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Application of the FUZZY TOPSIS method for selecting an underground mining method

Stojance MIJALKOVSKI [1]*, Dragi PELTECHKI [2], Zoran DESPODOV [1], Dejan MIRAKOVSKI [1], Vancho ADJISKI [1] and Nikolinka DONEVA [1]

Authors' affiliations and addresses:
[1] Faculty of Natural and Technical Sciences, Mining Engineering, Goce Delchev University, PO Box 201, 2000 Shtip, R. N. Macedonia
e-mail: stojance.mijalkovski@ugd.edu.mk,
e-mail: zoran.despodov@ugd.edu.mk,
e-mail: dejan.mirakovski@ugd.edu.mk,
e-mail: vanco.adziski@ugd.edu.mk,
e-mail: nikolinka.doneva@ugd.edu.mk

[2] Rudplan DOOEL, Karaman Atanas 36, 2400 Strumica, R. N. Macedonia,
e-mail: info@rudplan.com.mk

***Correspondence:**
Stojance Mijalkovski, Faculty of Natural and Technical Sciences, Mining Engineering, Goce Delchev University, PO Box 201, 2000 Shtip, R. N. Macedonia,
tel:++38971870394
e-mail: stojance.mijalkovski@ugd.edu.mk

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Abstract
Selecting the underground mining method is one of the most difficult decisions any mining engineer makes when designing a new mine or opening new parts in an existing mine. The underground mining method selection depends on many mining-geological, technical and economic factors. The process of selecting a mine excavation method is a multi-criteria decision-making process since many factors are considered when selecting a mining method. The selection of the most suitable mining method is of great importance for each mine, and this is especially evident in the phase of preparation and excavation of the ore deposit and optimization of the total cost of excavation.
This paper uses the Fuzzy TOPSIS method to select the underground mining method for metallic mineral resources as one of the most important multi-criteria decision-making methods. The methodology presented will enable the application of other fuzzy methods to solve problems related to the mining method selection.

Keywords
Underground mining method selection, fuzzy multi-criteria decision-making, fuzzy TOPSIS.

Introduction

When opening a new underground mine or opening new sections in an existing mine, it is very important to select the most appropriate mining method. The mining method has a major impact on the mine capital and operating costs, the safety and health conditions for employees, achieving the necessary rate of production, etc.

The mining method selection depends on many factors, which can be quantitative (can be measured or calculated) or qualitative (cannot be measured and defined by descriptive values; they need to be transformed into numerical values so that they can be used for calculation). The factors influencing the mining method selection can be divided into three groups (Hartman, 1992):

- The mining-geological factors, such as the geometry of the deposit (general shape, ore thickness, dip, plunge, depth), rock quality (ore zone, hangingwall and footwall, i.e., rock strength, fracture spacing, fracture shear strength, rock quality designation, presence and strength of faults or other structures, in-situ stress), ore variability (ore boundaries, ore uniformity, continuity, grade distribution), quality of the resource, etc.
- The mining-technical factors, such as the annual productivity, equipment used, health and safety of the workforce, environmental impact, ore dilution, mine recovery, the flexibility of the mining methods, the mining rate, and
- The economic factors, such as the capital cost, operating cost, mineable ore tons, orebody grades and ore value.

There are cases where mining and geological factors allow the application of a particular mining method, but its application is not economically justified. There are also cases where a particular mining method allows the application of certain mechanization, but this is not allowed by mining and technical factors (Hartman, 1992).

All the factors influencing the mining method selection are not the same weight – some are permanent, and some may change. The main factors for the mining method selection are the natural conditions of the deposit and its surroundings, on the basis of which the mining method is selected, the stope is constructed, the mining technology is adopted, the work is organized according to the planned production capacity and analyzes mining cost. The mining method selection is made between several possible variants, sometimes diametrically opposed to each other. The final decision on the mining method selection is made on the basis of economic analysis; that is, the mining method that provides the fastest return on investment and the highest profit is chosen.

The methodologies for mining method selection can be divided into three groups: qualitative methods, numerical methods, and decision-making methods (Nourali et al., 2012). This paper discusses the application of the fuzzy TOPSIS method for mining method selection, which is one of the most important methods in the group of multi-criteria decision-making methods.

Until today, many scientists dealt with the question of underground mining method selection by using different multi-criteria decision-making methods (MCDM), such as the Analytic Hierarchy Process (AHP), the Elimination Et Choix Traduisant la Realite (ELECTRE), the Preference Ranking Organization Method for Enrichment of Evaluations (PROMETHEE), the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Visekriterijumsko Kompromisno Rangiranje (VIKOR), fuzzy logic and others, separately and together. The combination of MCDM methods and the fuzzy extensions of methods are often applied for the same purpose. Bitarafan et al. (2004) used multiple criteria decision-making tools for mining method selection. Shahriar et al. (2007) used a new numerical Shahriar and Bakhtavar (Sh&B) approach and the AHP. The method is a combined and modified system of Nicholas, Modified Nicholas and UBC for mining method selection. Alpay et al. (2007) used a decision support system and AHP for the selection of an underground mining method at the Eskisehir–Karaburun chromite mine. Namin et al. (2008) used fuzzy TOPSIS for mining method selection and examined the model for GEG anomaly No. 3 at the Chahar Gonbad mine. Ataei et al. (2008b) used the TOPSIS method with 13 criteria to develop a suitable mining method for Golbini No. 8 of the Jajarm bauxite mine in Iran. Also, Ataei et al. (2008a) used the AHP method to select a mining method for the same mine. Karadogan et al. (2008) used fuzzy set theory to select the underground mining method. Namin et al. (2009) used AHP, TOPSIS and PROMETHEE to solve the mining method selection problem. Jamshidi et al. (2009) used the AHP to select the optimal underground mining method in the Jajarm bauxite mine. Alpay et al. (2009) have suggested a combination of AHP and fuzzy logic methods for underground mining method selection. Naghadehi et al. (2009) used fuzzy AHP for mining method selection at the Jajarm Bauxite mine. Mikaeil et al. (2009) developed a decision support system using Fuzzy AHP, and TOPSIS approaches to select the optimum underground mining method. Azadeh et al. (2010) used fuzzy AHP for mining method selection by modifying the Nicholas technique for the Choghart iron mine. Liu et al. (2010) used an optimization model of unascertained measurement for underground mining method selection and its application. Gupta et al. (2012) developed an AHP model for underground mining method selection. Bogdanovic et al. (2012) used the PROMETHEE and AHP methods to choose an appropriate mining method in the Coka Marin mine in Serbia. Chamzini et al. (2012)

used the integrated MCDM model for mining method selection in the presence of uncertainty (fuzzy AHP and fuzzy TOPSIS). Namin et al. (2012) used the hybrid fuzzy-based decision support system for MMS in order to estimate interrelationships between criteria. Mijalkovski et al. (2013) used AHP, PROMETHEE and AHP-PROMETHEE integrated methods for mining method selection for the Sasa mine in Macedonia. Shariati et al. (2013) used fuzzy AHP and TOPSIS for mining method selection for the Angouran mine in Iran. Ataei et al. (2013) proposed a Monte Carlo-based AHP (MAHP) technique for the mining method selection of a bauxite ore deposit in Iran. Gelvez et al. (2014) applied the AHP and VIKOR methods to select the optimum mining method in a coal mine in Colombia. Yavuz (2015) used AHP and Yager's method for the selection of an underground mining method for the Ciftalan lignite mine in Istanbul. Karimnia et al. (2015) used AHP to choose the better mining method at a salt mine in Iran. Chen et al. (2015) applied AHP and PROMETHEE methods to select the most suitable technique for mechanized mining in a thin coal mine in China. Mahase et al. (2016) gave a good overview of the MCDM methods applied in mine planning and similar cases. Javanshirgiv et al. (2017b) used fuzzy TOPSIS for mining method selection at the Kamar Mahdi fluorine mine in Iran. Dehghani et al. (2017) used a new model for mining method selection based on grey and TODIM methods. Balusa et al. (2018a) used fuzzy AHP for mining method selection at the Tummalapalle and Turamdih uranium mines in India. Chander et al. (2018) used AHP and VIKOR to select the optimal underground bauxite mining method. Balusa et al. (2018b) used AHP, WPM and PROMETHEE to determine the effective mining method for a bauxite mine. Liang et al. (2018) used the selection of optimal mining method with extended multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) approach. Ooriad et al. (2018) used a novel model for mining method selection in a fuzzy environment at the Tazareh coal mine, Semnan province, Iran. Balusa et al. (2019a) used AHP, TOPSIS, VIKOR, ELECTRE, PROMETHEE, and WPM for mining method selection at the Tummalapalle uranium mine, India. Balusa et al. (2019b) analyze the sensitivity in decision-making, which results in selecting an appropriate underground metal mining method using the fuzzy-AHP (FAHP) model. Wang et al. (2019) used Monte Carlo AHP for the selection of the longwall mining method in thin coal seams. Popovic et al. (2019) used the underground mining method selection methodology based on the Extended Pivot Pairwise Relative Criteria Importance Assessment (PIPRECIA-E) and group decision-making. Bajic et al. (2020) used fuzzy AHP for mining method selection at the Borska Reka copper mine in Serbia. Mijalkovski et al. (2021) used the ELECTRE, PROMETHEE, AHP and AHP-PROMETHEE methods for mining method selection.

As discussed earlier, this paper will apply fuzzy multi-criteria decision-making (the fuzzy TOPSIS method) to the mining method selection. The mining method selection will be made between 4 mining methods or alternatives, and the comparison will be made according to 22 parameters or criteria. The main advantage of this paper will be the application of fuzzy multi-criteria decision-making methods in mining to solve complex problems, such as the mining method selection while taking into account a large number of influential parameters.

Fuzzy multi-criteria decision-making

Over the years, numerous fuzzy multi-criteria decision-making (FMCDM) methods have been proposed in the literature (Liao and Xu, 2017), which differ in subject areas, such as the type of questions asked the theoretical background, and the type of obtained results. A number of methods have been designed for a particular problem. Therefore, they are not applicable to other problems. Recently, a number of FMCDM methods have been introduced to select the best compromise options (Liao and Xu, 2017; Kore et al., 2017). The FMCDM approaches have been developed not only by the motivation received from various real-life problems that require the consideration of multiple criteria but also by the desire of practitioners to enhance decision-making techniques through recent developments occurred in computer technology, scientific computing, and mathematical optimization (Mardani et al., 2015). All methods aim to make the decision-making process better informed and more formalized. The multi-criteria decision-making (MCDM) approach falls into two categories: classical MCDM and FMCDM (Bashiri et al., 2011). In the FMCDM approach, alternatives are ranked and selected from among a set of feasible alternatives. FMCDM can be categorized as fuzzy multi-objective decision-making (FMODM) and fuzzy multi-attribute decision-making (FMADM) approach (Liou et al., 2012).

Fuzzy TOPSIS method

The TOPSIS method was first proposed by Hwang et al. (1981). The basic concept of this method is that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). PIS minimizes the cost criteria and maximizes the benefit criteria, whereas the NIS minimizes the benefit criteria and maximizes the cost criteria. TOPSIS is an easy-to-apply method with uncomplicated computations (Lashgari et al., 2011). In the classical TOPSIS method, the criteria weights and the ratings of alternatives are known precisely, and crisp values are used in the evaluation process. In the TOPSIS method, decision-makers' judgments are represented with crisp values. According to the problems associated with determining the precise preference rating for an alternative for the criteria under consideration, decision-makers are keen on using fuzzy numbers instead of precise numbers. For this reason, the fuzzy TOPSIS method is appropriate for solving real-world problems under a fuzzy environment (Alavai et al., 2011; Chamzini et al., 2012; Ebrahimabadi et al., 2013; Kacprzak, 2018; Kore et al., 2017; Nadaban et al., 2016).

The TOPSIS and fuzzy TOPSIS methods have been extensively applied to engineering and management fields over the last two decades, and there have been plenty of studies related to the TOPSIS method in the literature (Huang et al., 2020; Javanshirgiv et al., 2017a; Kumar et al. 2013; Lashgari et al., 2012; Sitorus et al., 2019). The fuzzy TOPSIS method is based on the fuzzy theory used for decisions hindered by uncertainty. The fuzzy theory is one of the modern techniques which can deal with the impreciseness of input data and domain knowledge by giving quick, simple and often sufficiently good approximations of the desired solutions. This theory is able to convert most incorrect and enigmatic concepts, variables and systems into a mathematical form and set the context for reasoning, deduction and decision-making in uncertain conditions.

In this paper, the fuzzy TOPSIS method is considered, which involves the following steps (Alavi et al., 2011; Javanshirgiv et al., 2017b; Kacprzak, 2018; Kore et al., 2017; Kusumawardani et al., 2015; Nadaban et al., 2016; Sun, 2010):

Step 1. Identify the evaluation criteria and alternatives.

We assume that we have a decision group with K members. The fuzzy rating of the k^{th} decision maker about alternative A_i and criterion C_j is denoted: $\tilde{x}_{ij}^{k} = (a_{ij}^{k};\ b_{ij}^{k};\ c_{ij}^{k})$ and the weight of criterion C_j is denoted $\tilde{w}_j^{k} = (w_{j1}^{k};\ w_{j2}^{k};\ w_{j3}^{k})$.

Step 2. Choose the appropriate linguistic variable.

Triangular fuzzy numbers can be used to represent linguistic variables, which can be used for the importance weight of the criteria (Tab. 1) and the evaluation of alternatives with respect to each criterion (Tab. 2).

Tab. 1. Linguistic variable for the importance weight of criteria

Linguistic variables	Fuzzy triangular
Very Low (VL)	(0.1, 0.1, 0.3)
Low (L)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
High (H)	(0.5, 0.7, 0.9)
Very High (VH)	(0.7, 0.9, 0.9)

Tab. 2. Linguistic variable for the alternatives rating

Linguistic variables	Fuzzy triangular
Very Poor (VP)	(1, 1, 3)
Poor (P)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Good (G)	(5, 7, 9)
Very Good (VG)	(7, 9, 9)

Step 3. Construct the fuzzy decision matrix.

A fuzzy multi-criteria group decision-making problem can be concisely expressed in a matrix format as follows:

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{1j} & \tilde{x}_{1n} \\ \tilde{x}_{i1} & \tilde{x}_{ij} & \tilde{x}_{in} \\ \tilde{x}_{m1} & \tilde{x}_{mj} & \tilde{x}_{mn} \end{bmatrix} \end{array} \tag{1}$$

Where, $\tilde{x}_{ij}$ are linguistic variables that can be shown by triangular fuzzy numbers: $\tilde{x}_{ij} = (a_{ij};\ b_{ij};\ c_{ij})$

The aggregated fuzzy rating $\tilde{x}_{ij} = (a_{ij};\ b_{ij};\ c_{ij})$ of i^{th} alternative and j^{th} criterion is obtained as follows:

$$a_{ij} = \min_{k}\{a_{ij}^{k}\}, \quad b_{ij} = \frac{1}{k}\sum_{k=1}^{k} b_{ij}^{k}\ ,\ c_{ij} = \max_{k}\{c_{ij}^{k}\} \tag{2}$$

Step 4. Establish a criteria-weighted matrix.

It cannot be assumed that each evaluation criterion is equally important because the criteria have various weights.

$$\tilde{w} = \left[\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n\right] \tag{3}$$

where $\tilde{w}_j$ are linguistic variables that can be shown by fuzzy triangular numbers: $\tilde{w}_j = (w_{j1};\ w_{j2};\ w_{j3})$.

The aggregated fuzzy weights $\tilde{w}_j = (w_{j1};\ w_{j2};\ w_{j3})$ for the criterion C_j are calculated by these formulas:

$$w_{j1} = \min_k \{w_{j1}^k\}, \quad w_{j2} = \frac{1}{k}\sum_{k=1}^{k} w_{j2}^k\ ,\ w_{j3} = \max_k \{w_{j3}^k\} \tag{4}$$

Step 5. Normalize the fuzzy decision matrix.

The normalized fuzzy-decision matrix denoted by $\tilde{R}$ is shown as the following formula:

$$\tilde{R} = \left[\tilde{r}_{ij}\right] mxn\ , \quad i = 1,2,\dots,m\ ; \quad j = 1,2,\dots,n \tag{5}$$

Then, the normalization process can be performed by the following formula:

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right) \text{ and } c_j^+ = \max_i \{c_{ij}\} \text{ (benefit criteria)} \tag{6}$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) \text{ and } a_j^- = \min_i \{a_{ij}\} \text{ (cost criteria)} \tag{7}$$

Step 6. Compute the weighted normalized fuzzy decision matrix.

The weighted normalized fuzzy decision matrix is shown as the following matrix $\tilde{\boldsymbol{V}}$:

$$\tilde{\boldsymbol{V}} = \left[\tilde{v}_{ij}\right] mxn\ , \quad i = 1,2,\dots,m\ ; \quad j = 1,2,\dots,n \tag{8}$$

where: $\tilde{v}_{ij} = \tilde{r}_{ij} \cdot \tilde{w}_j$

Step 7. Compute the Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS).

The FPIS indicates the preferable alternative, and the negative ideal solution indicates the least preferable alternative. The FPIS and FNIS are calculated as follows:

$$A^+ = \left(\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+\right),\ where\ \tilde{v}_j^+ = \max_i \{v_{ij}\}, i = 1,2,..,m; j = 1,2,\dots,n \tag{9}$$

$$A^- = \left(\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-\right),\ where\ \tilde{v}_j^- = \min_i \{v_{ij}\}, i = 1,2,..,m; j = 1,2,\dots,n \tag{10}$$

Step 8. Compute the distance from each alternative to the FPIS and to the FNIS.

The distance of each alternative from the FPIS and FNIS is calculated using the following equations:

$$d_v\left(v_{ij}, v_j^+\right) = \sqrt{\frac{1}{3}\cdot\left(\sum\left(v_{ij} - v_j^+\right)^2\right)} \tag{11}$$

$$d_v\left(v_{ij}, v_j^-\right) = \sqrt{\frac{1}{3}\cdot\left(\sum\left(v_{ij} - v_j^-\right)^2\right)} \tag{12}$$

$$S_i^+ = \sum_{j=1}^{n} d\cdot\left(\tilde{v}_{ij}, \tilde{v}_j^+\right), i = 1,2,\dots,m \tag{13}$$

$$S_i^- = \sum_{j=1}^{n} d\cdot\left(\tilde{v}_{ij}, \tilde{v}_j^-\right), i = 1,2,\dots,m \tag{14}$$

where d (.,.) is the distance measured between two fuzzy numbers.

Step 9. Compute the closeness coefficient of each alternative.
For each alternative A_i we calculate the closeness coefficient CC_i as follows:

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+} \ , i = 1,2,\dots,m \tag{15}$$

Step 10. Rank the alternatives.
The alternative with the highest closeness coefficient represents the best alternative.

Case study

In this paper, the Fuzzy TOPSIS method will be used for mining method selection for an underground lead-zinc ore mine. The total proven reserves of lead-zinc ore are approximately 945,000 tonnes, with an average grade of 5 per cent of lead and 4 per cent of zinc, an average ore thickness of 10 to 15 meters, an average ore length of 800 to 900 meters and an average ore plunge of 40 degrees. The planned production capacity is 500,000 tonnes per year, the number of working days in a year is 350, and the number of shifts per day is 3. An average lead price of 1,650 US$ per tonne and an average zinc price of 1850 US$ per tonne is used.

The mining method selection will be based on several mining-geological, mining-technical and economic factors. The most important mining-geological characteristics (Darling, 2011; Nicholas, 1992) for this deposit are listed in Tab. 3.

Tab. 3. Mining and geological characteristics

Geometrical characteristics				
General shape	Ore thickness	Plunge	Depth below surface	Grade distribution
Platy-tabular	Intermediate (10÷30 m)	Intermediate (20°÷55°)	Constant depth (≈500 m)	Erratic
Rock mechanics characteristics				
Parameter		Ore	Hangingwall	Footwall
Rock substance strength (RSS)		Moderate (55÷110 MPa)	Moderate (55÷110 MPa)	Moderate (55÷110 MPa)
Rock quality designation (RQD)		Good (70÷100%)	Fair (40÷70%)	Fair (40÷70%)
Fracture shear strength		Weak	Weak	Weak

Alternatives

According to the mining-geological characteristics of the deposit, four mining methods are being considered for the selection of the mining method for underground mining of lead-zinc ore, which is considered as an alternative (Tab. 4). These mining methods have in the past been used in the same mine, and some of them are still used today to mine some parts of the mine.

Tab. 4. Alternatives for mining method selection

Alternatives	Symbol
Cut and Fill Stoping	A_1
Sublevel Stoping	A_2
Shrinkage Stoping	A_3
Sublevel Caving	A_4

Each mining method has advantages and disadvantages in terms of mining-technical and economic factors, as well as recommendations for application in certain mining-geological conditions. In Tab. 5, qualitative information on the conditions applicable to each mining method is given (Hartman, 1992; Nicholas, 1992; Darling, 2011; Tatiya, 2013).

Tab. 5. Suitable conditions for the application of mining methods (Hartman, 1992; Nicholas, 1992; Darling, 2011; Tatiya, 2013)

Mining-geological characteristics	Cut and Fill Stoping	Sublevel Stoping	Shrinkage Stoping	Sublevel Caving
Deposit shape	Any, regular to irregular	Tabular or lenticular, regular dip and boundaries	Tabular or lenticular, regular dip and boundaries	Tabular or massive
Size and thickness	Fairly large extent, thin to thick (2÷30 m)	Large extent, thickness not below 5 m and up to 30 m or more	Large extent, narrow to moderate thickness but not below 1 m and up to 30 m (up to 15 m is common)	Large extent along and across the dip, thickness > 6 m.
Deposit dip	Usually steep but can be applied for flat dips	Steep (preferably 60÷90°)	Steep (preferably 60÷90°)	Steep but can be applied to flat dips
Depth	Practised up to 2.5 km	Practised up to 1 km or even more	Practised up to 750 m	Moderate, practised up to a depth of 1.2 km

Ore grade	High but uniformity can be variable	Fairly uniform	Fairly uniform and high	Moderate but uniform, as sorting is not possible
Ore strength	Moderate to strong	Moderate to strong	Strong	Medium hard to strong
Rock strength	Weak	Fairly strong to strong	Strong to fairly strong	Weak to moderate but fractured, jointed and cavable

Tab. 6 gives the mining-technical and economic characteristics of the mining methods according to literary data (Darling, 2011; Tatiya, 2013) and hands-on data.

Tab. 6. Mining-technical and economic characteristics (Darling, 2011; Tatiya, 2013)

Mining-technical and economic characteristics	Cut and Fill Stoping	Sublevel Stoping	Shrinkage Stoping	Sublevel Caving
Productivity	Moderate (10÷20, maximum up to 30÷40 t/shift/man)	Moderate to high, not labour-intensive (OMS in the range of 15÷30 t/shift/man)	Low to moderate (OMS in the range of 5÷10 t/shift/man)	Fairly high (OMS in the range of 20÷40 t/shift/man)
Production rate	Moderate	Moderate to high	Small to moderate	High
Safety	High (Good safety records)	Moderate (Little exposure to unsafe conditions)	Low (Rough footing)	Moderate (Little exposure to unsafe conditions)
Stope development	Little	High	Moderate	High
Recovery	High (95÷100%, if pillar mined)	Moderate (during stoping 85÷95%, during pillar extraction 75÷80%, overall <75%)	Moderate (during stoping 85÷95%, during pillar extraction 75÷80%, overall <75%)	Comparatively high (80÷90%, but with dilution, sometimes it exceeds 100%)
Dilution	Low (5÷10%)	Moderate (< 20%)	Low (<10%)	High (10÷35%)
Mining cost	High (Relative cost 60%)	Moderate, (Relative cost 40%)	Comparatively high (Relative cost 50%)	Comparatively high (Relative cost 40÷60%)
Degradation of terrain and other environmental impacts	Low	Low	Low	Moderate

Criteria

As already mentioned, the mining method selection depends on many parameters or criteria (Bogdanovic et al., 2012). This paper presents a total of 22 mining-geological, mining-technical and economic characteristics, which are the criteria against which the alternatives will be compared (Tab. 7). Each criterion has a different weight, that is, influence on alternative solutions. In this paper, criterion weight was adopted by vote (Nourali et al., 2012), i.e., in consultation with a group of 15 underground mining experts, to minimize the subjectivity of optimization. Those values were then converted into equivalent fuzzy values. Also, Tab. 7 gives the criteria goal (max or min) and classification category (quantitative or qualitative).

Tab. 7. Criteria for mining method selection

Criteria	Symbol	Weight of criteria	Goal	Category
General shape	C_1	High (H)	max	Qualitative
Ore thickness	C_2	Very High (VH)	max	Quantitative
Ore plunge	C_3	Very High (VH)	max	Quantitative
Depth below surface	C_4	Medium (M)	min	Quantitative
Grade distribution	C_5	Very High (VH)	max	Qualitative
Rock substance strength (RSS) of ore	C_6	High (H)	min	Qualitative
Rock substance strength (RSS) of the hangingwall	C_7	Very High (VH)	min	Qualitative
Rock substance strength (RSS) of footwall	C_8	Very High (VH)	min	Qualitative
Rock quality designation (RQD) of ore	C_9	High (H)	max	Quantitative
Rock quality designation (RQD) of the hangingwall	C_{10}	High (H)	max	Quantitative
Rock quality designation (RQD) of footwall	C_{11}	High (H)	max	Quantitative
Fracture shear strength of ore	C_{12}	High (H)	max	Qualitative
Fracture shear strength of hangingwall	C_{13}	High (H)	max	Qualitative
Fracture shear strength of footwall	C_{14}	High (H)	max	Qualitative
Productivity	C_{15}	High (H)	max	Quantitative
Production rate	C_{16}	High (H)	max	Qualitative
Safety	C_{17}	Very High (VH)	max	Qualitative
Stope development	C_{18}	High (H)	min	Quantitative
Ore recovery	C_{19}	Very High (VH)	max	Quantitative
Ore dilution	C_{20}	Very High (VH)	min	Quantitative
Mining cost	C_{21}	Very High (VH)	min	Quantitative
Degradation of terrain and other environmental impacts	C_{22}	Very High (VH)	min	Qualitative

Numerical example

For the determination of the best mining method out of the four proposed alternatives, the fuzzy TOPSIS method involves the following steps:

In the first step of the fuzzy TOPSIS analysis, three decision makers use the linguistic variables (Tab. 1 and Tab. 2) to evaluate the relative importance or weights of criteria and the ratings of alternatives for various attributes. There may be more decision-makers. The final results of the outcome of decision makers' views are presented in the fuzzy decision matrix (Tab. 8) and normalized fuzzy decision matrix (Tab. 9).

By using Eq. 2, the combined normalized fuzzy decision matrix is obtained (Tab. 10). By using Eqs. 5 to 7, the combined normalized fuzzy decision matrix is normalized. The corresponding matrix is presented in Tab. 11. Next, by using Eq. 8, a weighted normalized fuzzy decision matrix is calculated, and the result is given in Tab. 12.

By using Eq. 9 and Eq. 10, the FPIS and the FNIS are calculated, and the result is given in Tab. 13. After determining the FPIS and FNIS, the distance of each alternative from the FPIS and FNIS was obtained as Eqs. 11 to 14. The results are presented in Tab. 14 and Tab. 15. By using Eq. 15, the closeness coefficient of each alternative is calculated, and the result is given in Tab. 16, which also shows the ranking of alternatives.

Tab. 8. Fuzzy decision matrix

Criteria	Decision Maker 1 Alternatives				Decision Maker 2 Alternatives				Decision Maker 3 Alternatives			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
C_1	VG	VG	VG	VG	G	VG	VG	G	VG	G	G	VG
C_2	VG	VG	VG	VG	VG	G	VG	VG	G	VG	G	G
C_3	F	VP	VP	F	G	P	P	G	F	VP	VP	F
C_4	VG	VG	G	VG	VG	G	VG	G	G	VG	G	VG
C_5	F	VP	VP	P	F	P	VP	P	G	VP	P	VP
C_6	G	G	F	G	G	G	F	G	VG	F	P	F
C_7	G	F	P	G	F	P	VP	G	G	P	VP	G
C_8	G	F	P	G	F	P	VP	G	G	P	VP	G
C_9	P	P	P	F	F	F	F	P	G	P	P	F
C_{10}	F	VP	VP	G	G	P	P	F	G	VP	VP	G
C_{11}	F	VP	VP	G	G	P	P	F	G	VP	VP	G
C_{12}	G	F	P	F	G	F	F	P	F	P	P	F
C_{13}	G	VP	VP	G	F	P	P	F	F	VP	VP	G
C_{14}	G	VP	VP	G	F	P	P	F	F	VP	VP	G
C_{15}	F	G	P	VG	F	G	VP	VG	F	G	P	VG
C_{16}	F	G	P	VG	F	G	F	VG	F	G	P	VG
C_{17}	VG	F	VP	F	VG	F	P	F	VG	F	VP	F
C_{18}	P	G	F	G	P	VG	F	VG	VP	VG	F	VG
C_{19}	VG	F	F	G	VG	P	P	G	VG	F	F	G
C_{20}	VP	F	P	VG	P	F	P	G	VP	F	P	G
C_{21}	VG	F	G	G	G	P	F	P	G	P	F	F
C_{22}	VP	VP	VP	F	VP	P	P	F	VP	VP	VP	P

Tab. 9. Normalized fuzzy decision matrix

Criteria	Decision Maker 1 Alternatives				Decision Maker 2 Alternatives				Decision Maker 3 Alternatives			
	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4	A_1	A_2	A_3	A_4
C_1	(7,9,9)	(7,9,9)	(7,9,9)	(7,9,9)	(5,7,9)	(7,9,9)	(7,9,9)	(5,7,9)	(7,9,9)	(5,7,9)	(5,7,9)	(7,9,9)
C_2	(7,9,9)	(7,9,9)	(7,9,9)	(7,9,9)	(7,9,9)	(5,7,9)	(7,9,9)	(7,9,9)	(5,7,9)	(7,9,9)	(5,7,9)	(5,7,9)
C_3	(3,5,7)	(1,1,3)	(1,1,3)	(3,5,7)	(5,7,9)	(1,3,5)	(1,3,5)	(5,7,9)	(3,5,7)	(1,1,3)	(1,1,3)	(3,5,7)
C_4	(7,9,9)	(7,9,9)	(5,7,9)	(7,9,9)	(7,9,9)	(5,7,9)	(7,9,9)	(5,7,9)	(5,7,9)	(7,9,9)	(5,7,9)	(7,9,9)
C_5	(3,5,7)	(1,1,3)	(1,1,3)	(1,3,5)	(3,5,7)	(1,3,5)	(1,1,3)	(1,3,5)	(5,7,9)	(1,1,3)	(1,3,5)	(1,1,3)
C_6	(5,7,9)	(5,7,9)	(3,5,7)	(5,7,9)	(5,7,9)	(5,7,9)	(3,5,7)	(5,7,9)	(7,9,9)	(3,5,7)	(1,3,5)	(3,5,7)
C_7	(5,7,9)	(3,5,7)	(1,3,5)	(5,7,9)	(3,5,7)	(1,3,5)	(1,1,3)	(5,7,9)	(5,7,9)	(1,3,5)	(1,1,3)	(5,7,9)
C_8	(5,7,9)	(3,5,7)	(1,3,5)	(5,7,9)	(3,5,7)	(1,3,5)	(1,1,3)	(5,7,9)	(5,7,9)	(1,3,5)	(1,1,3)	(5,7,9)
C_9	(1,3,5)	(1,3,5)	(1,3,5)	(3,5,7)	(3,5,7)	(3,5,7)	(3,5,7)	(1,3,5)	(5,7,9)	(1,3,5)	(1,3,5)	(3,5,7)
C_{10}	(3,5,7)	(1,1,3)	(1,1,3)	(5,7,9)	(5,7,9)	(1,3,5)	(1,3,5)	(3,5,7)	(5,7,9)	(1,1,3)	(1,1,3)	(5,7,9)
C_{11}	(3,5,7)	(1,1,3)	(1,1,3)	(5,7,9)	(5,7,9)	(1,3,5)	(1,3,5)	(3,5,7)	(5,7,9)	(1,1,3)	(1,1,3)	(5,7,9)
C_{12}	(5,7,9)	(3,5,7)	(1,3,5)	(3,5,7)	(5,7,9)	(3,5,7)	(3,5,7)	(1,3,5)	(3,5,7)	(1,3,5)	(1,3,5)	(3,5,7)
C_{13}	(5,7,9)	(1,1,3)	(1,1,3)	(5,7,9)	(3,5,7)	(1,3,5)	(1,3,5)	(3,5,7)	(3,5,7)	(1,1,3)	(1,1,3)	(5,7,9)
C_{14}	(5,7,9)	(1,1,3)	(1,1,3)	(5,7,9)	(3,5,7)	(1,3,5)	(1,3,5)	(3,5,7)	(3,5,7)	(1,1,3)	(1,1,3)	(5,7,9)
C_{15}	(3,5,7)	(5,7,9)	(1,3,5)	(7,9,9)	(3,5,7)	(5,7,9)	(1,1,3)	(7,9,9)	(3,5,7)	(5,7,9)	(1,3,5)	(7,9,9)
C_{16}	((3,5,7)	(5,7,9)	(1,3,5)	(7,9,9)	(3,5,7)	(5,7,9)	(3,5,7)	(7,9,9)	(3,5,7)	(5,7,9)	(1,3,5)	(7,9,9)
C_{17}	(7,9,9)	(3,5,7)	(1,1,3)	(3,5,7)	(7,9,9)	(3,5,7)	(1,3,5)	(3,5,7)	(7,9,9)	(3,5,7)	(1,1,3)	(3,5,7)
C_{18}	(1,3,5)	(5,7,9)	(3,5,7)	(5,7,9)	(1,3,5)	(7,9,9)	(3,5,7)	(7,9,9)	(1,1,3)	(7,9,9)	(3,5,7)	(7,9,9)
C_{19}	(7,9,9)	(3,5,7)	(3,5,7)	(5,7,9)	(7,9,9)	(1,3,5)	(1,3,5)	(5,7,9)	(7,9,9)	(3,5,7)	(3,5,7)	(5,7,9)
C_{20}	(1,1,3)	(3,5,7)	(1,3,5)	(7,9,9)	(1,3,5)	(3,5,7)	(1,3,5)	(5,7,9)	(1,1,3)	(3,5,7)	(1,3,5)	(5,7,9)
C_{21}	(7,9,9)	(3,5,7)	(5,7,9)	(5,7,9)	(5,7,9)	(1,3,5)	(3,5,7)	(1,3,5)	(5,7,9)	(1,3,5)	(3,5,7)	(3,5,7)
C_{22}	(1,1,3)	(1,1,3)	(1,1,3)	(3,5,7)	(1,1,3)	(1,3,5)	(1,3,5)	(3,5,7)	(1,1,3)	(1,1,3)	(1,1,3)	(1,3,5)

Tab. 10. Combined normalized fuzzy decision matrix

Criteria	Alternatives				Weight
	A_1	A_2	A_3	A_4	W
C_1	(5.00,8.33,9.00)	(5.00,8.33,9.00)	(5.00,8.33,9.00)	(5.00,8.33,9.00)	(0.5,0.7,0.9)
C_2	(5.00,8.33,9.00)	(5.00,8.33,9.00)	(5.00,8.33,9.00)	(5.00,8.33,9.00)	(0.7,0.9,0.9)
C_3	(3.00,5.67,9.00)	(1.00,1.67,5.00)	(1.00,1.67,5.00)	(3.00,5.67,9.00)	(0.7,0.9,0.9)
C_4	(5.00,8.33,9.00)	(5.00,8.33,9.00)	(5.00,7.67,9.00)	(5.00,8.33,9.00)	(0.3,0.5,0.7)
C_5	(3.00,5.67,9.00)	(1.00,1.67,5.00)	(1.00,1.67,5.00)	(1.00,2.33,5.00)	(0.7,0.9,0.9)
C_6	(5.00,7.67,9.00)	(3.00,6.33,9.00)	(1.00,4.33,7.00)	(3.00,6.33,9.00)	(0.5,0.7,0.9)
C_7	(3.00,6.33,9.00)	(1.00,3.67,7.00)	(1.00,1.67,5.00)	(5.00,7.00,9.00	(0.7,0.9,0.9)
C_8	(3.00,6.33,9.00)	(1.00,3.67,7.00)	(1.00,1.67,5.00)	(5.00,7.00,9.00)	(0.7,0.9,0.9)
C_9	(1.00,5.00,9.00)	(1.00,3.67,7.00)	(1.00,3.67,7.00)	(1.00,4.33,7.00)	(0.5,0.7,0.9)
C_{10}	(3.00,6.33,9.00)	(1.00,1.67,5.00)	(1.00,1.67,5.00)	(3.00,6.33,9.00)	(0.5,0.7,0.9)
C_{11}	(3.00,6.33,9.00)	(1.00,1.67,5.00)	(1.00,1.67,5.00)	(3.00,6.33,9.00)	(0.5,0.7,0.9)
C_{12}	(3.00,6.33,9.00)	(1.00,4.33,7.00)	(1.00,3.67,7.00)	(1.00,4.33,7.00)	(0.5,0.7,0.9)
C_{13}	(3.00,5.67,9.00)	(1.00,1.67,5.00)	(1.00,1.67,5.00)	(3.00,6.33,9.00)	(0.5,0.7,0.9)
C_{14}	(3.00,6.57,9.00)	(1.00,1.67,5.00)	(1.00,1.67,5.00)	(3.00,6.33,9.00)	(0.5,0.7,0.9)
C_{15}	(3.00,5.00,7.00)	(5.00,7.00,9.00)	(1.00,2.33,5.00)	(7.00,9.00,9.00)	(0.5,0.7,0.9)
C_{16}	(3.00,5.00,7.00)	(5.00,7.00,9.00)	(1.00,3.67,7.00)	(7.00,9.00,9.00)	(0.5,0.7,0.9)
C_{17}	(7.00,9.00,9.00)	(3.00,5.00,7.00)	(1.00,1.67,5.00)	(3.00,5.00,7.00)	(0.7,0.9,0.9)
C_{18}	(1.00,2.33,5.00)	(5.00,8.33,9.00)	(3.00,5.00,7.00)	(5.00,8.33,9.00)	(0.5,0.7,0.9)
C_{19}	(7.00,9.00,9.00)	(1.00,4.33,7.00)	(1.00,4.33,7.00)	(5.00,7.00,9.00)	(0.7,0.9,0.9)
C_{20}	(1.00,1.67,5.00)	(3.00,5.00,7.00)	(1.00,3.00,5.00)	(5.00,7.67,9.00)	(0.7,0.9,0.9)
C_{21}	(5.00,7.67,9.00)	(1.00,3.67,7.00)	(3.00,5.67,9.00)	(1.00,5.00,9.00)	(0.7,0.9,0.9)
C_{22}	(1.00,1.00,3.00)	(1.00,1.67,5.00)	(1.00,1.67,5.00)	(1.00,4.33,7.00)	(0.7,0.9,0.9)

Tab. 11. Normalized fuzzy decision matrix

Criteria	Alternatives				Weight
	A_1	A_2	A_3	A_4	W
C_1	(0.56,0.93,1.00)	(0.56,0.93,1.00)	(0.56,0.93,1.00)	(0.56,0.93,1.00)	(0.5,0.7,0.9)
C_2	(0.56,0.93,1.00)	(0.56,0.93,1.00)	(0.56,0.93,1.00)	(0.56,0.93,1.00)	(0.7,0.9,0.9)
C_3	(0.33,0.63,1.00)	(0.11,0.19,0.56)	(0.11,0.19,0.56)	(0.33,0.63,1.00)	(0.7,0.9,0.9)
C_4	(0.56,0.60,1.00)	(0.56,0.60,1.00)	(0.56,0.65,1.00)	(0.56,0.60,1.00)	(0.3,0.5,0.7)
C_5	(0.33,0.63,1.00)	(0.11,0.19,0.56)	(0.11,0.19,0.56)	(0.11,0.26,0.56)	(0.7,0.9,0.9)
C_6	(0.11,0.13,0.20)	(0.11,0.16,0.33)	(0.14,0.23,1.00)	(0.11,0.16,0.33)	(0.5,0.7,0.9)
C_7	(0.11,0.16,0.33)	(0.14,0.27,1.00)	(0.20,0.60,1.00)	(0.11,0.14,0.20)	(0.7,0.9,0.9)
C_8	(0.11,0.16,0.33)	(0.14,0.27,1.00)	(0.20,0.60,1.00)	(0.11,0.14,0.20)	(0.7,0.9,0.9)
C_9	(0.11,0.56,1.00)	(0.11,0.41,0.78)	(0.11,0.41,0.78)	(0.11,0.48,0.78)	(0.5,0.7,0.9)
C_{10}	(0.33,0.70,1.00)	(0.11,0.19,0.56)	(0.11,0.19,0.56)	(0.33,0.70,1.00)	(0.5,0.7,0.9)
C_{11}	(0.33,0.70,1.00)	(0.11,0.19,0.56)	(0.11,0.19,0.56)	(0.33,0.70,1.00)	(0.5,0.7,0.9)
C_{12}	(0.33,0.70,1.00)	(0.11,0.48,0.78)	(0.11,0.41,0.78)	(0.11,0.48,0.78)	(0.5,0.7,0.9)
C_{13}	(0.33,0.63,1.00)	(0.11,0.19,0.56)	(0.11,0.19,0.56)	(0.33,0.70,1.00)	(0.5,0.7,0.9)
C_{14}	(0.33,0.63,1.00)	(0.11,0.19,0.56)	(0.11,0.19,0.56)	(0.33,0.70,1.00)	(0.5,0.7,0.9)
C_{15}	(0.33,0.56,0.78)	(0.56,0.78,1.00)	(0.11,0.26,0.56)	(0.78,1.00,1.00)	(0.5,0.7,0.9)
C_{16}	(0.33,0.56,0.78)	(0.56,0.78,1.00)	(0.11,0.41,0.78)	(0.78,1.00,1.00)	(0.5,0.7,0.9)
C_{17}	(0.78,1.00,1.00)	(0.33,0.56,0.78)	(0.11,0.19,0.56)	(0.33,0.56,0.78)	(0.7,0.9,0.9)
C_{18}	(0.20,0.43,1.00)	(0.11,0.12,0.20)	(0.14,0.20,0.33)	(0.11,0.12,0.20)	(0.5,0.7,0.9)
C_{19}	(0.78,1.00,1.00)	(0.11,0.48,0.78)	(0.11,0.48,0.78)	(0.56,0.78,1.00)	(0.7,0.9,0.9)
C_{20}	(0.20,0.60,1.00)	(0.14,0.20,0.33)	(0.20,0.33,1.00)	(0.11,0.13,0.20)	(0.7,0.9,0.9)
C_{21}	(0.11,0.13,0.20)	(0.14,0.27,1.00)	(0.11,0.18,0.33)	(0.11,0.20,1.00)	(0.7,0.9,0.9)
C_{22}	(0.33,1.00,1.00)	(0.20,0.60,1.00)	(0.20,0.60,1.00)	(0.14,0.23,1.00)	(0.7,0.9,0.9)

Tab. 12. Weighted normalized fuzzy decision matrix

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	(0.28,0.65,0.90)	(0.28,0.65,0.90)	(0.28,0.65,0.90)	(0.28,0.65,0.90)
C_2	(0.39,0.83,0.90)	(0.39,0.83,0.90)	(0.39,0.83,0.90)	(0.39,0.83,0.90)
C_3	(0.23,0.57,0.90)	(0,08,0.17,0.50)	(0.08,0.17,0.50)	(0.23,0.57,0.90)
C_4	(0.17,0.30,0.70)	(0.17,0.30,0.70)	(0.17,0.33,0.70)	(0.17,0.30,0.70)
C_5	(0.23,0.57,0.90)	(0.08,0.17,0.50)	(0.08,0.17,0.50)	(0.08,0.23,0.50)
C_6	(0.06,0.09,0.18)	(0.06,0.11,0.30)	(0.07,0.16,0.90)	(0.06,0.11,0.30)
C_7	(0.08,0.14,0.30)	(0.10,0.25,0.90)	(0.14,0.54,0.90)	(0.08,0.13,0.18)
C_8	(0.08,0.14,0.30)	(0.10,0.25,0.90)	(0.14,0.54,0.90)	(0.08,0.13,0.18)
C_9	(0.06,0.39,0.90)	(0.06,0.29,0.70)	(0.06,0.29,0.70)	(0.06,0.34,0.70)
C_{10}	(0.17,0.49,0.90)	(0.06,0.13,0.50)	(0.06,0.13,0.50)	(0.17,0.49,0.90)
C_{11}	(0.17,0.49,0.90)	(0.06,0.13,0.50)	(0.06,0.13,0.50)	(0.17,0.49,0.90)
C_{12}	(0.17,0.49,0.90)	(0.06,0.34,0.70)	(0.06,0.29,0.70)	(0.06,0.34,0.70)
C_{13}	(0.17,0.44,0.90)	(0.06,0.13,0.50)	(0.06,0.13,0.50)	(0.17,0.49,0.90)
C_{14}	(0.17,0.44,0.90)	(0.06,0.13,0.50)	(0.06,0.13,0.50)	(0.17,0.49,0.90)
C_{15}	(0.17,0.39,0.70)	(0.28,0.54,0.90)	(0.06,0.18,0.50)	(0.39,0.70,0.90)
C_{16}	(0.17,0.39,0.70)	(0.28,0.54,0.90)	(0.06,0.29,0.70)	(0.39,0.70,0.90)
C_{17}	(0.54,0.90,0.90)	(0.23,0.50,0.70)	(0.08,0.17,0.50)	(0.23,0.50,0.70)
C_{18}	(0.10,0.30,0.90)	(0.06,0.08,0.18)	(0.07,0.14,0.30)	(0.06,0.08,0.18)
C_{19}	(0.54,0.90,0.90)	(0.08,0.43,0.70)	(0.08,0.43,0.70)	(0.39,0.70,0.90)
C_{20}	(0.14,0.54,0.90)	(0.10,0.18,0.30)	(0.14,0.30,0.90)	(0.08,0.12,0.18)

C_{21}	(0.08,0.12,0.18)	(0.10,0.25,0.90)	(0.08,0.16,0.30)	(0.08,0.18,0.90)
C_{22}	(0.23,0.90,0.90)	(0.14,0.54,0.90)	(0.14,0.54,0.90)	(0.10,0.21,0.90)

Tab. 13. Fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS)

Criteria	FPIS	FNIS
C_1	(0.28,0.65,0.90)	(0.28,0.65,0.90)
C_2	(0.39,0.83,0.90)	(0.39,0.83,0.90)
C_3	(0.23,0.57,0.90)	(0.08,0.17,0.50)
C_4	(0.17,0.33,0.70)	(0.17,0.30,0.70)
C_5	(0.23,0.57,0.90)	(0.08,0.17,0.50)
C_6	(0.07,0.16,0.90)	(0.06,0.09,0.18)
C_7	(0.14,0.54,0.90)	(0.08,0.13,0.18)
C_8	(0.14,0.54,0.90)	(0.08,0.13,0.18)
C_9	(0.06,0.39,0.90)	(0.06,0.29,0.70)
C_{10}	(0.17,0.49,0.90)	(0.06,0.13,0.50)
C_{11}	(0.17,0.49,0.90)	(0.06,0.13,0.50)
C_{12}	(0.17,0.49,0.90)	(0.06,0.29,0.70)
C_{13}	(0.17,0.49,0.90)	(0.06,0.13,0.50)
C_{14}	(0.17,0.49,0.90)	(0.06,0.13,0.50)
C_{15}	(0.39,0.70,0.90)	(0.06,0.18,0.50)
C_{16}	(0.39,0.70,0.90)	(0.06,0.29,0.70)
C_{17}	(0.54,0.90,0.90)	(0.08,0.17,0.50)
C_{18}	(0.10,0.30,0.90)	(0.06,0.08,0.18)
C_{19}	(0.54,0.90,0.90)	(0.08,0.43,0.70)
C_{20}	(0.14,0.54,0.90)	(0.08,0.12,0.18)
C_{21}	(0.10,0.25,0.90)	(0.08,0.12,0.18)
C_{22}	(0.23,0.90,0.90)	(0.10,0.21,0.90)

Tab. 14. Distance of each alternative from the FPIS

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	0.00	0.00	0.00	0.00
C_2	0.00	0.00	0.00	0.00
C_3	0.00	0.34	0.34	0.00
C_4	0.02	0.02	0.00	0.02
C_5	0.00	0.34	0.34	0.31
C_6	0.42	0.35	0.00	0.35
C_7	0.42	0.17	0.00	0.48
C_8	0.42	0.17	0.00	0.48
C_9	0.00	0.13	0.13	0.12
C_{10}	0.00	0.32	0.32	0.00
C_{11}	0.00	0.32	0.32	0.00
C_{12}	0.00	0.16	0.18	0.16
C_{13}	0.03	0.32	0.32	0.00
C_{14}	0.03	0.32	0.32	0.00
C_{15}	0.25	0.11	0.42	0.00
C_{16}	0.25	0.11	0.33	0.00
C_{17}	0.00	0.31	0.55	0.31
C_{18}	0.00	0.43	0.36	0.43
C_{19}	0.00	0.40	0.40	0.15
C_{20}	0.00	0.40	0.14	0.48
C_{21}	0.42	0.00	0.35	0.04
C_{22}	0.00	0.21	0.21	0.41
S^+	2.25	4.93	5.02	3.74

Tab. 15. Distance of each alternative from the FNIS

Criteria	Alternatives			
	A_1	A_2	A_3	A_4
C_1	0.00	0.00	0.00	0.00
C_2	0.00	0.00	0.00	0.00
C_3	0.34	0.00	0.00	0.34
C_4	0.00	0.00	0.02	0.00
C_5	0.34	0.00	0.00	0.04
C_6	0.00	0.07	0.42	0.07
C_7	0.07	0.42	0.48	0.00
C_8	0.07	0.42	0.48	0.00
C_9	0.13	0.00	0.00	0.03
C_{10}	0.32	0.00	0.00	0.32
C_{11}	0.32	0.00	0.00	0.32
C_{12}	0.18	0.03	0.00	0.03
C_{13}	0.30	0.00	0.00	0.32
C_{14}	0.30	0.00	0.00	0.32
C_{15}	0.18	0.34	0.00	0.42
C_{16}	0.09	0.23	0.00	0.33

C_{17}	0.55	0.24	0.00	0.24
C_{18}	0.43	0.00	0.08	0.00
C_{19}	0.40	0.00	0.00	0.26
C_{20}	0.48	0.08	0.43	0.00
C_{21}	0.00	0.42	0.07	0.42
C_{22}	0.41	0.19	0.19	0.00
S^-	4.90	2.45	2.17	3.46

Tab. 16. Closeness coefficient (CC) of each alternative and Rank of alternatives

Alternatives	Closeness coefficient	Rank
A_1	0.69	1
A_2	0.33	3
A_3	0.30	4
A_4	0.48	2

The ranking of alternatives is done according to the closeness coefficient and is presented in Tab. 16. It can be concluded that the most acceptable alternative is "A_1", i.e., Cut and Fill Stoping (Fig. 1). Alternative "A_4" is the second in the rank, followed by alternative "A_2", and last ranked alternative is A_3 ($A_1 \rightarrow A_4 \rightarrow A_2 \rightarrow A_3$).

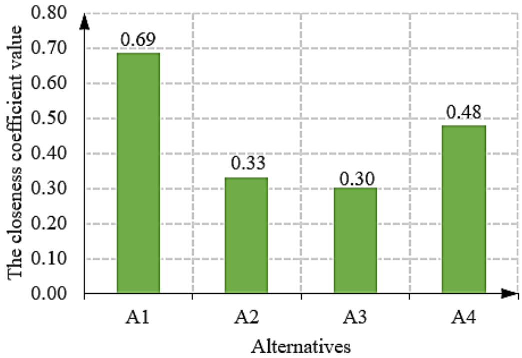

Fig. 1. The closeness coefficient (Rank) of alternatives

Conclusion

One of the biggest problems a mining engineer faces when designing a new mine or developing new parts in an existing mine is the selection of the mining method. It is very important to select the appropriate mining method selection as the mining method has a direct impact on the costs incurred during the deposit mining phase and thus directly impacts the financial operations of the mine itself.

When choosing the method of mining excavation, it is necessary to consider as many parameters as possible that influence the mining method selection because, in that case, the most suitable method of mining excavation in a specific case will be chosen. That was the main goal of the research in this paper as well as the contribution of the research.

From the previous research on solving the problem of mining method selection, we notice that several authors have applied different methods from the group of multi-criteria decision-making methods. A more advanced step in the research of decision-making methods is the application of fuzzy multi-criteria decision-making methods. The application of fuzzy multi-criteria decision-making methods enables the selection of the most appropriate way of mining excavation, considering a large number of influential parameters. For this purpose, several fuzzy multi-criteria decision-making methods can be applied, such as fuzzy AHP, fuzzy ELECTRE, fuzzy TOPSIS, fuzzy VIKOR and others.

In this paper, the underground mining method selection was carried out using the fuzzy TOPSIS method. The selection was made out of four mining methods, which were compared according to 22 criteria. According to the mining-geological characteristics of the ore deposit under consideration and the planned capacity, it was concluded that the most appropriate mining method is Cut and Fill Stoping.

The next step in researching this problem is the application of three or more fuzzy multi-criteria decision-making methods. After obtaining the ranking of the mining methods according to each fuzzy method, it is necessary to compare the results. In this way, the most suitable method will be mining method selection, which is important for solving this very complex issue.

References

Alavai, I. and Rokny, H.A. (2011). Comparison of fuzzy AHP and fuzzy TOPSIS methods for plant species selection (case study: Reclamation plan of Sungun copper mine, Iran), Australian journal of basic and applied sciences, 5(12), pp.1104-1113. http://www.ajbasweb.com/old/ajbas/2011/December-2011/1104-1113.pdf

Alpay, S. and Yavuz, M. (2007). A decision support system for underground mining method selection, 20th International conference on industrial, engineering and other applications of applied intelligent systems, Springer, Berlin, Heidelberg, pp.334-343. https://doi.org/10.1007/978-3-540-73325-6_33

Alpay, S. and Yavuz, M. (2009). Underground mining method selection by decision making tools, Tunnelling and Underground Space Technology, 24(2), pp.173–184. https://doi.org/10.1016/j.tust.2008.07.003

Ataei, M., Jamshidi, M., Sereshki, F. and Jalali, S.M.E. (2008a). Mining method selection by AHP approach, Journal of the Southern African Institute of Mining and Metallurgy, 108(12), pp.741-749. https://www.saimm.co.za/Journal/v108n12p741.pdf

Ataei, M., Shahsavany, H. and Mikaeil, R. (2013). Monte Carlo analytic hierarchy process (MAHP) approach to selection of optimum mining method, International journal of mining science and technology, 23(4), pp.573–578. https://doi.org/10.1016/j.ijmst.2013.07.017

Ataei, M., Sereshki, F., Jamshidi, M. and Jalali, S.M.E. (2008b). Suitable mining method for Golbini No.8 deposit in Jajarm (Iran) using TOPSIS method, Transactions of the Institutions of Mining and Metallurgy, 117(1), pp.1-5. https://doi.org/10.1179/174328608X343650

Azadeh, A., Osanloo, M. and Ataei, M. (2010). A new approach to mining method selection based on modifying the Nicholas technique, Applied Soft Computing, 10(4), pp.1040–1061. https://doi.org/10.1016/j.asoc.2009.09.002

Bajic, S., Bajic, D., Gluscevic, B. and Vakanjac, V.R. (2020). Application of fuzzy analytic hierarchy process to underground mining method selection, Symmetry, 12(1), pp.1-20. https://doi.org/10.3390/sym12020192

Balusa, B.C. and Gorai, A.K. (2019a). A comparative study of various multi-criteria decision-making models in underground mining method selection, Journal of the institution of engineers, 100(1), pp.105-121. https://doi.org/10.1007/s40033-018-0169-0

Balusa, B.C. and Gorai, A.K. (2018a). Design of a multi-criteria decision making model using fuzzy-AHP for selection of appropriate underground metal mining method, International journal of mining and mineral engineering, 9(4), pp.259-301. 10.1504/IJMME.2018.097425

Balusa, B.C. and Gorai, A.K. (2019b). Sensitivity analysis of fuzzy-analytic hierarchical process (FAHP) decision-making model in selection of underground metal mining method, Journal of sustainable mining, 18(1), pp.8-17. https://doi.org/10.1016/j.jsm.2018.10.003

Balusa, B.C. and Singam, J. (2018b). Underground mining method selection using WPG and PROMETHEE, Journal of the institution of engineers, 99, pp.165-171. 10.1007/s40033-017-0137-0

Bashiri, M., Badri, H. and Hejazi, T.H. (2011). Selection optimum maintenance strategy by fuzzy interactive linear assignment method, Applied mathematical modelling, 35(1), pp.152-164. https://doi.org/10.1016/j.apm.2010.05.014

Bitarafan, M.R. and Ataei, M. (2004). Mining method selection by multiple criteria decision making tools, Journal of the Southern African Institute of Mining and Metallurgy, 104(9), pp.493-498. https://hdl.handle.net/10520/AJA0038223X_2888

Bogdanovic, D., Nikolic, D. and Ilic, I. (2012). Mining method selection by integrated AHP and PROMETHEE method, Annals of the Brazilian Academy of Sciences, 84(1), pp.219–233. https://doi.org/10.1590/S0001-37652012005000013

Chamzini, A.Y., Yakchali, S.H. and Zavadskas, E.K. (2012). Using an integrated MCDM model for mining method selection in presence of uncertainty, Economic Research-Ekonomska Istraživanja, 25(4), pp.869-904. https://doi.org/10.1080/1331677X.2012.11517537

Chander, B.B., Gorai, A.K. and Jayantu, S. (2018). Design of Decision-Making Techniques Using Improved AHP and VIKOR for Selection of Underground Mining Method, Recent Findings in Intelligent Computing Techniques, 2, pp.495–504. https://doi.org/10.1007/978-981-10-8636-6_52

Chen, W. and Shihao, T. (2015). Selection of an appropriate mechanized mining technical process for thin coal seam mining, Mathematical Problems in Engineering, 2015, ID 893232, pp.1-10. https://doi.org/10.1155/2015/893232

Darling, P. (2011). SME Mining Engineering Handbook Third Edition. USA: Society for Mining, Metallurgy, and Exploration, Inc., Part 13, pp.1327-1454.

Dehghani, H., Siami, A. and Haghi, P. (2017). A new model for mining method selection based on grey and TODIM methods, Journal of Mining and Environment, 8(1), 49-60. https://doi.org/10.22044/jme.2016.626

Ebrahimabadi, A. and Alavi, I. (2013). Plant type selection for reclamation of Sarcheshmeh copper mine using fuzzy-topsis approach, Archives of mining sciences, 58(3), pp.953-968. https://doi.org/10.2478/amsc-2013-0067

Gelvez, J.I.R. and Aldana, F.A.C. (2014). Mining method selection methodology by multiple criteria decision analysis-case study in Colombian coal mining, International Journal of the Analytic Hierarchy Process, 3(1), pp.1–11. http://www.isahp.org/uploads/p744104.pdf

Gupta, S. and Kumar, U. (2012). An analytical hierarchy process (AHP)-guided decision model for underground mining method selection, International journal of mining, reclamation and environment, 26(4), pp.324-336. https://doi.org/10.1080/17480930.2011.622480

Hartman, H.L., (1992). SME Mining Engineering Handbook Second Edition. Society for Mining, Metallurgy, and Exploration, Inc., Part 23.4, pp. 2090-2106.

Huang, W., Hossain, G.M.S., Sultana, A., Rasel, M. and Rahman, M.S. (2020). Applying Fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in the Selection of Best Candidate: A Case Study on Interview Performance, British Journal of Economics, Finance and Management Sciences, 17(1), pp.36-49. http://www.ajournal.co.uk/EFpdfs/EFvolume17(1)/EFVol.17%20(1)%20Article%204.pdf

Hwang, C.L. and Yoon, K. (1981). Multiple Attribute Decision Making, Methods and Applications A State-of-the-Art Survey, Springer-Verlag Berlin Heidelberg. https://www.springer.com/gp/book/9783540105589

Javanshirgiv, M., Moghadder, M.T. and Safari, M. (2017a). The selection of appropriate mining method for the Deh Gheybi granite quarry mine using the FTOPSIS method, International journal of mining and mineral engineering, 8(2), pp.113-130. https://www.inderscience.com/offer.php?id=84203

Javanshirgiv, M. and Safari, M. (2017b). The selection of an underground mining method using the fuzzy TOPSIS method: a case study in the Kamar Mahdi II fluorine mine, Mining science, 24, pp.161-181. https://doi.org/10.5277/msc172410

Jamshidi, M.,Ataei, M., Sereshki, F., Jalali, S. and Esmaeil, M. (2009). The application of AHP approach to selection of optimum underground mining method, case study: Jajarm Bauxite Mine (Iran), Archives of mining sciences, 54(1), pp.103–117.

Kacprzak, D. (2018). Fuzzy TOPSIS method for group decision making, Multiple criteria decision making, 13, pp.116-132. 10.22367/mcdm.2018.13.07

Karadogan, A., Kahriman, A. and Ozer, U. (2008). Application of fuzzy set theory in the selection of underground mining method, Journal of The Southern African Institute of Mining and Metallurgy, 108(2), pp.73-79. https://www.saimm.co.za/Journal/v108n02p073.pdf

Karimnia, H. and Bagloo, H. (2015). Optimum mining method selection using fuzzy analytical hierarchy process–Qapiliq salt mine, Iran, International Journal of Mining Science and Technology, 25(2), pp.225–230. https://doi.org/10.1016/j.ijmst.2015.02.010

Kore, N.B., Ravi, K. and Patil, S.B. (2017). A simplified description of fuzzy TOPSIS method for multi-criteria decision making, International research journal of engineering and technology, 4(5), pp.1-4, https://www.irjet.net/archives/V4/i5/IRJET-V4I5548.pdf

Kumar, S., Singh, B., Qadri, M.A., Kumar, Y.V.S. and Haleem, A. (2013). A framework for comparative evaluation of lean performance of firms using fuzzy TOPSIS, International Journal of productivity and quality management, 11(4), pp.371-392. 10.1504/IJPQM.2013.054267

Kusumawardani, R.P. and Agintiara, M. (2015). Application of fuzzy AHP-TOPSIS method for decision making in human resource manager selection process, Procedia computer science, 72, pp.638-464. https://doi.org/10.1016/j.procs.2015.12.173

Lashgari, A., Chamzini, A.Y., Fouladgar, M.M., Zavadskas, E.K., Shafiee and Abbate, N. (2012). Equipment selection using fuzzy multi criteria decision making model: Key study of Gole Gohar iron mine, Inzinerine Ekonomika-Engineering Economics, 23(2), pp.125-136. https://doi.org/10.5755/j01.ee.23.2.1544

Lashgari, A., Fouladgar, M.M. Chamzini, A.Y. and Skibniewski, M.J. (2011). Using an integrated model for shaft sinking method selection, Journal of civil engineering and management, 17(4), pp.569-580. http://dx.doi.org/10.3846/13923730.2011.628687

Liang, W., Zhao, G. and Hong, C. (2018). Selecting the optimal mining method with extended multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) approach, Neural Computing and Applications, 31(10), pp.5871-5886. https://doi.org/10.1007/s00521-018-3405-5

Liao, H. and Xu, Z. (2017). Hesitant Fuzzy Decision Making Methodologies and Applications, Springer Nature Singapore Pte Ltd. 2017. https://link.springer.com/book/10.1007/978-981-10-3265-3

Liou, J.J.H. and Tzeng, G.H. (2012). Comments on "multiple criteria decision making (MCDM) methods in economics: an overview", Technological and economic development of economy, 18(4), pp.672-695. https://doi.org/10.3846/20294913.2012.753489

Liu, A.H., Dong, L. and Dong, L.J. (2010). Optimization model of unascertained measurement for underground mining method selection and its application, Journal of Central South University of Technology, 17(4), pp.744-749. https://doi.org/10.1007/s11771-010-0550-0

Mahase, M.J., Musingwini, C. and Nhleko, A.S. (2016). A survey of applications of multi-criteria decision analysis methods in mine planning and related case studies. Journal of the Southern African Institute of Mining and Metallurgy, 116(11), pp.1051-1056. http://dx.doi.org/10.17159/2411-9717/2016/v116n11a7
Mardani, A., Jusoh A. and Zavadskas, E.K. (2015). Fuzzy multiple criteria decision-making techniques and applications-Two decades review from 1994 to 2014, Expert systems with applications, 42(8), pp.4126-4148. https://doi.org/10.1016/j.eswa.2015.01.003
Mijalkovski, S., Despodov, Z., Mirakovski, D., Hadzi-Nikolova, M., Doneva, N. and Gocevski, B. (2013). Mining method selection for deeper parts of "Svinja Reka" ore deposit – "SASA" mine, 5th Balkan mining congress, pp.133-136.
Mijalkovski, S., Peltecki, D., Despodov, Z., Mirakovski, D., Adjiski, V. and Doneva, N. (2021). Methodology for underground mining method selection, Mining science, 28, pp.201-216. https://doi.org/10.37190/MSC212815
Mikaeil, R., Naghadehi, M.Z., Ataei, M. and Khalokakaei, R. (2009). A decision support system using fuzzy analytical hierarchy process (FAHP) and TOPSIS approaches for selection of the optimum underground mining method, Archive of mining sciences, 54(2), pp.349-368.
Nadaban, S., Dzitac, S. and Dzitac. I. (2016). Fuzzy TOPSIS: A general view, Procedia computer science, 91, pp.823-831. https://doi.org/10.1016/j.procs.2016.07.088
Naghadehi, M.Z., Mikaeil, R. and Ataei, M. (2009). The application of fuzzy analytic hierarchy process (FAHP) approach to selection of optimum underground mining method for Jajarm Bauxite Mine, Iran, Expert Systems with Applications, 36(4), pp.8218–8226. https://doi.org/10.1016/j.eswa.2008.10.006
Namin, F.S., Shahriar, K., Ataee, M. and Dehghani, H. (2008). A new model for mining method selection of mineral deposit based on fuzzy decision making, Journal of the Southern African Institute of Mining and Metallurgy, 108(7), pp.385-395. https://www.saimm.co.za/Journal/v108n07p385.pdf
Namin, F.S., Shahriar, K., Bascetin, A. and Ghodsypour, S. H. (2012). FMMSIC: a hybrid fuzzy based decision support system for MMS (in order to estimate interrelationships between criteria), Journal of the Operational Research Society, 63(2), 218-231. https://doi.org/10.1057/jors.2011.24
Namin, F.S., Shahriar, K., Bascetin, A. and Ghodsypour, S.H. (2009). Practical applications from decision-making techniques for selection of suitable mining method in Iran, Mineral Resources Management, 25(3), pp. 57-77. https://gsm.min-pan.krakow.pl/Practical-applications-from-decision-making-techniques-for-selection-of-suitable,96840,0,2.html
Nicholas, D.E. (1992). Selection method, SME Mining Engineering Handbook, Howard L. Hartman (ed.), 2nd edition, Society for Mining Engineering, Metallurgy and Exploration, Inc., 2090–2106.
Nourali, H., Nourali, S., Ataei, M. and Imanipour, N. (2012). A hierarchical preference voting system for mining method selection problem, Archives of mining sciences, 57(4), pp.925-938. 10.2478/v10267-012-0070-x
Ooriad, F.A., Yari, M., Bagherpour, R. and Khoshouei, M. (2018). The development of a novel model for mining method selection in a fuzzy environment; Case study: Tazareh coal mine, Semnan province, Iran, The Mining-Geology-Petroleum Engineering Bulletin, 33(1), pp.45-53. https://doi.org/10.17794/rgn.2018.1.6
Popovic, G., Dordevic, B. and Milanovic, D. (2019). Multiple criteria approach in the mining method selection, Industrija, 47(4), pp.47-62. https://doi.org/10.5937/industrija47-24128
Tatiya, R.R. (2013). Surface and Underground Excavation 2nd Edition: Methods, Techniques and Equipment, Taylor & Francis Group, London, UK, Chapter 16, pp.527-684. https://doi.org/10.1201/b14818
Shahriar, K., Bakhtavar, E., Saeedi, G. and Shirazi, M.A. (2007). A new numerical method and AHP for mining method selection, Aachen international mining symposia (AIMS), pp.289-305.
Shariati, S., Yazdani-Chamzini, A. and Pourghaffari Bashari, B. (2013). Mining Method Selection by Using an Integrated Model, International Research Journal of Applied and Basic Sciences, 6(2), pp.199-214. http://www.irjabs.com/files_site/paperlist/r_1672_131014105923.pdf
Sitorus, F., Cilliers, J.J. and Brito-Parada, P.R. (2019). Multi-criteria decision making for the choise problem in mining and mineral processing: Applications and trends, Expert systems with applications, 121, pp.393-417. https://doi.org/10.1016/j.eswa.2018.12.001
Sun, C.C. (2010). A performance evaluation model by integrating fuzzy AHP and fuzzy TOPSIS methods, Expert systems with applications, 37(12), pp.7745-7754. https://doi.org/10.1016/j.eswa.2010.04.066
Wang, C., Yang, S., Jiang, C.Y., Wu, G.Y. and Liu, Q.Z. (2019). Monte Carlo analytic hierarchy process for selection of the longwall mining method in thin coal seams, Journal of the Southern African Institute of Mining and Metallurgy, 119(12), pp.1005-1012. https://www.saimm.co.za/Journal/v119n12p1005.pdf
Yavuz, M. (2015). The application of the analytic hierarchy process (AHP) and Yager's method in underground mining method selection problem, International Journal of Mining, Reclamation and Environment, 29(6), pp.453–475. https://doi.org/10.1080/17480930.2014.895218