,1]$ is called the correlation coefficient. The most commonly used correlation coefficient is Pearson's correlation coefficient. It shows the variance of random variables X and Y with finite and positive variances. The reverse implication of the Pearson correlation coefficient is not true (Edelmann et al., 2021).

Armstrong (2019) studied the use of the Pearson correlation coefficient and related statistical methods in the ophthalmological literature. It is important to focus on the nonlinear relationship between two variables, on bivariate normal data, on r (correlation coefficient) representing a significant part of the variance (Y), on outlying data values, on appropriate sample size and on the causality indicated by significant correlation. The challenges and limitations of r require a more cautious approach to its use and the application of alternative methods.

In today's smart home, the ability to recognise everyday activities primarily depends on the strategy used to select the appropriate functions related to these activities. The aim was to use a strategy for selecting daily activity functions based on the Pearson correlation coefficient. The results of the experiment suggest that the proposed approach ensures a higher recognition and achieves an average F-rate improvement of 1.56% and 2.7%, respectively (Liu et al., 2020).

Šverko et al. (2022) argue that there are sets of connections between different neurons or groups of neurons in the background of all human thinking or actions and reactions. These connections were examined and evaluated by brain signal electroencephalography (EEG) using the complex Pearson correlation coefficient (CPCC). It was identified that CPCC contains information on the other two rates balanced in a single complex numbered index.

The Pearson correlation coefficient can be used as a decision-making tree (PCC-Tree), and its parallel implementation was developed within the Map-Reduction (MR-PCC-Tree). The proposed methods use the Pearson correlation coefficient as a new measure of the quality of elements to confirm the optimum attributes of the distribution and, at the same time, the distribution points in the growth of decision-making trees. Unlike several traditional decision-making tree classifiers in 17 data sets, the proposed PCC tree is no worse than traditional models as a whole. In addition, experimental results from another 8 data sets show the feasibility of the proposed MR-PCC tree and its good parallel performance (MU et al., 2018).

The correlation coefficient comparing the final prices of copper and aluminium in USD / tonne for the period 2011-2021 was identified at the level of 0.872853309. Using the annual production of copper and aluminium in millions of tonnes for the period 2011-2020, the correlation coefficient shows a value of 0.935856776.

The prediction of the development of copper mining and aluminium production. The relationship between the annual copper mining and aluminium production is shown in Figure 2, where the polynomial trend line in the fourth stage with a reliability value of R^2 of 0.9322 seems to be the most adequate.
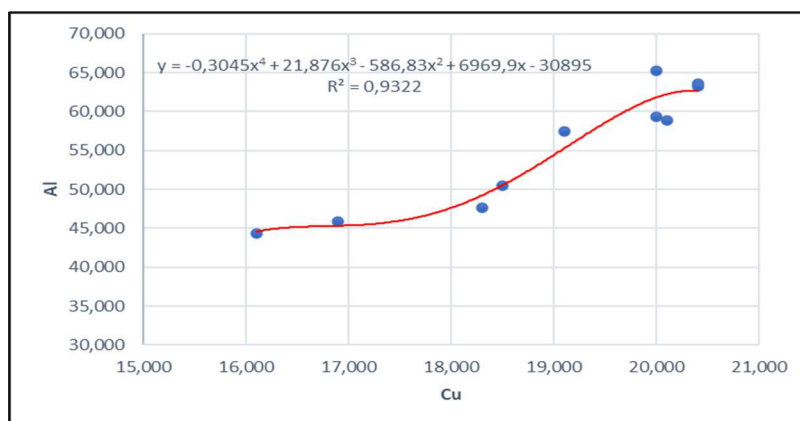

Fig. 2. Relationship between copper and aluminium production – trend link

Copper mining has had a linear trend in the last five years, and a similar development can be expected in the near future. Therefore, the LINTREND function was used for prediction until 2023, and the above polynomial function was used to predict aluminium production with an R^2 value of 0.9082. A graphical illustration of the relationship between copper mining and aluminium production, including the forecast period 2021-2023, is shown in Figure 3.
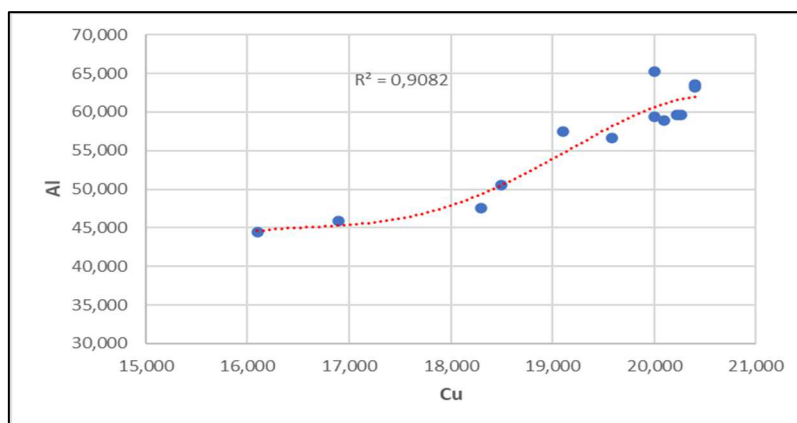

Fig. 3. Prediction of the relationship between copper and aluminium production – trend link

Prediction of the development of aluminium prices. The basic statistics of the current aluminium price time series and the smoothed time series are provided in Table 2. The statistics show the minimum and maximum values, the mean, the standard deviation and the variance. It can be concluded from the results shown in Table 2 that the best performance was achieved by the 1NN30L network, i.e. the network that considered a 30-day delay.

Table 2. Statistics of actual and smoothed time series

Network	Minimum	Maximum	Mean	Standard Deviation	Variance
Aluminium Price in USD	1435.00	3167.70	1963.24	303.530	92130.70
1NN30L	1498.86	3139.27	2003.80	297.049	88237.90
2NN30L	1431.79	3153.89	1946.66	306.631	94022.60
3NN30L	1468.67	3145.01	1977.03	301.737	91045.40
4NN30L	1446.58	3090.94	1936.89	294.503	86731.90
5NN30L	1507.24	3122.76	2001.08	294.336	86633.60

Figure 4 shows the development of aluminium prices in USD in red, while the other curves represent the time series smoothed by the top five neural networks with a 30-day delay. It is not obvious from the chart which of the smoothed time series best illustrates the development of the real price of aluminium; however, 1NN30L was evaluated as the best network.
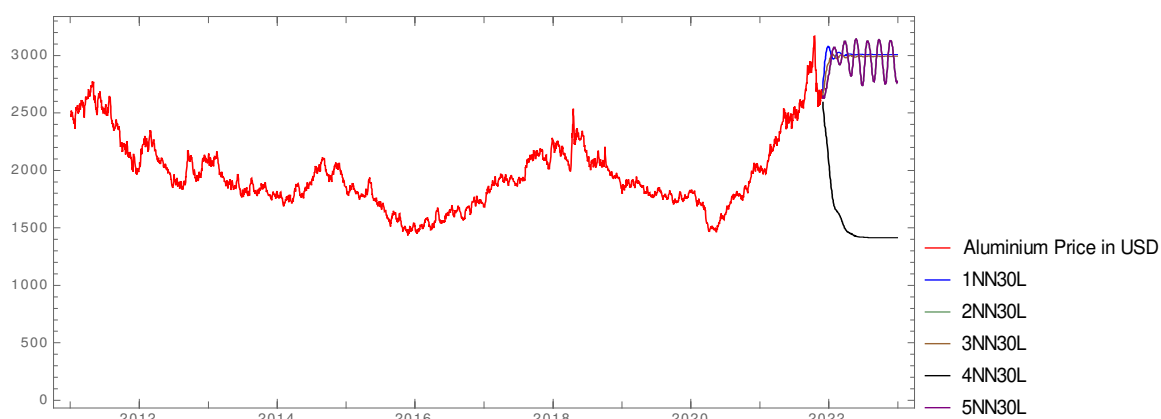

Fig. 4. Smoothing time series using all networks

The 1NN30L network was chosen as the best network for smoothing the time series, and we will pay attention to it, especially its residues. Figure 5 illustrates the development of 10 residues.
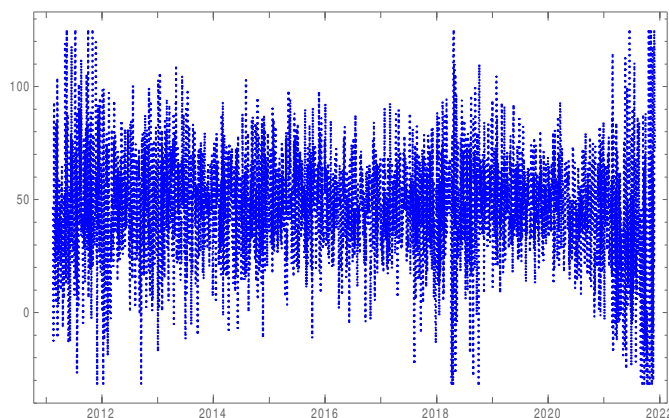

Fig. 5. 1NN30L residuals

The prediction of future development for each trading day until 31 December 2022 using all five selected NN is shown in Figure 6. If we look at the blue curve of the predicted development using the 1NN30L network, we can observe the best prediction of development, i.e. development predicted by the best performing NN.
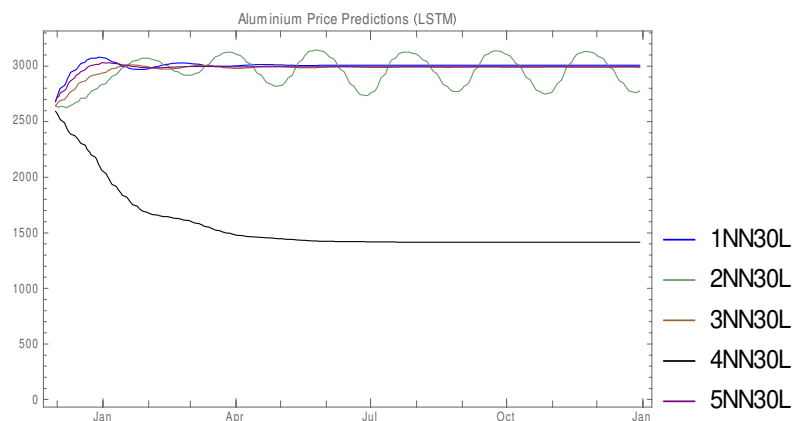
Fig. 6. Predictions of all networks

The basic characteristics of future development prediction for each trading day until 31 December 2022 for individual networks are presented in Table 3. The basic statistics include minimum, maximum, mean values, standard deviation and variance.

Tab. 3. Prediction statistics

Network	Minimum	Maximum	Mean	Standard Deviation	Variance
1NN30L	2678.28	3080.55	3003.81	39.6206	1569.79
2NN30L	2625.90	3145.30	2947.73	140.0520	19614.70
3NN30L	2642.43	3010.83	2976.20	56.7119	3216.24
4NN30L	1414.72	2593.35	1554.89	269.0620	72394.50
5NN30L	2687.14	3032.11	2987.30	41.0218	1682.79

Let us focus again on the 1NN30L prediction as to the best-performing network. Figure 7 shows the actual development of the time series since 2011. The following blue curve represents the development predicted by 1NN30L until 31 December 2022.
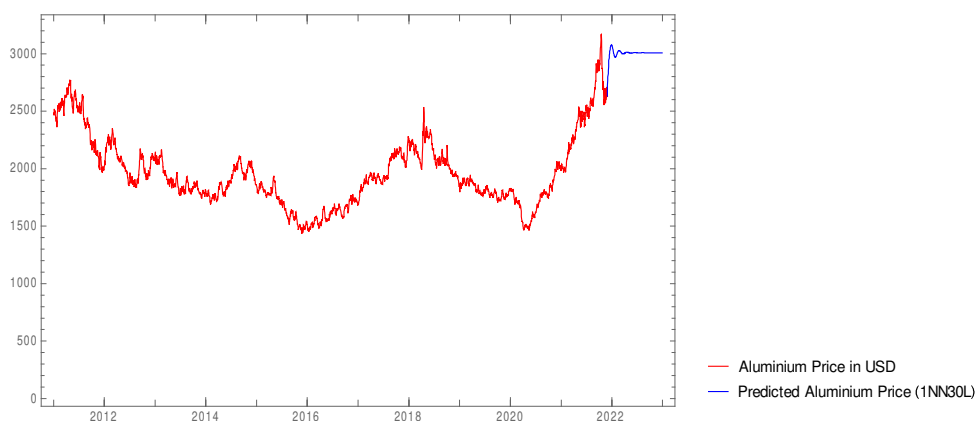

Fig. 7. 1NN30L prediction

More details of the development are provided in Figure 8, which illustrates the development predicted between 30 October 2021 and 31 January 2023.
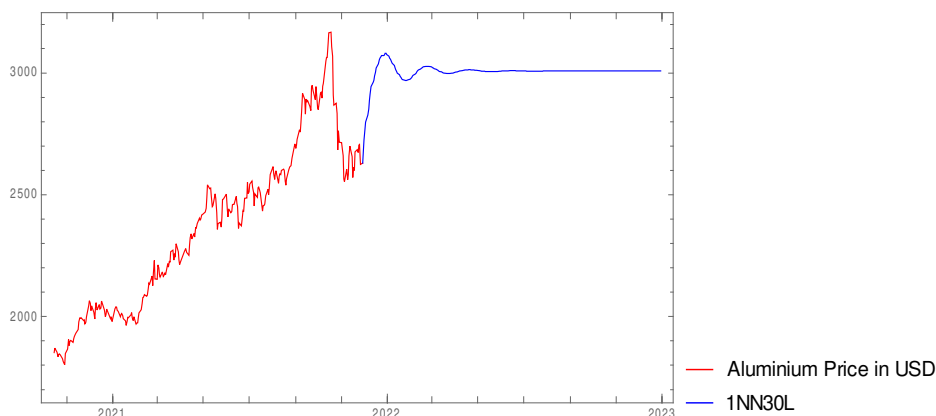
Fig. 8. Development from 30 October 2021 to 31 January 2023

Prediction of copper price development. The essential statistics of the current copper price time series and the smoothed time series are provided in Table 4. The statistics show the minimum and maximum, the mean, standard deviation and variance. It can be concluded from the results shown in Table 4 that the best performance was achieved by the 3NN30L network, i.e. the network that considered a 30-day delay.

Table 4. Statistics of actual and smoothed time series

Network	Minimum	Maximum	Mean	Standard Deviation	Variance
Copper Price in USD	4310.50	10724.50	6841.37	1399.72	$1.95923*10^6$
1NN30L	4358.79	10308.30	6746.11	1347.26	$1.81511*10^6$
2NN30L	4368.31	10333.20	6783.78	1374.37	$1.88891*10^6$
3NN30L	4543.18	10325.90	6897.86	1307.70	$1.71007*10^6$
4NN30L	4232.69	10417.30	6706.16	1390.50	$1.93350*10^6$
5NN30L	4354.32	10312.80	6743.82	1331.45	$1.77277*10^6$

Figure 9 shows the development of copper prices in USD in red, while the other curves represent time series smoothed by the five best neural networks with a 30-day delay. It is not obvious from the chart which of the smoothed time series best illustrates the development of the real price of aluminium; however, 3NN30L was evaluated as the best network.
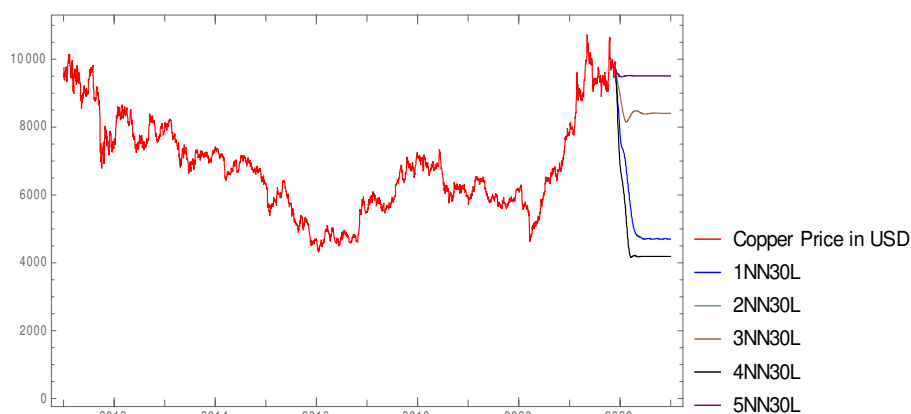

Fig. 9. Smoothing time series using all networks

The 3NN30L network was chosen as the best network for smoothing the time series, and we will pay attention to it, especially its residues. Figure 10 illustrates the development of 10 residues.
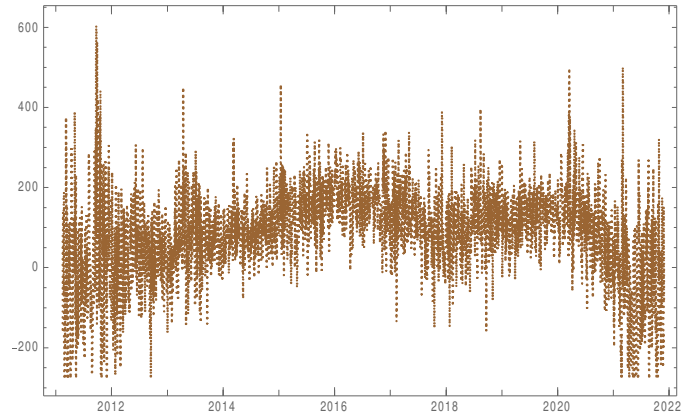
*Fig.10. 3*NN30L residuals

The prediction of future development for each trading day until 31 December 2022 using all five selected NN is shown in Figure 11. If we look at the brown curve of the predicted development using the 3NN30L network, we can observe the best prediction of development, i.e. development predicted by the best performing NN.
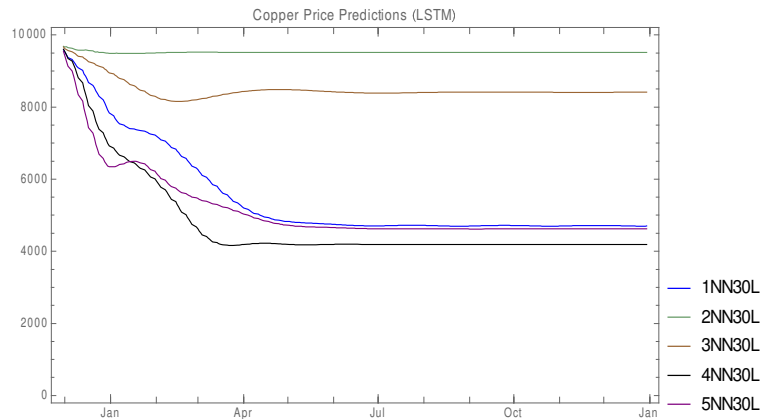

Fig. 11. Predictions of all networks

The essential characteristics of the prediction of future development for each trading day until 31 December 2022 for each network are provided in Table 5. The statistics include the minimum, maximum, mean, standard deviation and variance.

Tab. 5. Prediction statistics

Network	Minimum	Maximum	Mean	Standard Deviation	Variance
1NN30L	4696.87	9595.33	5506.86	1319.280	$1.7405*10^6$
2NN30L	9484.12	9681.80	9518.03	24.060	578.883
3NN30L	8156.72	9661.26	8483.84	278.195	77392.600
4NN30L	4163.81	9599.98	4789.56	1266.350	$1.60365*10^6$
5NN30L	4619.05	9539.37	5176.33	1003.150	$1.00632*10^6$

Let us focus now on the 2NN30L prediction as to the best-performing network. Figure 12 shows the actual development of the time series since 2011. The following green curve represents the development predicted by 2NN30L until 31 December 2022.
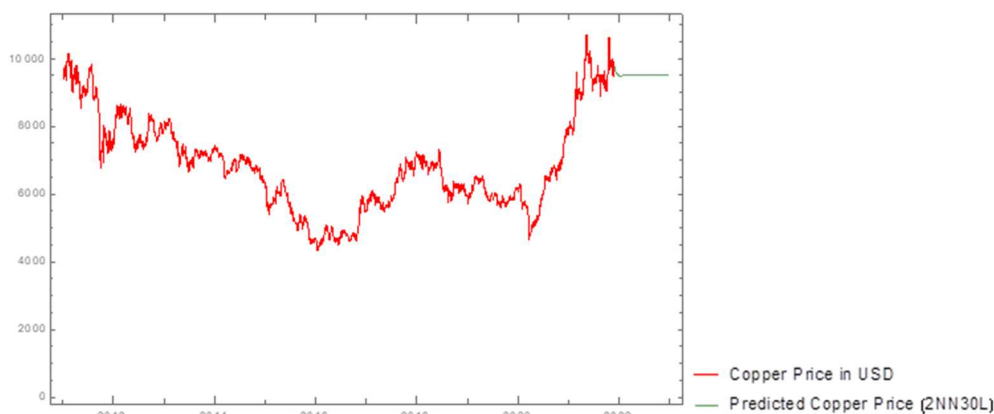
Fig. 12. 2NN30L prediction

More details of the development are provided in Figure 13, which shows the development predicted between 30 October 2021 and 31 January 2023
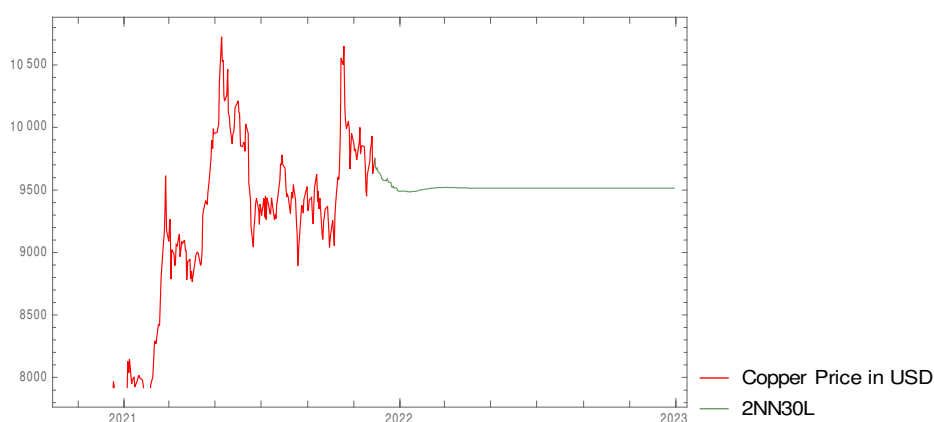

Fig. 13. Development from 30 October 2021 to 31 January 2023

Discussion

RQ1: Can copper and aluminium be classified as economically imperfect substitutes?

Due to the recent high demand for both metals, we will study the volume and price development over the last 10 years.

The answer to this research question and partly to the second and third questions can be found in the first part of the presented results that graphically illustrate the development of the time series of copper and aluminium; the table shows the statistics concerning the matrix of copper mining and aluminium production. In the last reference year, the price of copper and aluminium rose rapidly. The correlation coefficient comparing the final prices of copper and aluminium in USD / tonnes for the period of 2011-2021 at the level of 0.872853309 confirms that prices of both commodities are highly linearly interdependent. Using the annual production of copper and aluminium in millions of tonnes for the period 2011-2020, the correlation coefficient has the value of 0.935856776, confirming the very high linear dependence of these two commodities.

Considering the data obtained from the available literature and the results of correlation coefficients of price and production of both commodities, we can conclude that copper and aluminium may be imperfect substitutes in some respects, but they can generally be considered complementary.

RQ2: How has the volume of copper and aluminium production changed from 1.1.2011 to the present?

RQ3: How has the price of copper and aluminium changed from 1 January 2011 to the present?

Based on the knowledge of the volume and price changes over the last 10 years, we can predict the development in the coming years

The answer to the second and third research questions can be found in the first part of the presented results, which graphically illustrates the development of the time series of copper and aluminium prices. The table shows statistics concerning the annual copper mining and aluminium production in 2011-2020.

The most comprehensive part is the prediction of aluminium and copper prices using neural networks. The collected data are provided in tables and then graphically illustrated. The year 2021 was marked by a significant increase in copper and aluminium prices, culminating on 30 October 2021. The development of copper and

aluminium prices is illustrated in the figures included in the copper price forecast and aluminium price forecast, which also deal with the development of the past ten years. The following is a prediction of the development of copper mining and aluminium production using regression analysis where the most suitable polynomial trend line in the fourth stage showed a reliability R^2 of 0.9322.

RQ4: What development and changes in the production volumes and prices of copper and aluminium can be expected in the upcoming years? And how will they be affected by the fact that substitutes are/are not economically imperfect?

The research has previously identified that the forecasting of shorter time series is more appropriate because it is not affected by more distant historical developments, while the use of copper and aluminium in the industry has multiplied in recent decades. Time series forecasting (2011-2021) thus provides an adequate basis for predicting the future development of copper and aluminium prices in the period from October 2021 to January 2023. The results have shown that the 1NN30L neural network with an LSTM layer and considered a 30-day delay is the most suitable network for the forecasting of future copper price values , and the 3NN30L neural network with an LSTM layer and considered a 30-day delay is the most suitable network for the forecasting of future aluminium price values.

The forecast has confirmed that the price of copper will fall at the end of 2021, and the trend will be constant in the next planned period. Similar to copper, there was a significant decrease in the price of aluminium at the end of 2021; price growth is forecasted in early 2022, reaching below the level of 30 October 2021, and the trend will then be almost constant.

The resulting predicted development of daily copper and aluminium prices in our research can be described as mostly constant. Copper, specifically copper oxide, is used as a virus and bacteria deactivator; compared to silver, it is more readily available and less expensive to use. The demand for copper is thus increasing in the Covid 19 crisis, and so is its price. Due to the very high price of copper, many researches aim to replace copper with cheaper aluminium not only in mechanical engineering, which leads to an increased demand for aluminium and consequently to an increase in its price. These factors are also related to the development of copper mining and aluminium production.

Copper mining has stabilised in recent years as, despite the increased demand, copper is being replaced by aluminium in certain sectors. Aluminium production has increased significantly in the last decade, and it can be expected to grow in the near future. The development of copper mining and aluminium production is forecasted using regression analysis where the most suitable polynomial trend line in the fourth stage showed a reliability R2 of 0.9322.

Conclusions

The paper aimed to explain the relationship between two economically imperfect substitutes, especially regarding their production volumes and prices, using the example of metal commodities of copper and aluminium. The experiment included forecasting models based on artificial neural networks - NN with LSTM layer with a 30-day delay, for the development and prediction of the selling price of copper and aluminium for tradable days 2011-2021. Regression analysis was used for copper mining and aluminium production data of annual totals for the period 2011-2020. Neural networks allowed us to reliably analyse the development and forecast/predict changes in copper and aluminium prices. Thus, we could successfully find the answer to the third or the fourth research question. The second or the fourth research question was about the time series trend and forecasting of copper and aluminium production. The answer to the first research question was obtained from published scholarly articles where it was identified that copper and aluminium might be imperfect substitutes in some respects, but they can generally be considered complementary.

After a detailed study of the development of copper and aluminium prices from 2011 to 2021, we have forecasted a trend until the end of January 2023, concluding that copper and aluminium prices will continue to rise in the coming years, also due to the challenges of the past few years affecting the global economy. The prediction of aluminium production has an ever-increasing trend; on the contrary, copper mining will stagnate slightly in the coming years due to its high price and limited quantities. Donga, Tukker & Van der Voet (2019) argue that Asia's most populous nation greatly impacts copper prices due to rapid economic development in recent decades, including a high production capacity and high copper consumption. China's share of global copper demand increased from 20% in 2006 to 46% in 2016, including incremental year-on-year growth.

The forecasted development of copper and aluminium prices obtained from the research indicates a large increase in 2021, where the price culminated in November 2021. Overall, the price of copper will fall at the end of 2021, and the development will be more or less constant in the next predicted period; the price of aluminium also fell sharply at the end of 2021, but it will rise early in 2022 almost to the level of November 2021, and in the next predicted period, the development will almost be constant. Copper mining has stabilised in recent years and has shown a linear trend over the last five years; despite increased demand, copper is being replaced by aluminium

in certain sectors. Aluminium production has increased significantly in the last decade and can be expected to grow in the near future.

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